

Theoretical prediction of cosmological constant Λ in Veneziano ghost theory of QCD

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Based on the Veneziano ghost theory of QCD, we predict the cosmological constant Λ , which is related to energy density of cosmological vacuum by $\Lambda = \frac{8\pi G}{3}\rho_\Lambda$. In the Veneziano ghost theory, the vacuum energy density ρ_Λ is expressed by absolute value of the product of quark vacuum condensate and quark current mass: $\rho_\Lambda = \frac{2N_f H}{m_q} c |m_q \langle 0 | : \bar{q} q : | 0 \rangle|$. We calculate the quark local vacuum condensates $\langle 0 | : \bar{q} q : | 0 \rangle$ by solving Dyson–Schwinger Equations for a fully dressed confining quark propagator $S_f(p)$ with an effective gluon propagator $G_{\mu\nu}^{ab}(q)$. The quark current mass m_q is predicted by use of chiral perturbation theory. Our theoretical result of Λ , with the resulting $\langle 0 | : \bar{q} q : | 0 \rangle = -(235 \text{ MeV})^3$ and light quark current mass $m_q \simeq 3.29\text{--}6.15 \text{ MeV}$, is in a good agreement with the observable of the $\Lambda \approx 10^{-52} \text{ m}^{-2}$ used widely in a great amount of literatures.

Keywords cosmological constant Λ , Veneziano ghost theory of QCD, local quark vacuum condensate, quantum chromodynamics (QCD)

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1 The problem of cosmological constant Λ

The starting point of the cosmological study is Einstein Equation which is a set of ten equations in Einstein's theory of general relativity. The original Einstein field equations can be written in the form [1–3]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu} \quad (1)$$

in units of $\hbar = c = 1$, where G is the gravitational constant $G = 6.7087(10) \times 10^{-39} \text{ GeV}^{-2}$ (sometime one calls Newton's constant), $R_{\mu\nu}(\mu, \nu = 0, \dots, 3)$ is the Ricci tensor, R is the trace of Ricci tensor (it is like the radius of curvature of space–time), $g_{\mu\nu}(x)$ represents the metric tensor which is a function of position x in spacetime. $T^{\mu\nu}$ is energy-momentum tensor, which describes the distribution of matter and energy. Eq. (1) describes a non-static universe. However, Einstein believed, in that time, that our universe should be static. In order to get a static

universe, in 1917 Einstein introduced a new term of $\Lambda g_{\mu\nu}$ in Eq. (1) to balance the attractive force of gravity, and then he got his new modified equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu} \quad (2)$$

Λ in Eq. (2) is the so-called cosmological constant which is a dimensional parameter with units of $(length)^{-2}$. Indeed, Eq. (2) allows a static universe [4], called Einstein's universe, which is one of Friedmann's solution [5] of Einstein's field equation for appearance of Λ term.

However, the recent cosmological observations have strongly shown that our present universe is expanding, and it is not a stationary Einstein universe [6–9]. For example, the redshift—distance relation derived by Hubble [10, 11] led to the static universe being abandoned in favor of the now familiar expanding universe cosmology. Accordingly, there seems to be no point in introducing Λ term into Eq. (2), namely, Λ might be vanishing. This led Einstein to declare his static cosmological model, and es-

pecially the introduction of the Λ term in to his original field equation theory, his “biggest blunder”.

Whilst the original Einstein’s motivation for a cosmological constant evaporated, it was later appreciated that there were other, more fundamental reasons for its presence [12–15]. There is nothing inconsistent with the presence of such a term in the equations, Eq. (2). Indeed, recent improved astronomical techniques have found that a positive value of Λ is needed to explain some observations [16–19]. The universe has an unknown form of energy density, named the dark energy, about 74% of the total energy density. The simplest possibility is the existence of the vacuum energy or cosmological constant. In the standard model of particle physics, we have Higgs field that produces electroweak phase transition, which changes the value of the vacuum energy. It has also been known that the vacuum fluctuations in quantum field theory naturally induce such a vacuum energy. In fact, the cosmological constant Λ is related to vacuum energy density ρ_Λ , and turns out to be a measurement of the energy density of the cosmological vacuum [20, 21] by

$$\Lambda = \frac{8\pi G}{3}\rho_\Lambda \quad (3)$$

Therefore, the existence of Λ is an intrinsic property of cosmological vacuum since $\rho_\Lambda = \frac{3}{8\pi G}\Lambda$. In other words, Eq. (3) shows that Λ is a potentially important contributor to the dynamical history of the universe evolution. The reason is that in the general theory of relativity, any form of energy affects the gravitational field. A sufficiently large energy density will cause galaxies to appear to accelerate away from us, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects.

Although the existence of a non-zero vacuum energy would, in principle, have an effect on gravitational physics on all scales, the value of Λ in our present universe is not known, and it is an empirical issue that will ultimately be settled by observation. A precise determination of this number (Λ) will be one of the primary goals of observational cosmology in the near future. Theoretical physicists would like to develop an understanding of why energy density of the cosmological vacuum has the value of $\rho_\Lambda = \frac{3}{8\pi G}\Lambda$.

A major outstanding problem is that most quantum field theories predict a huge cosmological constant Λ from the energy of the quantum vacuum. This conclusion also follows from dimensional analysis and effective field theory down to the Planck scale, by which we would expect a cosmological constant of the order of M_{Pl}^4 (M_{Pl} is Planck mass with $M_{\text{Pl}} = G^{-1/2} = 1.22 \times 10^{19}$ GeV. The Planck energy is thought to be the energy where conventional physical theories break down and a new theory of quantum gravity is required). We know that the measured value is in the order of 10^{-35} s^{-2} ,

or 10^{-47} GeV^4 , or 10^{-29} g/cm^3 , or about 10^{-120} in reduced planck units (M_{Pl}). That is, there is a large difference between the magnitude of the vacuum energy expected from zero-point fluctuations and scalar potential, $\rho_\Lambda^{\text{theory}} = 2 \times 10^{110} \text{ erg/cm}^3$, and the value we apparently observe, $\rho_\Lambda^{\text{observe}} = 2 \times 10^{-10} \text{ erg/cm}^3$, a discrepancy as a factor of 10^{120} . This is largest discrepancy — the worst theoretical prediction in the history of physics. At the same time, some supersymmetric theories require a cosmological constant that is exactly zero. Therefore, we face a big difficulty in understanding the observational $\rho_\Lambda^{\text{observe}}$. This problem has been referred to as the long-standing cosmological constant problem.

Based on Veneziano ghost theory [20, 21] of QCD, we calculate the vacuum energy density ρ_Λ in this paper by solving the Dyson–Schwinger equations (DSEs) [22, 23] of quark propagator in QCD to predict quark local vacuum condensates $\langle 0 | : \bar{q}q : | 0 \rangle$, and evaluate quark current mass m_q using Chiral Perturbation Theory [24–26] of low energy QCD (χPT). We then predict the numerical value of the cosmological constant Λ in terms of our calculating results of quark vacuum condensate in QCD vacuum given by the solutions of DSEs and quark current mass predicted by χPT according to the formulations of QCD Ghost theory [18, 27–29]. In Section 2, we briefly introduce QCD Veneziano ghost theory and give its formalism for calculating the cosmological constant Λ . We calculate quark local vacuum condensate $\langle 0 | : \bar{q}q : | 0 \rangle$ by solving DSEs in Section 3. In Section 4 we evaluate quark current mass m_q from χPT which is a very good approximation to determine the quark masses. Our numerical prediction of Λ and comparisons with empirical values are presented in Section 5. Finally, we reserve summary and concluding remarks for Section 6.

2 The QCD Veneziano ghost theory and cosmological constant Λ

The recent cosmological observations have confirmed the existence of the early inflationary epoch (it occurred in the first 10^{-32} seconds of the existence of our universe. During this time the metric itself changed exponentially, causing space to change from smaller than an atom to around 100 million light years across.) and accelerated expansion of the universe. Observational results tell us that the universe is permeated with an unknown form of energy density, now named the dark energy. The simplest possibility for the origin of dark energy is the existence of the vacuum energy. Vacuum energy is an underlying background energy that exists in space even when it is devoid of matter (known as free space), which is deduced from the concept of virtual particles created and destroyed out of the vacuum, and are themselves derived from the energy–time uncertainty principle, $\Delta E \cdot \Delta t \simeq \hbar$.

This vacuum fluctuations naturally induce such a vacuum energy. These particles are always created out of the vacuum in particle–antiparticle pairs, which shortly annihilate each other and disappear. However, these particles and antiparticles interact with others before disappearing. “Empty space” can possess its own energy because this energy is a property of space itself, and it would not be diluted as space expands. As more space comes into existence, more of this energy of space would appear.

An additional contribution to the vacuum energy comes from spontaneous symmetry breaking in quantum field theory. The effects of these vacuum energy can be observed in various phenomena, and must have consequences for the behavior of the universe on cosmological scales. In other words, vacuum energy must exist and a non-zero vacuum energy is expected to contribute to the cosmological constant, and affects the expansion of the universe.

Vacuum energy can be easily incorporated in the quantum field theory. In the standard model of particle physics, we have Higgs field that produces electroweak phase transition, masses of massive particles, and changes the value of the vacuum energy.

Recently, a very interesting proposal on the origin of a cosmological constant has been made [20, 21], without an introduction of new degrees of freedom that are already known, with the cosmological constant being of just the right magnitude to describe the observed expansion. In this proposal, it is claimed that the cosmological constant arises from the contribution of the ghost field that are supposed to be present in the low energy effective theory of QCD. The ghosts are required to exist for the resolution of the $U(1)$ problem [18, 27–29], but are completely decoupled from the physical sector. The above claim is that the ghosts are decoupled from physical states and make no contributions in flat Minkowski space, but once they are in the curved space or time-dependent background the cancelation of their contribution to the vacuum energy is off-set, leaving a small energy density $\rho_A \sim H \Lambda_{\text{QCD}}^3$, where H is the Hubble parameter and Λ_{QCD} is the QCD mass scale of order a few hundred (~ 200 MeV). With $H \sim 10^{-33}$ eV, this gives the right magnitude of ρ_A^{observe} .

The evolution of the universe is determined to a large extent by the same microphysics laws that govern high energy physics phenomena. Hence, any progress in particle physics has a large impact on the cosmological model(s) and, conversely, any new step taken towards the understanding of the past, present and future of our universe might provide a hint of high energy physics beyond what we currently know.

We closely follow the work of Urban and Zhitnitsky [20, 21] which thought that the gravitation cannot be a truly fundamental interaction, but rather, it is a low

energy effective quantum field theory. In such a case, the corresponding gravitons should be treated as quasi-particles that rather than feel all the microscopic degrees of freedom, are sensitive only to the “relevant excitation”.

It is customary to associate the “dark” energy density with vacuum fluctuations. Our crucial observation is that while naively all QCD degrees of freedom can propagate only to very short distances $\Lambda_{\text{QCD}}^{-1}$, there is a unique (unphysical) degree of freedom which is exactly massless and propagates to arbitrary large distances [20, 21]. This is the justly celebrated Veneziano ghost degree of freedom. Indeed, despite its being unphysical it leads to physically observable consequences: It plays a crucial role in explaining the value of the η' mass as a result of the mixing between the “would be” Nambu–Goldstone boson and the ghost itself. We note that this mechanism does not violate unitarity. Nowadays, this mechanism has been considered as the standard Witten–Veneziano resolution of the famous $U(1)_A$ problem [18, 27–29].

If we accept this philosophy, the cosmological constant Λ can be written down as being proportional to an absolute value of a product of the local quark vacuum condensate $\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle$ and quark current mass m_q . In other words, the cosmological vacuum energy density ρ_A can be expressed in terms of QCD parameters for $N_f = 2$ light flavors as follows:

$$\rho_A = c \frac{2HN_f}{m_{\eta'}} |m_q \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle| \quad (4)$$

where $c = c_{\text{QCD}} \times c_{\text{grav}}$. The first factor c_{QCD} is a dimensionless coefficient with value of $c_{\text{QCD}} \simeq 1$ [20, 21], which is entirely of QCD origin and is related to the definition of QCD on a specific finite compact manifold such as a torus, $\rho_A \simeq c_{\text{QCD}} \frac{2N_f |m_q \langle \bar{q}q \rangle|}{L m_{\eta'}}$ with L being the size of the manifold and $m_{\eta'}$ the mass of η' meson. A precise computation of c_{QCD} has been Conducted in a conventional lattice QCD approach by studying corrections of order $1/s$ to the vacuum energy [20, 21]. It is obvious that c_{QCD} depends on the manifold where the theory is defined. The second factor c_{grav} has a purely gravitational origin and is defined as the relation between the size L of the manifold we live in and the Hubble constant H , $L = (c_{\text{grav}} H_0)^{-1}$. Practically, one can define this size of the manifold as $L \simeq 17H_0^{-1}$ where $H_0 = 2.1 \times 10^{-42} \times h$ GeV and $h = 0.71$ (H_0 , Hubble constant today). Therefore, one can explicitly obtain an estimate for the linear length L of the torus, and then obtain the value of c_{grav} with $c_{\text{grav}} = 0.0588$.

In the derivation of the energy density ρ_A from the Veneziano ghost theory, the key starting point is the topological susceptibility χ defined as

$$\chi = i \int dx \langle 0 | T \{ Q(x), Q(0) \} | 0 \rangle \quad (5)$$

$$Q \equiv \frac{\alpha_s}{16\pi} \varepsilon^{\mu\nu\lambda\sigma} G_{\mu\nu}^a G_{\lambda\sigma}^a \equiv \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu a} \equiv \partial_\mu K^\mu$$

$$K^\mu \equiv \frac{g_s^2}{16\pi^2} \varepsilon^{\mu\nu\lambda\sigma} A_\nu^a (\partial_\lambda A_\sigma^a + \frac{g_s}{3} f^{abc} A_\lambda^b A_\sigma^c)$$

where A_μ^a are the conventional QCD color gluon fields and Q is the topological charge density, and $\alpha_s = \frac{g_s^2}{4\pi}$. g_s is the strong coupling constant of QCD.

Therefore, we can calculate the cosmological constant Λ according to Eq. (3) by use of Eq. (4). To this end, we briefly discuss the QCD vacuum and evaluate vacuum condensate in the following section.

3 Quark vacuum condensates in non-perturbative QCD vacuum state

Quantum mechanics dictates that even “empty” space is not empty, but rather filled with quantum fluctuations of all possible kinds. In many contexts, such as in atomic physics, these vacuum fluctuations are subtle effects that can only be observed by precision experiments. In other situations, especially when interactions of sufficient strength are involved, the vacuum fluctuations can be of substantial magnitude and can even “condense” into a non-vanishing vacuum expectation value of some quantum fields, called vacuum condensates. These vacuum condensates can act as a medium [30–32] that influences the properties of particles propagating through it.

An important example of such a vacuum condensate is the Higgs vacuum expectation value [33–36], which is introduced in the Standard Model of particle physics to generate the masses of quarks, leptons, and the gauge bosons ($W^\pm$, Z^0) of the weak interaction. The vacuum expectation value of the Higgs field, $\langle\phi\rangle = 246$ GeV, is uniquely determined in the Standard Model. The quark and lepton masses differ from one another only due to the different strength of the coupling of each fermion to the Higgs field. At the same time, the quark masses also receive additional contributions from the quark and gluon condensates in the QCD vacuum. In fact, the contribution of the QCD vacuum condensates to the masses for the three light quarks (u,d,s) considerably exceed the mass believed to be generated by the Higgs field [33–36].

The non-vanishing value of chiral quark vacuum condensates signals the spontaneous breaking of chiral symmetry in QCD, and quantitatively it is related to the pseudo-Goldstone bosons mass spectrum [37–39]. Due to non-perturbative effects of QCD, the vacuum of QCD has a nontrivial structure. The vacuum condensates are very important in the elucidation of the QCD structure and in description of hadron properties. If the vacuum acts as a medium and influences the properties of fundamental particles and their interactions, its properties can conceivably change. This idea has important implications in

many aspects of physics, for example, production of massive particles in the early stage of the universe after Big Bang.

3.1 Definition of vacuum condensates of non-perturbative QCD

The non-perturbative vacuum of QCD is densely populated by long-wave fluctuations of quark and gluon fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, such as

$$\langle 0 | : \bar{q} q : | 0 \rangle, \quad \langle 0 | : \bar{q} [i g_s \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2}] q : | 0 \rangle$$

$$\left\langle 0 \left| : \bar{q} \gamma_\mu \frac{\lambda^a}{2} q \bar{q} \gamma_\mu \frac{\lambda^a}{2} q : \right| 0 \right\rangle$$

$$\langle 0 | : G_{\mu\nu}^a G_{\mu\nu}^a : | 0 \rangle, \quad \langle 0 | : f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c : | 0 \rangle, \dots \quad (6)$$

which are called vacuum condensates of QCD, where $q(x)$ is the quark field, and $G_{\mu\nu}^a$ represents the gluon field strength tensor with a being the color index ($a = 1, 2, \dots, 8$), and it can be expressed as

$$G_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g_s f^{abc} A_\mu^b(x) A_\nu^c(x) \quad (7)$$

λ^a in Eq. (6) is the $SU_C(3)$ Gell–Mann matrix, f^{abc} in Eqs. (6), (7) represent the $SU_C(3)$ structure constants, and g_s is the coupling constant related to the so-called QCD running coupling constant α_s by $\alpha_s(Q) = \frac{g_s^2(Q)}{4\pi}$. A_μ^a in Eq. (7) is the gluon field, and $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ in Euclidean space with γ_μ being Dirac matrix.

In QCD, by condensates we mean the vacuum mean values $\langle 0 | O_i | 0 \rangle$ of the local operators $O_i(x)$, which arise due to non-perturbative effects. The latter point is very important and needs clarification. When determining vacuum condensates one implies the averaging only over non-perturbative fluctuations. If for some operators O_i the non-zero vacuum mean value appears also in the perturbative theory, it should not be taken into account in determination of the condensate. In other words, in determination of condensates the perturbative vacuum mean values should be subtracted in calculation of the vacuum averages.

Separation of perturbative and non-perturbative contributions to the quark propagator, $S_q(x)$, has some arbitrariness. For the nonperturbative propagator, one makes an expansion of vacuum expectation values involving antiquark–quark fields that would vanish in a perturbative vacuum. The nonzero local quark vacuum condensate $\langle 0 | : \bar{q}(0) q(0) : | 0 \rangle$ is responsible for the spontaneous breakdown of chiral symmetry.

The quantities $\langle 0 | : \bar{q}(x) q(0) : | 0 \rangle$ are non-local vacuum condensates, which describe the distribution of quarks in the non-perturbative vacuum [37–39]. As is seen from Section 2.2, the cosmological constant Λ is pro-

portional to the QCD local ($x = 0$) vacuum condensate $|\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle|$ in the QCD ghost theory. Therefore, to predict Λ we have to calculate the quark local vacuum condensate. We do this calculation by studying the fully dressed confining quark propagator $S_f(p)$ in the following subsection.

3.2 DSEs for a fully dressed confining quark propagator $S_f(p)$

Quark propagator $S_f(p)$ is presented by DSEs [22, 23], which are a tower of coupling integral equations that relate the Green's functions of QCD to one another, and are non-perturbative in nature. Solving these equations provides the solution of quark propagator in non-perturbative QCD vacuum. This tower of the equations includes the Bethe-Salpeter (BS) equations [40–44], the solution of which is the quark-antiquark bound state amplitude. The mathematical expression of the DSEs can be written as

$$i(S_f(p))^{-1} = i(S_f^0(p))^{-1} + \frac{4}{3}g_s^2 \int \frac{d^4k}{(4\pi)^4} \gamma^\mu S_f(k) \Gamma^\nu(k, p) G_{\mu\nu}(q) \quad (8)$$

In Eq. (8), $G_{\mu\nu}(q)$ denotes fully dressed gluon propagator and has the form of

$$G_{\mu\nu}(q) = \frac{1}{q^2} \left[\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{1}{1 - \Pi} + \xi \frac{q_\mu q_\nu}{q^2} \right] \quad (9)$$

with ξ being a gauge parameter ($\xi = 0$ denotes the Landau gauge and $\xi = 1$ stands for Feynman gauge). $\delta_{\mu\nu} = \text{diag}(1, 1, 1, 1)$ is the Euclidean metric. $\Pi(q)$ can be expressed by the following identity [45, 46]

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) q^2 \Pi(q) \\ &= -g_s^2 \int d^4x e^{iqx} \langle 0 | T[J^\mu(x) J^\nu(0)] | 0 \rangle_{pp} \end{aligned} \quad (10)$$

where the subscript “pp” indicates the proper part, i.e., only one vector-meson-irreducible graphs contribute to $\Pi_{\mu\nu}(q)$.

$\Gamma^\nu(k, p)$ in Eq. (8) is Bethe-Salpeter (BS) amplitude describing the dressed gluon-quark coupling vertex. Gauge covariance requires that the BS amplitude, $\Gamma^\nu(k, p)$, must satisfy the Ward-Takahashi identity [47–49]

$$(k - p)_\mu i\Gamma_\mu(k, p) = S_f^{-1}(k) - S_f^{-1}(p) \quad (11)$$

In the quantum field theory, the BS amplitude for a two-body quark-antiquark bound state is obtained as a solution of the homogeneous BS equation:

$$\begin{aligned} \Gamma_M^{rs}(k, p) &= \int \frac{d^4k'}{(4\pi)^4} K_{ab,cd}^{rs,tu}(k', k, p) \\ &\cdot \left[S_f \left(k' - \frac{1}{2}p \right) \Gamma_M \left(k' + \frac{1}{2}p \right) \right]_{cd}^{tu} \end{aligned} \quad (12)$$

where Γ_M is the proper meson-quark vertex and $K_{ab,cd}^{rs,tu}$ is the kernel. A commonly used approximation for the kernel, $K_{ab,cd}^{rs,tu}$, is the generalized ladder approximation, in which

$$K_{ab,cd}^{rs,tu}(k', k, p) = g_s^2 G^{\mu\nu}(k' - p) \cdot \left(\frac{\lambda^a}{2} \right)_{ac} \left(\frac{\lambda^a}{2} \right)_{db} (\gamma^\mu)_{rt} (\gamma^\nu)_{us} \quad (13)$$

Solving Eq. (12) for Γ_M yields so-called dressed quark core of the bound state.

As it is impossible to solve the complete set of DSEs of Eq. (8), one has to truncate this infinite tower in a physically acceptable way to reduce them to something that is soluble. To begin with, we make a further simplification by replacing the full vertex $\Gamma^\nu(k, p)$ in Eq. (8) in terms of its bare one γ^ν ,

$$\Gamma^\nu = \gamma^\nu \quad (14)$$

Thus, Eq. (8) then becomes to

$$i[S_f(p)]^{-1} = i(S_f^0(p))^{-1} + \frac{4}{3}g_s^2 \int \frac{d^4k}{(4\pi)^4} \gamma^\mu S_f(k) \gamma^\nu G_{\mu\nu}(q) \quad (15)$$

This procedure is called as “Rainbow” approximation of DSEs and used widely in literature.

3.3 Effective gluon propagator $G_{\mu\nu}$ and general form of quark propagator $S_f(p)$

Since the fully dressed gluon propagator $G_{\mu\nu}(q)$ in Eqs. (8) and (15) is completely unknown at non-perturbative region, we must use a modeling gluon propagator to solve the DSEs. The Feynman-like gauge ($\xi = 1$) used in Eq. (9) leads to the choice of the following empirical form of the $G_{\mu\nu}(q)$

$$G_{\mu\nu}^{ab}(q) = \delta^{ab} \delta_{\mu\nu} G(q) \quad (16)$$

for the model gluon propagator, where the Greek letters $\mu\nu$ represent Lorentz indices and the letters a and b stand for color indices. Using the results of Refs. [50–53] for the propagator $G(q)$ leads to

$$g_s^2 G(s) = \frac{4\pi\alpha(s)}{s} \quad (17)$$

where $\alpha(s)$ is formulated phenomenally in terms of parameters χ and Δ as follows [54–57]:

$$\alpha(s) = 3\pi s \frac{\chi^2}{4\Delta^2} e^{-s/\Delta} + \frac{\pi d}{\ln(s/\Lambda_{\text{QCD}}^2 + \epsilon)} \quad (18)$$

which determines quark-quark interaction through a strength χ and a range parameter Δ . The first term in Eq. (18) simulates the infrared enhancement and confinement, and the second term matches the leading log renormalization group results. The QCD scale parameter

$\Lambda_{\text{QCD}} = 0.20 \text{ GeV}$, and $d = 12/(33 - 2N_f) = 12/27$ for flavor number $N_f = 3$. The ϵ in the second term of Eq. (18) varies in the range of $1.0 - 2.50$ [56–61]. However, we take ϵ to be 2.0, in this work. The parameters Δ and χ are given by Refs. [56–61] as shown in Table 1.

Table 1 The values of the strength parameter χ and range parameter Δ of the quark-quark interaction used in our present calculations.

Set No.	Δ	χ
Set 1	0.40	1.84
Set 2	0.20	1.65
Set 3	0.02	1.50

An important observation is that the general form of the inverse quark propagator $S_f^{-1}(p)$ can be rewritten in momentum space [62–64] as

$$S_f^{-1}(p) = i\not{p} \cdot A_f(p^2) + B_f(p^2) \quad (19)$$

in a covariant gauge, and $\not{p} = \gamma^\mu p_\mu$. The propagator is renormalized at space-like point μ_p^2 according to $A_f(\mu_p^2) = 1$ and $B_f(\mu_p^2) = m_f(\mu_p^2)$ with $m_f(\mu_p^2)$ being the current quark mass whose value is empirically about $m_{u,d} \simeq 5.1 \text{ MeV}$ for u, d quarks, and $m_s = 127.5 \text{ MeV}$ for s quark.

Except for the current quark mass and perturbative corrections, the functions $A_f(p^2)$ and $B_f(p^2)$ are non-perturbative quantities, and satisfy the new set of DSEs in the Feynman gauge [55–62]

$$\begin{aligned} & [A_f(p^2) - 1]p^2 \\ &= \frac{8}{3}g_s^2 \int \frac{d^4q}{(4\pi)^4} G(p-q) \frac{A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} p \cdot q \quad (20) \\ B_f(p^2) &= \frac{16}{3}g_s^2 \int \frac{d^4q}{(4\pi)^4} G(p-q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \quad (21) \end{aligned}$$

For practical calculations, we can go further to write the DSEs, Eqs. (20), (21), as follows:

$$\begin{aligned} [A(s) - 1]s &= \frac{1}{3\pi^3} g_s^2 \int_0^\infty s' ds' \\ &\cdot \int_0^\pi \sin^2 x D(s, s') \frac{\sqrt{ss'} A(s') \cos x}{s' A^2(s') + B^2(s')} dx \quad (22) \end{aligned}$$

$$\begin{aligned} B(s) &= \frac{2}{3\pi^3} g_s^2 \int_0^\infty s' ds' \\ &\cdot \int_0^\pi \sin^2 x D(s, s') \frac{B(s')}{s' A^2(s') + B^2(s')} dx \quad (23) \end{aligned}$$

where $s = p^2$, $g_s^2 D(s, s') = g_s^2 D(s + s' - 2\sqrt{ss'} \cos x)$. Therefore, using the modeling gluon propagator $G(p-q)$ in Eq. (17) with $\alpha(s)$ in Eq. (18) we can solve these two coupled integral equations, Eqs. (22) and (23), and obtain the self-energy functions A_f and B_f which will be used in the following calculations of vacuum condensates.

3.4 Quark vacuum condensate determined by solutions of the DSEs, $A_f(p^2)$ and $B_f(p^2)$

Using the approximations needed for the DSEs, discussed in detail by Roberts and Williams [50, 52, 60], the non-local quark vacuum condensate $\langle 0 | : \bar{q}(x)q(0) : | 0 \rangle$ is then given by the scalar part of Fourier transformed inverse quark propagator [63],

$$\begin{aligned} & \langle 0 | : \bar{q}(x)q(0) : | 0 \rangle \\ &= (-4N_c) \int \frac{d^4p}{(2\pi)^4} \frac{B_f(p^2) e^{ipx}}{p^2 A_f^2(p^2) + B_f^2(p^2)} \\ &= -\frac{3}{4\pi^2} \int_0^\infty s ds \frac{B_f(s)}{s A_f^2(s) + B_f^2(s)} \frac{2J_1(\sqrt{sx^2})}{\sqrt{sx^2}} \quad (24) \end{aligned}$$

where the color number $N_c = 3$. Note that our work depends on making a short-distance expansion. Using the expansion of the J_1 Bessel function, $2J_1(\sqrt{sx^2})/\sqrt{sx^2} = 1 - sx^2/8 + \dots$, and the definitions of the condensates given in Eq. (24), one finds that the local quark condensate is

$$\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle = -\frac{3}{4\pi^2} \int_0^{s_o} ds \frac{s B_f(s)}{s A_f^2(s) + B_f^2(s)} \quad (25)$$

Note that the upper limit of the integrals over s for the DSEs, Eq. (24), is infinite, while for the condensates, Eq. (25), there is a finite limit, s_o . That is because the effective gluon propagator, Eqs. (17) and (18), provides a natural cutoff for the DSEs integrals, while the range for the condensates is given by the renormalization point, μ . For light quarks perturbative QCD begins to dominate non-perturbative QCD at about 3–4 GeV, so a renormalization point of $\mu^2 = 10 \text{ GeV}^2$ is expected. Therefore, we use $s_o = 10 \text{ GeV}^2$. This has been explained in detail in our previous publication of Ref. [62].

Using the solutions of DSEs of Eqs. (22) and (23), A_f and B_f , with three different sets of the quark-quark interaction parameters (Δ , χ) given in Table 1, leads to our following theoretical predictions for the local quark vacuum condensates $\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle_f$ from Eq. (25). The corresponding theoretical results are listed in Table 2.

Table 2 Predictions of local quark vacuum condensates in QCD vacuum, $\langle 0 | : \bar{q}q : | 0 \rangle_\mu^f$ with f standing for quark flavor and μ denotes renormalization point, $\mu^2 = 10 \text{ GeV}^2$.

Set No. of quark interactions	$\langle 0 : \bar{q}q : 0 \rangle_{u,d}$ for u and d quarks
Set 1	$-0.0130 \text{ GeV}^3 \sim -235 \text{ MeV}^3$
Set 2	$-0.0078 \text{ GeV}^3 \sim -198 \text{ MeV}^3$
Set 3	$-0.0027 \text{ GeV}^3 \sim -139 \text{ MeV}^3$

These results are consistent with many other calculations, such as QCD sum rules [64–66], Lattice QCD [67–69] and Instanton model predictions [70–72]. These numerical results will be used to calculate Λ in Section

5.

4 Dynamically running masses m_q of light quarks

As we have seen from formulae, Eqs. (3) and (4) in Section 2.2, to predict Λ , we need to know the quark current mass m_q except the local quark vacuum condensate $\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle$. Together with the strong coupling constant, quark masses are also fundamental QCD input parameters of the standard model, and thus their precise determination is of paramount importance for present day particle phenomenology and for our determination of cosmological constant Λ . In the light quark sector, the mass of the strange quark m_s deserves particular interest, because its present uncertainty severely limits the precision of current predictions of the CP — violating observable ε'/ε . The ratios of light quark masses are known quite precisely from the chiral perturbation theory [73, 74], and thus once the absolute scale is set by m_s , the masses of the lighter up and down quarks can be determined. We assess the values of light quark current masses in this section.

4.1 Definition of quark mass in QCD

Non-perturbative definition of quark mass is provided by the chiral Ward identities of QCD [75]. This definition allows us to express the renormalized quark mass, in a given scheme and at a given renormalization scale, in terms of the renormalization constants and bare quantities. Three independent definitions for quark mass (vector Ward identity definition, axial Ward identity definition, and definition from the quark propagator) have been proposed [46, 76], which are all equivalent in principle, in the sense that they lead to the same value of the renormalized mass.

As free quarks do not exist, the definition of their masses is somewhat problematic and ambiguous. One could, for example, take the quark mass as the average energy of the quark bound in a hadron in its ground state, or as the probable mass it would have were it to be free. This “free mass” is the mass that appears in the expressions describing quark currents. All these mass values are indirectly determined in experiment in one way or another. At present no one knows how to calculate quark masses exactly from first principles QCD since the quark mass is an input parameter of QCD. In the following subsection, we predict quark current mass by chiral perturbation theory (χPT), which is a precise and powerful tool [46] in predicting quark mass ratio, and further, light quark masses: m_u , m_d and m_s .

4.2 Light quark current masses m_q in the χPT

The sum of light quarks u and d current mass, $m_u + m_d$,

can be obtained from the Gell–Mann–Oakes–Renner relationship [77]

$$(m_u + m_d)\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle = f_\pi^2 M_\pi (1 - \delta_\pi) \quad (26)$$

where $M_\pi = 139$ MeV is the mass of pion meson having been determined by experimental measurements, $\delta_\pi = 0.05 \pm 0.05$ [77] and f_π is the pion decay constant given [78, 79] theoretically by

$$f_\pi^2 = \frac{3}{4\pi^2} \int ds s \left\{ \left[\frac{B_f^2(s)}{sA_f^2(s) + B_f^2(s)} \right]^2 [A_f^2(s) + sA_f(s)A_f'(s)s^2(A_f'(s))^2 + s(B_f'(s))^2] - \frac{B_f(s)B_f'(s)\frac{s}{2}[(B_f'(s))^2 + B_f(s)B_f''(s)]}{sA_f^2(s) + B_f^2(s)} \right\} \quad (27)$$

As we mentioned before $\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle$ is the local quark vacuum condensate determined by Eq. (25) and shown in Table 2. The functions $A_f'(s)$ and $B_f'(s)$ in Eq. (27) are the first derivatives of $A_f(s)$ and $B_f(s)$ with respect to s . Our calculations from Eq. (27) at the cut-off parameter $\mu_p^2 = 10$ GeV² show that $f_\pi = 93$ MeV, which perfectly matches its experimental measurement of 93 MeV.

Evidently, putting our predictions of the quark vacuum condensates given in Table 2, the pion decay constant $f_\pi = 93$ MeV predicted by Eq. (27), and the measured pion mass of $M_\pi = 139$ MeV into Gell–Mann–Oakes–Renner relation, Eq. (26) leads to

$$m_u + m_d = 9.437 \text{ MeV} \quad (28)$$

According to the χPT , the ratios of two light quark masses can be given in an excellent approximation by the following two identities [80–82]:

$$\frac{m_s}{\bar{m}} = 24.4 \pm 1.5, \quad \frac{m_s^2 - \bar{m}^2}{\bar{m}^2 - m_u^2} = (22.7 \pm 0.8)^2 \quad (29)$$

with $\bar{m} = (m_u + m_d)/2$. From Eqs. (28, 29) we further arrive at

$$\frac{m_u}{m_d} = 0.535 \pm 0.049, \quad \frac{m_s}{m_d} = 19.180 \pm 1.30 \quad (30)$$

Comparing Eq. (28) with Eq. (30) leads the current masses of the light quarks, u , d and s to

$$m_u = 3.29 \text{ MeV}, \quad m_d = 6.15 \text{ MeV} \\ m_s = 117.96 \text{ MeV} \quad (31)$$

These resulting values of the light quark current masses are consistent with those given by QCD sum rules predictions and many other theoretical calculations [82–85].

5 Prediction of cosmological constant Λ in the QCD Veneziano theory

Combining Eq. (3) with Eq. (4) we have get the following

Table 3 Cosmological constant Λ with 3 sets of different quark interactions at renormalization point, $\mu^2 = 10 \text{ GeV}^2$.

Quark interact.	$\langle 0 : \bar{q}q : 0 \rangle_{u,d}$	ρ_Λ	$\Lambda/(10^{-35} \text{ s}^{-2})$	$\Lambda/(10^{-52} \text{ m}^{-2})$
Set 1	-0.0130 GeV^3	$(2.20 \times 10^{-3} \text{ eV})^4$	0.31	0.35
Set 2	-0.0078 GeV^3	$(1.94 \times 10^{-3} \text{ eV})^4$	0.18	0.20
Set 3	-0.0027 GeV^3	$(1.49 \times 10^{-3} \text{ eV})^4$	0.06	0.07

identity:

$$\Lambda = \frac{8\pi G}{3} c \frac{2HN_f}{m_{\eta'}} |m_q \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle| \quad (32)$$

where $N_f = 2$ for two flavors u and d system, $c = c_{QCD} \dot{c}_{\text{grav}} \simeq 1 \times \frac{1}{17} = 0.0588$, $m_{\eta'} = 958 \text{ MeV}$, $H = 2.1 \times 10^{-42} h \text{ GeV}$ with $h = 0.71$ and $\frac{8\pi G}{3} = 1.11 \times 10^{-38} \text{ fm} \cdot \text{GeV}^{-1}$ as we previously discussed. Using these numbers we obtain the value of cosmological constant Λ for three different quark-quark interactions. The results are shown in Table 3.

Our predicted results are in a good agreement with experimental observable $\Lambda = (1.26 \pm 0.05) \times 10^{-56} \text{ cm}^{-2}$ in Refs. [86, 87]. This matches strongly indicates that the Veneziano ghost theory of QCD can solve the big discrepancy between theoretical predictions and observables without introducing any new degrees of freedom. It can also be seen from Table 3 that the prediction for Λ is not affected by different sets of the parameters of quark-quark interaction in Eq. (18) and shown in table 1, at least the order of magnitude of the predicted Λ remains the same.

6 Summary and concluding remarks

The cosmological constant Λ is an important physical quantity with the dimensions of an inverse area introduced for the first time by Einstein, who modified the field equations of his general theory of relativity to obtain a stationary universe. The constant has recently been used to explain the observed accelerated expansion of the universe. But its observational value is about 120 orders of magnitude smaller than the one theoretically computed in the framework of the currently accepted quantum field theories. Namely, quantum field theory predicted that the vacuum energy density is on the order of M_{Pl}^4 , where M_{Pl} is the reduced Planck mass ($M_{\text{Pl}} = 1.22 \times 10^{19} \text{ GeV}$), which is about 120 orders of magnitude larger than the measured value of $\rho_\Lambda = (2.3 \times 10^{-3} \text{ eV})^4$. This difference is so called cosmological constant problem, the worst problem of fine-tuning in physics.

Based on the Veneziano ghost theory of QCD, we predict, in this paper, the cosmological constant Λ , which is closely related to the energy density of cosmological vacuum ρ_Λ by $\Lambda = \frac{8\pi G}{3} \rho_\Lambda$. Thus, Λ is a measurement of the cosmological vacuum energy density. In other words, Λ fully represents the energy density in the cosmological

vacuum, and is an intrinsic property of the vacuum.

Our underlying philosophy here is that gravitation can not be a truly fundamental interaction, instead it is a low energy effective interaction. In such a case, the corresponding gravitons can be treated as quasiparticles that do not feel all the microscopic degrees of freedom, but are sensitive only to the “relevant excitation”. We claim that this new quasiparticle should be celebrated Veneziano ghost of QCD, which is exactly massless and can propagate to arbitrary large distance. This Veneziano ghost theory plays a crucial role in explaining the value of the η' mass as a result of the mixing between the “would be” Nambu-Goldstone boson and the ghost itself. It should be also emphasized that this theory does not violate unitarity, and nowadays it has been considered as the standard Witten-Veneziano resolution of the famous $U_A(1)$ problem.

The vacuum energy density ρ_Λ is expressed by the absolute value of the product of quark vacuum condensate and quark current mass: $\rho_\Lambda = c \frac{2N_f H}{m_{\eta'}} |m_q \langle 0 | : \bar{q}q : | 0 \rangle|$. We calculate the local quark vacuum condensates $\langle 0 | : \bar{q}q : | 0 \rangle$ in this paper by solving DSEs of a fully dressed confining quark propagator $S_f(p)$ with an effective gluon propagator $G_{\mu\nu}^{ab}(q)$. The quark current mass m_q is predicted by use of chiral perturbation theory. Our theoretical result of Λ , with our predicted $\langle 0 | : \bar{q}q : | 0 \rangle = -(235 \text{ MeV})^3$ and light quark current mass $m_{u,d} \simeq 3.29 \text{ MeV}$, is in a good agreement with the empirical value of the $\Lambda \approx 10^{-52} \text{ m}^{-2}$ used widely in a great amount of literatures.

The cosmological constant Λ is a potentially important contributor to the dynamical history of the universe. Unlike an ordinary matter, which can clump together or disperse as it evolves, the energy density in a cosmological constant is a property of spacetime itself, and is the same everywhere under ordinary circumstances. A sufficiently large cosmological constant will cause galaxies to appear to accelerate away from us, in contrast to the tendency of ordinary forms of energy to slow down the recession of distant objects. The value of Λ in our present universe is not known as we mentioned before, and may be zero, although there are some evidences for a nonzero value. A precise determination of this constant will be one of the primary goals of both theoretical and observational cosmology in the near future.

As we have emphasized, the existence of the cosmological constant Λ is a property of spacetime itself, which measures the energy density of cosmological vacuum.

There are now considerable evidences that the universe began as a fireball in cosmological vacuum, in the form of the so-called “Big Bang”, with extremely high temperature and high energy density. We may claim that before the Big Bang the universe could be a physical vacuum densely populated with long-wave fluctuations of quark and gluon fields. The order parameters of this complicated physical vacuum state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, called vacuum condensate, as we discussed in Section 3. These condensates are conventionally considered to be properties of the QCD vacuum, and hence to be constant throughout the spacetime. A consequence of the existence of such QCD vacuum condensates is a contribution to the cosmological constant. Therefore, we believe Eq. (32), equivalently Eq. (4), is of validity. Its meaning in physics is also evident.

One might doubt the correctness of the Veneziano QCD ghost theory that we used in this work, since it is an analogue of two-dimensional theory based on the Schwinger model [88, 89], replacing the vector gauge field by two scalar fields. These scalar fields have positive and negative norms and cancel with each other, leaving no trace in the physical subspace. They have small contributions to the vacuum energy in the curved space. It is known that the QCD ghost must be intrinsically vector field in order for the $U(1)$ problem to be consistently resolved within the framework of QCD. It seems to be necessary to examine if the Veneziano mechanism works in terms of the vector ghost fields instead of the scalar fields used here. However, Ohta *et al.* in Refs. [27, 90–92] has discussed the same problem in a more realistic four dimensional model, and shows that, indeed, QCD ghost produces vacuum energy density ρ_A proportional to the Hubble parameter which has the right magnitude $\sim (3 \times 10^{-3} \text{ eV})^4$.

Probably, Λ is dependent upon the temperature T and momentum p of virtual particles that produce vacuum condensates. Therefore, based on Eq. (32) we can also predict the Λ dependence on temperature T and momentum p by solving the temperature dependent Dyson–Schwinger Equations. In this case, Λ is a function of T and p . The new study could show the behavior of Λ during the evolution of the universe. This work is under its way to come up soon.

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