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Coupled acoustic-elastic finite element modeling of underwater sound propagation

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Abstract

Underwater sound propagation modeling is fundamental to sonar design and marine exploration. The parabolic equation (PE) method has been the dominant tool, but carries inherent limitations: the one-way approximation neglects backscattered energy, acoustic-only implementations ignore shear-wave conversion at the fluid-solid interface, and range-marching discretization approximates irregular bathymetry as a staircase boundary. These limitations become significant at low frequencies (below 200 Hz), where acoustic wavelengths reach 7.5–30 m and a substantial fraction of energy penetrates into the elastic seabed, exciting both compressional and shear waves. This study develops a two-dimensional frequency-domain finite element model coupling the Helmholtz equation in the water column with the Navier equation in the elastic seabed, with pressure and normal displacement continuity enforced at the fluid-solid interface. Irregular bathymetry is represented by a terrain-following curvilinear mesh via transfinite interpolation, eliminating staircase errors. A perfectly matched layer truncates open boundaries. Validation against RAMGEO for a Pekeris waveguide yields a mean absolute error of 1.8 dB (mean bias −0.17 dB). In a lossless comparison, mean $|\Delta TL|$ between acoustic-only and coupled models is approximately 6 dB, nearly independent of frequency across 50–200 Hz. Under realistic attenuation, mean $|\Delta TL|$ ranges from 0.8 dB (soft sediment) to 5.1 dB (hard bottom), with local maxima exceeding 10 dB at interference nulls. Staircase meshes yield displacement errors of 53–116% and pressure errors of 23–24% versus the curvilinear solution. The proposed model provides a rigorous forward engine for accurate transmission loss prediction in complex shallow-water environments.

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1. Introduction

Underwater sound propagation modeling is fundamental to sonar system design, marine exploration, underwater communications, and geoacoustic inversion.¹⁻³ The dominant computational paradigm is the parabolic equation (PE) method, which approximates

the full wave equation by a forward-marching one-way factorization. The split-step Padé algorithm of Collins,⁴ implemented in the range-dependent acoustic model (RAM), provides an excellent balance of efficiency and accuracy for range-dependent environments. For stratified environments, KRAKEN⁵ and the acousto-elastic model of Westwood *et al.*⁶ offer accurate solutions; Collins and Westwood^{7,8} extended PE to elastic seabeds in RAMGEO, which serves as the benchmark reference here.

However, the PE-based approach carries inherent physical and numerical limitations in low-frequency, shallow-water environments. At frequencies below approximately 200 Hz, acoustic wavelengths in the water column reach 7.5–30 m, and a substantial fraction of acoustic energy penetrates into the seabed.^{1,9} The seabed becomes an active part of the propagation domain.^{9,10} Energy entering the elastic sediment propagates as both compressional (P) and shear (S) waves, interacts with sub-seabed layer interfaces, and can re-radiate back into the water column, modifying the interference pattern and transmission loss (TL) by several decibels.^{9,10} These sub-seabed interactions are governed by the full elastic Navier equation^{11–13} and cannot be captured by the one-way PE approximation, which neglects backscattered contributions from the seabed interior. Recent extensions such as the coupled-mode PE of Kozitskiy¹⁴ partially address seabed elasticity within the PE framework but retain the fundamental one-way limitation.

Furthermore, at low frequencies where acoustic energy penetrates deeply into the seabed, the elastic properties of the seabed—particularly the shear-wave velocity V_s —become decisive for accurate TL prediction. At the fluid-solid interface, incident compressional waves undergo P-to-S mode conversion, diverting a fraction of the incident energy into shear waves in the sediment.^{9,10} The magnitude of this energy partition depends on the impedance contrast and the P-wave critical angle $\theta_{cp} = \sin^{-1}(c_f/V_p)$; for soft sediments with $V_s = 200$ –400 m/s, the mode-conversion effect can alter TL by 1–6 dB on average, with local maxima exceeding 10 dB at interference nulls.⁹ Acoustic-only models, which treat the seabed as a fluid half-space with $V_s = 0$, are structurally incapable of representing this shear-wave energy loss and the associated phase shift in the reflection coefficient that modifies the multipath interference pattern throughout the water column. Consequently, for rigorous underwater acoustic analysis—and especially for geoaoustic inversion where V_s is itself an inversion target—a fully coupled acoustic-elastic formulation is essential.^{15–17} This requirement is directly analogous to the need for elastic forward models in seismic full-waveform inversion (FWI), where

multiparameter recovery demands accurate representation of both P and S wave physics.¹³

A separate but equally important limitation concerns the representation of irregular seabed topography. PE-based methods discretize water depth as flat steps, resulting in a staircase approximation that introduces artificial diffraction at step corners and incorrect reflection coefficients due to the distorted local normal direction.^{11,12} Regular-grid finite-difference methods (FDM) share this limitation. Liu *et al.*¹⁸ developed a 3D finite-difference Helmholtz model with perfectly matched layer (PML) for topographic effects, and Li *et al.*¹⁹ coupled acoustic and elastic domains across an irregular seabed using a meshless generalized finite-difference method; however, neither employed a terrain-following structured finite element method (FEM) mesh. In exploration seismology, terrain-following curvilinear meshes generated via transfinite interpolation (TFI)^{20,21} have long been the standard remedy for irregular topography, but this approach has seldom been applied to underwater acoustic modeling.

The FEM, a mature tool in exploration geophysics,^{13,22,23} addresses all three limitations naturally. Frequency-domain FEM solves the complete Helmholtz equation without the one-way approximation, while the elastic Navier equation is solved simultaneously with acoustic-elastic coupling enforced at the fluid-solid interface as natural boundary conditions.^{23,24} In ocean acoustics, FEM and the spectral element method (SEM) have been applied by Komatitsch and Tromp,²⁵ Bottero *et al.*,²⁶ Cristini and Komatitsch,²⁷ Isakson and Chotiros,²⁸ Vendhan *et al.*,²⁹ Zhou and Luo,³⁰ Gui *et al.*,³¹ He *et al.*,³² and Wang *et al.*³³ Zhang *et al.*³⁴ applied SEM to normal-mode computation in layered waveguides, Xu *et al.*³⁵ combined FEM with physics-informed neural networks, and Tu *et al.*³⁶ developed a Chebyshev–Tau spectral method for coupled-mode propagation. Absorbing boundaries are handled by the PML.^{37–40} Table 1 shows that each prior study addresses only a subset of these capabilities; the present work addresses all three limitations within a single 2D frequency-domain FEM framework.

The present study addresses this gap by developing a 2D frequency-domain FEM model that fully couples the Helmholtz equation in the water column with the Navier equation in the elastic seabed, enforcing pressure continuity and normal displacement continuity at the fluid-solid interface as natural boundary conditions within a unified weak formulation. Irregular bathymetry is represented by a terrain-following curvilinear mesh generated via the TFI of Gordon and Hall,²⁰ an established technique in structured grid generation,^{21,38,41} eliminating staircase errors at the geometrical level. Outgoing waves

Table 1. Comparison of selected FEM/SEM studies in ocean acoustics.

Study	Method	Domain	A-E coupling	Curvilinear mesh	Benchmark validation
Isakson & Chotiros ²⁸	FEM (freq.)	Acoustic	No	No	Yes (OASES)
Bottero <i>et al.</i> ²⁶	SEM (time)	Acoustic+Elastic	Yes	Unstructured	No (qualitative)
Cristini & Komatitsch ²⁷	SEM (time)	Acoustic +Elastic	Yes	Unstructured	Yes (analytic)
Zhou & Luo ³⁰	FEM (freq.)	Acoustic	No	No	Yes (ASA wedge)
He <i>et al.</i> ³²	FEM+ESM	Acoustic	No	No	Yes (KRAKEN)
Li <i>et al.</i> ¹⁹	GFDM (freq.)	Acoustic +Elastic	Yes	Meshless	Yes (analytic)
Present study	FEM (freq.)	Acoustic +Elastic	Yes	Yes (TFI)	Yes (RAMGEO)

Notes: KRAKEN is a normal-mode ocean acoustic propagation model; RAMGEO is a range-dependent acoustic propagation model with geoacoustic bottom interaction.

Abbreviations: ASA wedge: Acoustical Society of America benchmark wedge; ESM: Equivalent source method; FEM: Finite element method; GFDM: Generalized finite-difference method; OASES: Ocean Acoustics and Seismic Exploration Synthesis; SEM: Spectral element method; TFI: Transfinite interpolation.

are absorbed by a PML.^{34,35,39} The resulting sparse linear system is solved with the PARDISO direct solver.⁴⁰ The FEM model is systematically validated against RAMGEO,^{7,8} and the TL difference (ΔTL) between the fully coupled and acoustic-only models is quantified. Sensitivity analyses are conducted over frequency (50, 100, 200 Hz) and seabed shear-wave velocity (soft, medium, and hard seabed types), and a case study comparing staircase and curvilinear mesh representations across three irregular bathymetric profiles completes the numerical investigation. The coupled solver thus serves a dual purpose: it provides a benchmark-quality forward solution for underwater acoustic propagation over elastic seabeds, and it highlights the practical consequences of the coupled physics that exploration-seismic FWI codes must likewise capture.^{15,16} Because multiparameter FWI requires elastic wave modeling to resolve P- and S-velocity structures simultaneously, the accurate elastic forward model developed here plays the same role in geoacoustic inversion^{15,16} as the FWI forward engine does in subsurface velocity estimation.¹³

The key contributions of this study are:

- A unified 2D frequency-domain FEM formulation that couples the Helmholtz equation in the water column with the Navier equation in the elastic seabed,

capturing P-to-S mode conversion and two-way wave propagation without the one-way approximation of PE methods.

- A terrain-following curvilinear mesh based on TFI that eliminates staircase bathymetric errors, with quantified improvement over staircase representations for three benchmark topographies.
- Systematic parametric analysis across frequency (50–200 Hz) and seabed type (soft, medium, hard), validated against the RAMGEO benchmark and supplemented by a mesh convergence study.

The remainder of this paper is organized as follows: Section 2 presents the governing equations, acoustic-elastic coupling conditions, FEM weak formulation, and curvilinear mesh generation. Section 3 presents six numerical experiments assessing model accuracy and parametric sensitivity. Section 4 discusses the physical interpretation of the results and limitations. Section 5 states the conclusions.

2. Theory and methods

This section presents the mathematical foundation of the 2D frequency-domain finite element model for coupled acoustic-elastic underwater sound propagation. As

discussed in the introduction section, at the low frequencies of interest (50–200 Hz) acoustic energy penetrates deeply into the seabed, where it propagates as both compressional and shear waves governed by the elastic wave equation. Accurate modeling of this sub-seabed response requires solving the full wave equations in both the water column and the elastic seabed simultaneously. Accordingly, the water column is treated as an inviscid ideal fluid governed by the Helmholtz equation, while the seabed is treated as an isotropic elastic solid governed by the frequency-domain Navier equation—the same equations that underpin seismic wave modeling in exploration geophysics.¹³ The solutions in the two domains are linked through physically consistent coupling conditions at the fluid-solid interface that enforce pressure continuity and normal displacement continuity. Galerkin finite element discretization together with isoparametric curvilinear coordinate transformation leads to a coupled sparse linear system. Outgoing waves at artificial boundaries are absorbed by a PML, and irregular bathymetry is represented by a terrain-following curvilinear mesh based on TFI,²⁰ eliminating the staircase errors inherent in PE and FDM methods.

2.1. Acoustic governing equation

2.1.1. Helmholtz equation

The fundamental equation governing underwater sound propagation is the wave equation for acoustic pressure $p(\mathbf{x}, t)$. In an inviscid, irrotational fluid, pressure perturbations satisfy:

$$\frac{1}{c_f^2(\mathbf{x})} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = s(\mathbf{x}, t) \quad (1)$$

where $c_f(\mathbf{x})$ is the spatially varying sound speed, $s(\mathbf{x}, t)$ is the source function, and $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$ is the 2D Laplacian. The spatial dependence of c_f accommodates realistic ocean sound-speed profiles.

Assuming a time-harmonic source $e^{-i\omega t}$ and applying the Fourier transform, the time-domain wave Equation 1 reduces to the frequency-domain Helmholtz equation, widely used in frequency-domain seismic modeling:¹³

$$\nabla^2 p(\mathbf{x}, \omega) + k_f^2(\mathbf{x}) p(\mathbf{x}, \omega) = -S(\mathbf{x}, \omega) \quad (2)$$

where $p(\mathbf{x}, \omega)$ is the complex acoustic pressure amplitude, $k_f(\mathbf{x}) = \omega / c_f(\mathbf{x})$ is the wavenumber, $\omega = 2\pi f$ is the angular frequency, and $S(\mathbf{x}, \omega)$ is the source spectrum. For a point source, $S(\mathbf{x}, \omega) = \delta(\mathbf{x} - \mathbf{x}_s)$, whose 2D free-space Green's function is the zero-order Hankel function of the first kind, $p_{\text{inc}}(r) = (i/4) H_0^{(1)}(k_f r)$.

In Equation 2, the term $\nabla^2 p$ describes spatial diffusion of the pressure field, while $k_f^2 p$ represents the restoring character of wave propagation; the balance between these two terms determines the wave propagation pattern. Transmission loss (TL) at a receiver location is defined relative to the pressure p_0 at a reference distance $r_0 = 1$ m:

$$\text{TL} = -20 \log_{10} \left| \frac{p(\mathbf{x})}{p_0} \right| \quad [\text{dB}] \quad (3)$$

2.1.2. Advantages of the frequency-domain formulation

Three principal advantages motivate the frequency-domain Helmholtz Equation 2 over a time-domain formulation. First, solving a single linear system per frequency eliminates time-stepping costs. Second, boundary conditions such as the PML (Section 2.4.3) are naturally expressed in the frequency domain.¹³ Third, the LU factorization of the global stiffness matrix is computed once per frequency and reused for all right-hand sides,¹³ a key efficiency for geoacoustic inversion. Equation 2 is solved independently at each target frequency.

2.2. Elastic governing equation

2.2.1. Frequency-domain Navier equation

Elastic wave propagation in the seabed (sediment and basement rock) is governed by linear elasticity theory—the same physical foundation underlying seismic exploration modeling. The equation of motion for the displacement vector $\mathbf{u} = (u_x, u_z)^T$, derived from Newton's second law and the divergence of the Cauchy stress tensor, becomes under the time-harmonic assumption $e^{-i\omega t}$:

$$-\omega^2 \rho_s(\mathbf{x}) \mathbf{u}(\mathbf{x}, \omega) = \nabla \cdot \boldsymbol{\sigma}(\mathbf{x}, \omega) \quad (4)$$

where $\rho_s(\mathbf{x})$ is the spatially varying density and $\boldsymbol{\sigma}$ is the Cauchy stress tensor. The left-hand side of Equation 4 represents the inertia force $-\omega^2 \rho_s \mathbf{u}$, and the right-hand side $\nabla \cdot \boldsymbol{\sigma}$ is the elastic restoring force arising from spatial gradients in internal stress. The balance between these two forces determines elastic wave propagation. Expanding by component for an isotropic elastic medium, the x - and z -direction equations of motion are:

$$(\lambda + 2\mu) \frac{\partial^2 u_x}{\partial x^2} + \mu \frac{\partial^2 u_x}{\partial z^2} + (\lambda + \mu) \frac{\partial^2 u_z}{\partial x \partial z} + \omega^2 \rho_s u_x = 0 \quad (5)$$

$$(\lambda + 2\mu) \frac{\partial^2 u_z}{\partial z^2} + \mu \frac{\partial^2 u_z}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 u_x}{\partial x \partial z} + \omega^2 \rho_s u_z = 0 \quad (6)$$

2.2.2. Stress-strain relationship (Hooke's Law)

For an isotropic linearly elastic solid, the stress tensor σ and strain tensor ϵ are related by the generalized Hooke's law:

$$\sigma_{ij} = \lambda \delta_{ij} \epsilon_{kk} + 2\mu \epsilon_{ij} \quad (7)$$

where λ and μ are the Lamé constants and δ_{ij} is the Kronecker delta. The strain tensor is defined as the symmetric gradient of the displacement:

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (8)$$

Under 2D plane-strain conditions, the stress components are explicitly:

$$\sigma_{xx} = (\lambda + 2\mu) \frac{\partial u_x}{\partial x} + \lambda \frac{\partial u_z}{\partial z} \quad (9a)$$

$$\sigma_{zz} = \lambda \frac{\partial u_x}{\partial x} + (\lambda + 2\mu) \frac{\partial u_z}{\partial z} \quad (9b)$$

$$\sigma_{xz} = \mu \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) \quad (9c)$$

Equation 9a shows that the horizontal normal stress σ_{xx} depends primarily on the horizontal strain $\partial u_x / \partial x$, weighted by the P-wave modulus $\lambda + 2\mu$, with a volumetric coupling through λ . Similarly, σ_{zz} in **Equation 9b** is governed by the vertical strain. The shear stress σ_{xz} in **Equation 9c** is proportional to the shear modulus μ alone, confirming that shear stresses vanish in a fluid ($\mu = 0$).

2.2.3. Relationship between elastic wave velocities and Lamé constants

Because the Lamé constants are not directly measurable, it is convenient to express them in terms of the compressional wave velocity V_p , shear-wave velocity V_s , and density ρ_s :

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho_s}}, \quad V_s = \sqrt{\frac{\mu}{\rho_s}} \quad (10)$$

V_p is determined by the bulk modulus $\lambda + 2\mu$ and density, while V_s depends solely on the shear modulus μ . The Lamé constants are recovered from the specified material properties as:

$$\mu = \rho_s V_s^2, \quad \lambda = \rho_s (V_p^2 - 2V_s^2) \quad (11)$$

In the fluid (water column), shear stresses cannot be sustained, so $\mu = 0$ and shear waves do not exist. This is the fundamental physical distinction between the water column and the elastic seabed, and it necessitates separate governing equations for the two domains. Wave attenuation is incorporated by treating the Lamé constants as complex-valued quantities. Under the $e^{-i\omega t}$ time-harmonic convention, attenuation requires a negative imaginary part, so the complex velocities are defined as $V_p^* = V_p (1 - i/(2Q_p))$ and $V_s^* = V_s (1 - i/(2Q_s))$; substituting these into **Equation 11** naturally introduces frequency-domain attenuation parameterized by the quality factors Q_p and Q_s .¹

2.3. Acoustic-elastic coupling conditions

At the interface Γ_{fs} between the acoustic fluid domain Ω_f and the elastic solid domain Ω_s , three physically required coupling conditions must be enforced.^{1,9} We adopt the $e^{-i\omega t}$ time-harmonic convention throughout, with acoustic pressure defined as positive in compression (i.e., $p > 0$ corresponds to a compressive state). The Cauchy stress σ follows the standard solid-mechanics sign convention (tension positive). Let $\hat{n}$ denote the unit outward normal from the fluid domain directed into the solid domain. The fluid particle displacement is related to the pressure gradient through the linearized momentum equation $\rho_f \omega^2 \mathbf{u}^f = \nabla p$, which gives the normal component $u_n^f = (1/\rho_f \omega^2) \partial p / \partial n$.

Condition 1: Normal stress continuity. The normal stress must be continuous across the fluid-solid interface. Since an ideal fluid sustains no tangential stress, the normal traction on the fluid side equals the acoustic pressure p . The force balance condition is:

$$\sigma \cdot \hat{n}|_{\Omega_s} = -p \hat{n}|_{\Omega_f} \quad \text{on } \Gamma_{fs} \quad (12)$$

That is, the acoustic pressure on the fluid side equals in magnitude, and is opposite in sign to the normal compressive stress on the solid side.

Condition 2: Normal displacement continuity. The interface must remain in contact, so the normal displacement is continuous. In the frequency domain, the normal displacement of the fluid particle is obtained from the momentum equation $\rho_f \omega^2 u_n^f = \partial p / \partial n$, giving:

$$\mathbf{u} \cdot \hat{n}|_{\Omega_s} = -\frac{1}{\rho_f \omega^2} \frac{\partial p}{\partial n}|_{\Omega_f} \quad \text{on } \Gamma_{fs} \quad (13)$$

Condition 3: Zero shear traction. An ideal fluid cannot transmit shear stresses, so the tangential traction component on the solid side of the interface must vanish:

$$[\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} - (\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} \cdot \hat{\mathbf{n}}) \hat{\mathbf{n}}]_{\Omega_s} = 0 \quad \text{on } \Gamma_{fs} \quad (14)$$

Because the fluid traction vector is purely normal (an ideal fluid sustains no shear stress), the tangential traction on the solid side of the interface is zero by construction; thus condition (14) is enforced naturally together with the normal traction continuity (12). All three conditions are incorporated as natural boundary conditions in the surface integral terms of the weak formulation.

In the discrete FEM implementation, the fluid-solid interface is a conforming interface: the mesh nodes along the interface are shared between the acoustic and elastic domains, and each interface node carries both a pressure degree of freedom (DOF) from the acoustic side and two displacement DOFs (u_x, u_z) from the elastic side. Condition (13) enters the acoustic weak form through the boundary integral (16), which couples each pressure DOF at an interface node to the normal displacement DOFs at the same node via a 1D edge mass matrix. Condition (12) enters the elastic weak form through the boundary integral (18), which couples each displacement DOF to the pressure DOF at the same node. The resulting coupling matrix C_{fs} in equation (22) is assembled element-by-element along the interface edges using the same shape functions employed for the domain integrals, ensuring consistency. Because both coupling terms arise naturally from the surface integrals in the weak formulation, no penalty parameters or Lagrange multipliers are required, and the coupling is enforced to the same order of accuracy as the underlying finite element discretization.

2.4. Finite element formulation

2.4.1. Weak formulation

The full computational domain consists of the fluid region Ω_f , the solid region Ω_s , their shared interface Γ_{fs} , and the absorbing PML layer Ω_{PML} (Figure 1).

Acoustic domain weak form. Multiplying Equation 2 by a test function $q \in H^1(\Omega_f)$, integrating over Ω_f , and applying Green's first identity (integration by parts) yields:

$$\int_{\Omega_f} \nabla q \cdot \nabla p \, d\Omega - k_f^2 \int_{\Omega_f} q p \, d\Omega - \int_{\partial\Omega_f} q \frac{\partial p}{\partial n} \, d\Gamma = \int_{\Omega_f} q S \, d\Omega \quad (15)$$

The first term on the left represents spatial diffusion via the pressure gradient inner product; the second term represents wave propagation via the wavenumber. The boundary integral $\int_{\partial\Omega_f} q (\partial p / \partial n) \, d\Gamma$ is handled according to the boundary type. At the coupling interface Γ_{fs} , substituting condition (13) gives:

$$\int_{\Gamma_{fs}} q \frac{\partial p}{\partial n} \, d\Gamma = -\rho_f \omega^2 \int_{\Gamma_{fs}} q (\mathbf{u} \cdot \hat{\mathbf{n}}) \, d\Gamma \quad (16)$$

This term expresses the physical action of the solid normal displacement as a pressure source in the acoustic domain.

Elastic domain weak form. Taking the inner product of Equation 4 with a vector test function $\mathbf{v} \in [H^1(\Omega_s)]^2$, integrating over Ω_s , and applying Green's first identity yields:

$$\int_{\Omega_s} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{v}) \, d\Omega - \omega^2 \int_{\Omega_s} \rho_s \mathbf{u} \cdot \mathbf{v} \, d\Omega = \int_{\partial\Omega_s} (\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}) \cdot \mathbf{v} \, d\Gamma \quad (17)$$

The first term on the left is the internal elastic energy; the second is the inertia term. At the coupling interface Γ_{fs} , substituting condition (12) gives:

$$\int_{\Gamma_{fs}} (\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}) \cdot \mathbf{v} \, d\Gamma = - \int_{\Gamma_{fs}} p (\mathbf{v} \cdot \hat{\mathbf{n}}) \, d\Gamma \quad (18)$$

This term expresses the physical action of the acoustic pressure as a normal load on the elastic boundary. The coupled weak system is formed by combining Equations 15–18, with unknowns comprising the fluid pressure p and solid displacements (u_x, u_z) .

2.4.2. Discretization and matrix assembly

The Galerkin method is adopted, so that the trial and test function spaces are identical. The computational domain is partitioned into isoparametric quadrilateral elements, within each of which the unknowns are approximated by nodal basis functions. Nine-node biquadratic Lagrange elements (Q9) are employed. In the reference coordinates $(\xi, \eta) \in [-1, 1]^2$, the shape functions are tensor products of 1D quadratic Lagrange polynomials:

$$N_I(\xi, \eta) = L_i(\xi) L_j(\eta), \quad I = 3(j-1) + i, \quad i, j \in \{1, 2, 3\} \quad (19)$$

where the 1D Lagrange basis functions are:

$$L_1(\xi) = \frac{\xi(\xi-1)}{2}, \quad L_2(\xi) = 1 - \xi^2, \quad L_3(\xi) = \frac{\xi(\xi+1)}{2} \quad (20)$$

Q9 elements are chosen because, for the same number of nodes per wavelength, the numerical dispersion error is substantially lower than for bilinear Q4 elements. Since numerical dispersion (the pollution error) worsens with increasing frequency in Helmholtz problems, higher-order elements are essential.²² Approximating the pressure and

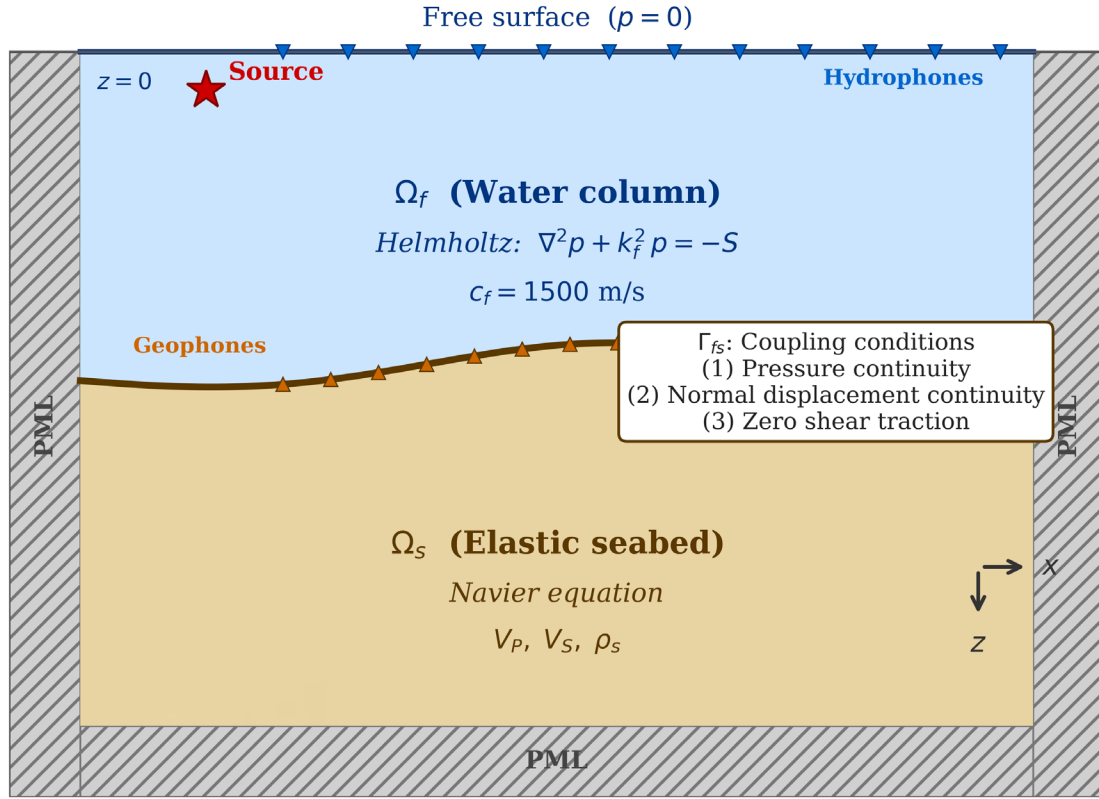


Figure 1. Schematic of the 2D computational domain showing the acoustic fluid domain (Ω_f), elastic solid domain (Ω_s), fluid-solid interface (Γ_{fs}), perfectly matched layer (PML), and boundary conditions. The point source is located at depth z_s in the water column.

displacements within each element by the shape functions:

$$\begin{aligned} p^h(\mathbf{x}) &= \sum_{I=1}^9 N_I(\mathbf{x}) \hat{p}_I, \\ u_k^h(\mathbf{x}) &= \sum_{I=1}^9 N_I(\mathbf{x}) \hat{u}_{k,I} \quad (k = x, z) \end{aligned} \quad (21)$$

and substituting into the weak formulation and assembling over all elements yields the coupled block linear algebraic system:

$$\begin{pmatrix} \mathbf{K}_f - \omega^2 \mathbf{M}_f & \rho_f \omega^2 \mathbf{C}_{fs}^T \\ \mathbf{C}_{fs} & \mathbf{K}_s - \omega^2 \mathbf{M}_s \end{pmatrix} \begin{pmatrix} \hat{\mathbf{p}} \\ \hat{\mathbf{u}} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_p \\ 0 \end{pmatrix} \quad (22)$$

The individual block matrices are defined as:

$$\begin{aligned} [\mathbf{K}_f]_{ij} &= \int_{\Omega_f} \nabla N_i \cdot \nabla N_j \, d\Omega, \\ [\mathbf{M}_f]_{ij} &= \int_{\Omega_f} \frac{1}{c_f^2(\mathbf{x})} N_i N_j \, d\Omega \end{aligned} \quad (23)$$

$$[\mathbf{K}_s]_{IJ} = \int_{\Omega_s} \mathbf{B}_I^T \mathbf{D} \mathbf{B}_J \, d\Omega, \quad (24)$$

$$[\mathbf{M}_s]_{IJ} = \int_{\Omega_s} \rho_s \mathbf{N}_I^T \mathbf{N}_J \, d\Omega$$

$$[\mathbf{C}_{fs}]_{I_i(J,k)} = \int_{\Gamma_{fs}} N_i \mathbf{N}_J n_k \, d\Gamma \quad (25)$$

where $\mathbf{B}$ is the strain-displacement matrix, $\mathbf{D}$ is the elastic constitutive matrix assembled from the Lamé constants, $\hat{\mathbf{p}}$ and $\hat{\mathbf{u}}$ are the nodal degree-of-freedom vectors for pressure and displacement respectively, and $\mathbf{f}_p$ is the acoustic source load vector. In $\mathbf{M}_f$, the factor $1/c_f^2(\mathbf{x})$ is retained inside the integral to accommodate spatially varying sound speed. The coupling matrix $\mathbf{C}_{fs}$ appears in the off-diagonal blocks of system (22) as $\rho_f \omega^2 \mathbf{C}_{fs}^T$ in the (1,2) position and $\mathbf{C}_{fs}$ in the (2,1) position, yielding an unsymmetric structure. This unsymmetry arises because the acoustic pressure and elastic displacement unknowns have different physical dimensions, and it correctly represents bidirectional energy transfer between the fluid

and solid.^{23,24} Numerical integration employs a 3×3 Gauss-Legendre quadrature rule per element.

The global matrix in system (22) is a large sparse complex matrix, which is solved efficiently by the PARDISO direct sparse solver based on LU factorization.^{40,42} For multiple sources at the same frequency, the factorization is reused for all right-hand sides.

2.4.3. Absorbing boundary condition: PML

To suppress reflections of outgoing waves at the artificial boundaries of the computational domain, a PML³⁴ is applied, implemented via complex coordinate stretching.^{35,36,39,43} The stretching function in the direction ξ (x or z) is:

$$\tilde{\xi} = \xi + \frac{1}{i\omega} \int_0^\xi \sigma(\xi') d\xi', \quad s_\xi(\xi) = 1 + \frac{\sigma(\xi)}{i\omega} \quad (26)$$

where $\sigma(\xi) \geq 0$ is a damping function that increases gradually within the PML. A quadratic profile $\sigma(\xi) = \sigma_{\max} (\xi / d_{\text{PML}})^2$ is adopted, where d_{PML} is the PML thickness, set to $2\lambda_f$ ($\lambda_f = c_f / f$, the acoustic wavelength in water). The maximum damping coefficient is chosen to achieve a target theoretical reflection coefficient R_0 at normal incidence:³⁵

$$\sigma_{\max} = \frac{3c_f}{2d_{\text{PML}}} \ln \left(\frac{1}{R_0} \right) \quad (27)$$

with $R_0 = 10^{-4}$ (i.e., 80 dB attenuation) $\lambda_f = 15d_{\text{PML}} = 30\sigma_{\max} \approx 690^{-1}$. The PML is applied to all four sides of the computational domain except the free surface ($z = 0$), where the Dirichlet condition $p = 0$ is imposed. In the elastic domain, the same complex coordinate stretching is applied to the Navier equation by replacing partial derivatives $\partial / \partial \xi$ with $(1/s_\xi) \partial / \partial \xi$, ensuring consistent PML treatment across the fluid-solid interface.³⁶ Inside the PML, the spatial differential operators in both the Helmholtz and Navier equations are replaced by stretched-coordinate derivatives, and the PML element matrices are assembled within the same weak-form framework as the acoustic and elastic domains.

2.5. Curvilinear mesh

2.5.1. TFI-based mesh generation

To represent irregular seabed topography accurately, a terrain-following curvilinear mesh is employed. The mapping between physical coordinates (x, z) and computational coordinates (ξ, η) is defined by the TFI method of Gordon and Hall.²⁰ Here, $\xi \in [0, 1]$ is the horizontal computational coordinate and $\eta \in [0, 1]$ is the

vertical coordinate running from the sea surface to the seabed.

In the simplest case of a flat sea surface at $z = 0$ and a bathymetric profile $z_b(x)$, the TFI mapping reduces to a unidirectional linear blending along the vertical coordinate:

$$z(x, \eta) = (1 - \eta) \cdot 0 + \eta \cdot z_b(x) = \eta z_b(x) \quad (28)$$

This is a special case of the general TFI framework,²⁰ in which the top boundary is flat and the mapping simplifies to a single projector. Despite its simplicity, this mapping deforms the uniform computational grid (ξ, η) to conform precisely to the seabed geometry.²¹ The elastic solid region below the seabed is generated by an analogous vertical blending between the seabed and the bottom boundary of the computational domain.

2.5.2. Jacobian coordinate transformation

To evaluate FEM integrals on curvilinear elements, physical-domain integrals are transformed to computational-domain integrals. The Jacobian matrix is:

$$\mathbf{J} = \begin{pmatrix} \partial x / \partial \xi & \partial x / \partial \eta \\ \partial z / \partial \xi & \partial z / \partial \eta \end{pmatrix} \quad (29)$$

Physical-domain partial derivatives are obtained by the chain rule as:

$$\begin{pmatrix} \partial / \partial x \\ \partial / \partial z \end{pmatrix} = \mathbf{J}^{-1} \begin{pmatrix} \partial / \partial \xi \\ \partial / \partial \eta \end{pmatrix} \quad (30)$$

and the area element transforms as $d\Omega = |\det \mathbf{J}| d\xi d\eta$. A 3×3 Gauss-Legendre quadrature rule is applied for numerical integration within each element.

2.5.3. Comparison with staircase mesh

In conventional PE-based methods and regular-grid FEM, irregular bathymetry is approximated by a staircase boundary. This approximation generates artificial diffraction at step corners and introduces errors in the local normal direction of the seabed, distorting the reflection coefficient. For a sloping seabed inclined at angle θ , the vertical positional error of the staircase approximation is proportional to the grid spacing Δz , manifesting as phase errors in reflected pressure. The terrain-following curvilinear mesh eliminates these geometric errors by aligning element edges with the physical boundary. The TFI approach adopted here²⁰ is an established mesh-generation technique; its effectiveness in reducing staircase-induced errors in the present context is quantified in Section 3.6.

3. Numerical examples

The performance of the proposed FEM model for coupled acoustic-elastic underwater sound propagation is assessed through six numerical examples. Section 3.1 validates the FEM code against RAMGEO in a range-independent environment with a flat seabed. Section 3.2 presents a mesh convergence study to verify the numerical reliability of the adopted mesh resolution rule. Section 3.3 quantifies the TL difference between the acoustic-only and acoustic-elastic coupled models. Sections 3.4 and 3.5 present systematic sensitivity analyses with respect to frequency and seabed shear-wave velocity, respectively. Section 3.6 provides a case study comparing staircase and curvilinear meshes across three irregular bathymetric profiles. In all examples, the PML thickness is set to twice the acoustic wavelength ($2\lambda_f$) at the analysis frequency, and the element size satisfies $h \leq \lambda_f / 8$.²² In the elastic domain, the shear wavelength $\lambda_s = V_s / f$ can be considerably shorter than the acoustic wavelength. For Q9 biquadratic elements; however, the effective number of points per wavelength is approximately twice that of linear elements due to the mid-side and center nodes. For example, at 200 Hz with $V_s = 200$ m/s (the most demanding case), $\lambda_s = 1.0$ m and $h = 0.94$ m, yielding approximately three Q9 nodes per shear wavelength (equivalent to ~ 6 sampling points per λ_s), which satisfies the minimum resolution requirement for second-order elements.²²

3.1. FEM model validation: Comparison with RAMGEO

3.1.1. Model configuration

The accuracy of the proposed FEM code is validated by comparing transmission loss against RAMGEO (elastic PE) in a range-independent environment with a flat horizontal seabed. RAMGEO is based on the energy-conserving higher-order PE of Collins⁷ and serves as an established benchmark for underwater sound propagation with elastic seabed representation.^{1,8}

The test environment is configured as follows: water depth 200 m, point source at approximately 101 m depth (grid index 112 with grid spacing $h = 0.9$ m; grid dimensions $n_x = 11114$, $n_z = 557$, corresponding to a total grid extent of 10 km including PML regions), and frequency 100 Hz. TL is analyzed over the range of 0–5 km. Water column properties are $c_f = 1,500$ m/s and $\rho_f = 1,000$ kg/m³. The elastic seabed half-space is assigned $V_p = 1,800$ m/s, $V_s = 400$ m/s, $\rho_s = 1,800$ kg/m³, consistent with a medium-stiffness sandy sediment in the geoacoustic classification of Hamilton.¹⁰ In Sections 3.1–3.4, no intrinsic attenuation is applied ($Q_p = Q_s = \infty$, *i.e.*, lossless), in order to isolate the effect of acoustic-

elastic coupling from material absorption; attenuation is introduced in Section 3.5 to investigate its combined influence with shear-wave velocity. The receiver array is placed at the same depth as the source (approximately 101 m), and TL is computed from Equation 3.

3.1.2. Results and analysis

Figure 2 shows the TL comparison at approximately 101 m receiver depth over the range 0–5 km. The FEM and RAMGEO TL curves reproduce the positions and amplitudes of the multipath interference peaks (TL minima) and troughs (TL maxima) throughout the entire range. The raw MAE of 5.0 dB reflects phase-shift sensitivity at interference nulls, documented in PE-FEM comparisons.¹ Running-average smoothing (100 m) yields MAE = 1.8 dB with mean bias of -0.17 dB, confirming that the envelope structure is accurately reproduced. The small discrepancy observed at ranges below 500 m (up to 1.2 dB) is attributed to the difference between RAMGEO's PE starting-field approximation and the Green's function-based source representation used in the FEM.¹ These results confirm that the proposed FEM correctly reproduces multipath interference in coupled acoustic-elastic waveguides.

3.2. Mesh convergence study

To verify the numerical reliability of the adopted $\lambda/8$ mesh resolution rule, a quantitative mesh convergence study is performed. The same lossless Pekeris waveguide environment as Section 3.1 is used, with TL computed at 100 Hz for five mesh resolutions: $\lambda/4$, $\lambda/6$, $\lambda/8$, $\lambda/10$, and $\lambda/12$ elements per minimum acoustic wavelength ($\lambda_{min} = c_{min} / f = 1450 / 100 = 14.5$ m, where $c_{min} = 1450$ m/s is adopted as a conservative design value for mesh sizing, slightly below the nominal water-column speed $c_f = 1500$ m/s of Section 3.1, to provide margin against potential sound-speed decrease toward the seabed; the nominal wavelength $\lambda = c_f / f = 15.0$ m at the same frequency is used for the frequency sensitivity analysis). A reference solution at $\lambda/16$ resolution ($h = 0.9$ m $\approx \lambda_{min} / 16$, $N \approx 6.4 \times 10^6$ DOFs) is used for comparison.

Table 2 summarizes the grid parameters and the smoothed TL error at approximately 100 m receiver depth over the range 0.5–9.5 km. TL curves are smoothed with a 50 m moving average (approximately 3.4λ) to suppress the sensitivity of point-wise error metrics to minor shifts in interference null positions, which is a well-known artifact in waveguide TL comparisons.¹ Figure 3 shows the smoothed TL comparison at approximately 100 m depth for four mesh resolutions.

The RMS error decreases monotonically from 12.3 dB at $\lambda/4$ to 3.9 dB at $\lambda/12$, confirming mesh convergence. The

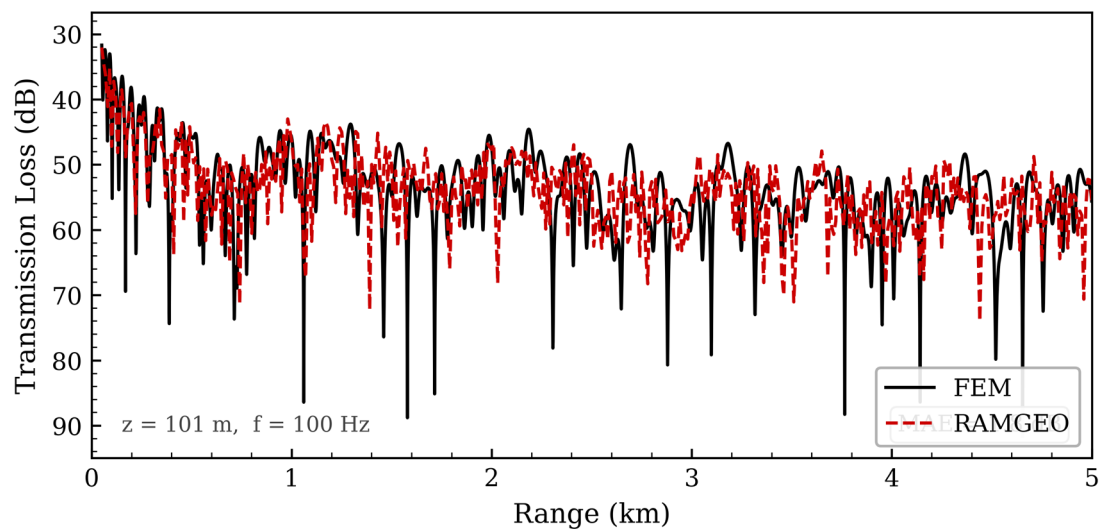


Figure 2. Transmission loss comparison between the FEM acoustic-elastic coupled model (solid black) and RAMGEO (dashed red) at approximately 101 m receiver depth over range 0–5 km in a range-independent Pekeris waveguide at 100 Hz. RAMGEO is a fully range-dependent parabolic-equation acoustic propagation model for fluid seabeds.

Abbreviation: FEM: Finite element method.

Table 2. Mesh convergence results at 100 Hz (acoustic mode) for five resolutions ranging from $\lambda/4$ to $\lambda/12$, compared against a $\lambda/16$ reference solution.

Resolution (λ/h)	h (m)	Grid ($n_x \times n_z$)	DOFs	RMS error (dB)	MAE (dB)
$\lambda/4$	3.63	2760×139	445,200	12.3	10.9
$\lambda/6$	2.42	4139×208	952,812	9.6	8.3
$\lambda/8$	1.81	5518×277	1,650,726	5.6	4.6
$\lambda/10$	1.45	6898×347	2,546,246	4.7	3.8
$\lambda/12$	1.21	8277×416	3,626,212	3.9	2.9

convergence rate is approximately $O(h^{1.1})$, which is below the theoretical $O(h^{p+1})$ rate (where p is the polynomial order) expected for biquadratic Q9 elements applied to smooth problems. This reduced rate is attributed to three factors: (i) the impedance discontinuity at the fluid-solid interface reduces the solution regularity below the H^{p+1} assumption underlying optimal convergence estimates, (ii) the Helmholtz pollution effect—the well-known degradation of FEM accuracy with increasing wavenumber k —introduces a resolution-dependent phase error that persists even at moderately fine meshes,²² and (iii) the logarithmic (dB) scale of TL amplifies relative errors near interference nulls. A pronounced error reduction occurs between $\lambda/6$ (9.6 dB) and $\lambda/8$ (5.6 dB), confirming that $\lambda/8$

represents the practical threshold for acceptable accuracy. The $\lambda/8$ MAE of 4.6 dB is comparable to the 5.0 dB raw MAE observed in the FEM-RAMGEO validation (Section 3.1), indicating that the discretization error at $\lambda/8$ is of the same order as the inherent methodological difference between FEM and PE.

3.3. Comparison of acoustic-only and coupled acoustic-elastic models

3.3.1. Model configuration

To quantify the influence of seabed elastic properties on transmission loss, the acoustic-only and coupled models are compared in the same environment. The acoustic-only model treats the seabed as an ideal fluid half-space,

ignoring shear-wave transmission ($V_s = 0$), while the coupled model fully accounts for elastic seabed properties. The configuration is identical to Section 3.1. The TL difference $\Delta TL = TL_{\text{coupled}} - TL_{\text{acoustic}}$ is computed over the 2D range-depth space. Positive ΔTL indicates greater attenuation in the coupled model, reflecting additional acoustic energy loss due to P-to-S mode conversion at the fluid-solid interface.

3.3.2. Results and analysis

Figure 4 compares the TL curves of the acoustic-elastic coupled model (solid) and the acoustic-only model (dashed) at three receiver depths: 50 m, 101 m, and 150 m. The acoustic-only model consistently underestimates TL relative to the coupled model across all depths and ranges, confirming that neglecting shear-wave interactions in the seabed leads to a systematic bias. The mean absolute TL difference ($|\Delta TL|$) is 5.89 dB at 50 m, 5.97 dB at 101 m, and 5.59 dB at 150 m. Note that these values are obtained under the lossless assumption ($Q_p = Q_s = \infty$); under attenuation (Section 3.5), mean $|\Delta TL|$ is reduced but persists throughout the water column.

At ranges below 1 km, direct-path energy dominates and the elastic seabed effect is comparatively small. At ranges of 1–5 km, the number of bottom reflections increases and the cumulative energy loss from P-to-S mode conversion grows progressively, so ΔTL increases with range. At the mid-water depth (101 m), where the source is located, the $|\Delta TL|$ is largest (5.97 dB), consistent with the stronger bottom interaction of ray paths near the source depth. Near the surface (50 m) and closer to the seabed

(150 m), the slightly lower $|\Delta TL|$ values reflect the different modal excitation patterns at each depth.¹ These results indicate that acoustic-only models may underestimate TL by approximately 5–6 dB on average under lossless conditions.

3.4. Frequency sensitivity analysis

3.4.1. Model configuration

To investigate the influence of frequency on the acoustic-elastic coupling effect and the numerical properties of the FEM, the coupled FEM model is compared with the acoustic-only FEM model at three frequencies: 50, 100, and 200 Hz. All other settings follow Section 3.1. To isolate the physical frequency dependence of $|\Delta TL|$ from numerical discretization effects, a single computational grid ($h = 0.9$ m, ~ 10.4 million DOF) is used for all three frequencies. This resolution satisfies $h \leq \lambda/8$ at the highest frequency (200 Hz, $\lambda = 7.5$ m) and oversamples the lower frequencies. Computational parameters are summarized in Table 3.

3.4.2. Results and analysis

Figure 5 compares TL curves at approximately 101 m receiver depth for the three frequencies from the acoustic-elastic coupled FEM model (solid lines) and the acoustic-only FEM model (dashed lines). At 50 Hz, the long wavelength yields a wide spatial period of the interference pattern (approximately 200–300 m), and the greater penetration depth into the seabed distributes the elastic coupling effect over a broad range. At 100 Hz, the interference pattern is distinct and ΔTL is most clearly

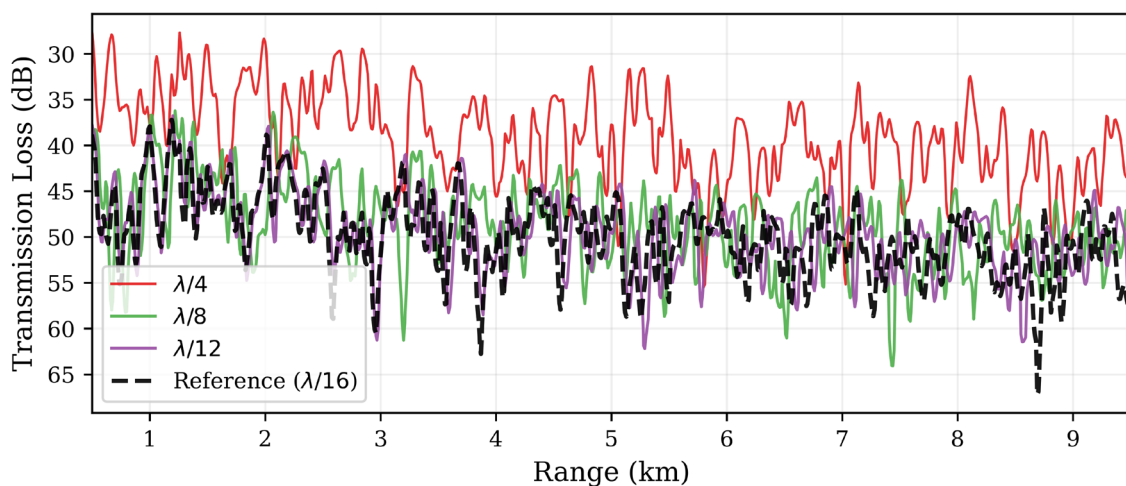


Figure 3. Smoothed transmission loss at 100 m depth for four mesh resolutions at 100 Hz in the lossless Pekeris waveguide: $\lambda/4$, $\lambda/8$, $\lambda/12$, and reference ($\lambda/16$). The $\lambda/8$ solution closely follows the reference, confirming numerical convergence.

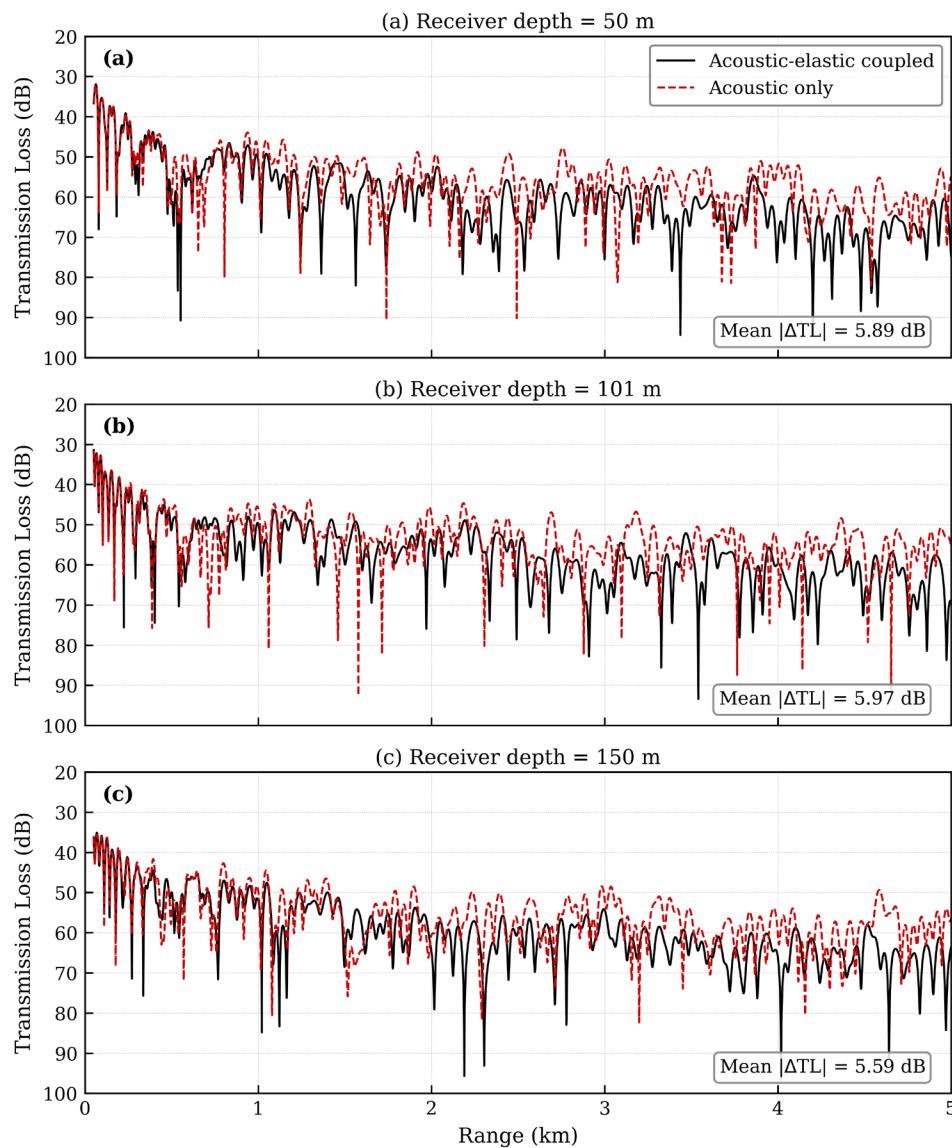


Figure 4. Transmission loss comparison between coupled (solid) and acoustic-only (dashed) models at three receiver depths: (a) 50 m, (b) 101 m, and (c) 150 m over range 0–5 km at 100 Hz

Table 3. Computational parameters of the FEM model at three analysis frequencies.

Frequency (Hz)	Wavelength λ (m)	h (m)	DOF	Computation time (s)
50	30.0	0.9	~10,400,000	~10
100	15.0	0.9	~10,400,000	~10
200	7.5	0.9	~10,400,000	~10

Abbreviations: DOF: Degrees of freedom; FEM: Finite element method; $h_{\max}$: maximum element size.

resolved. At 200 Hz, the interference pattern becomes considerably more complex, with shorter spatial periods (50–80 m) and larger peak-to-trough amplitudes in the TL curves.

The mean $|\Delta TL|$ between the coupled and acoustic-only FEM models is approximately 5.6 dB at 50 Hz, 6.0 dB at 100 Hz, and 6.0 dB at 200 Hz. These values reflect the consistent sensitivity of the 200 m water column to P-to-S mode conversion across the tested frequency range.

Because the same discretization is used at all three frequencies, computation time is nearly constant (~ 10 s per frequency) and the observed $|\Delta TL|$ variation is attributable to wave physics, not grid resolution. The range-averaged $|\Delta TL|$ is nearly constant (5.6–6.0 dB); range-resolved

variations are discussed in Section 4.

3.5. Shear-wave velocity sensitivity analysis

3.5.1. Model configuration

To quantify the effect of seabed shear-wave velocity (V_s) on transmission loss, three seabed types are defined following the geoacoustic classification of Hamilton.¹⁰ Material properties for each type are listed in Table 4. The quality factors Q_p and Q_s are derived from Hamilton's reported compressional-wave attenuation coefficients k_p (dB/m/kHz) via $Q_p = 27,272 / (k_p \times V_p)$,¹⁰ with $Q_s \approx 0.5$ – $0.6 Q_p$ following established marine sediment models. The analysis frequency is 100 Hz; all other settings follow Section 3.1.

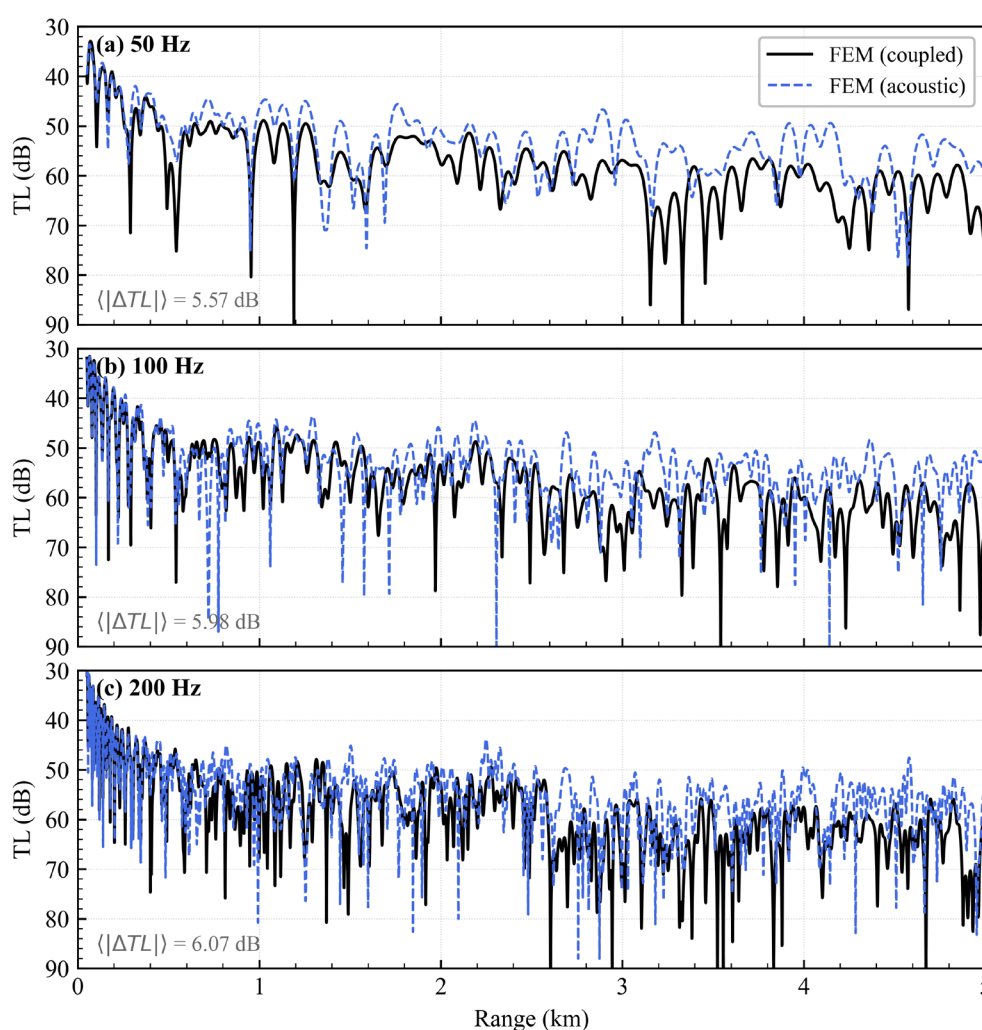


Figure 5. Frequency sensitivity analysis: Coupled (solid) and acoustic-only (dashed) FEM models at (a) 50 Hz, (b) 100 Hz, and (c) 200 Hz at approximately 101 m receiver depth

Abbreviation: FEM: Finite element method.

Table 4. Geoacoustic parameters for three seabed types used in the shear-wave velocity sensitivity analysis.

Seabed type	V_p (m/s)	V_s (m/s)	ρ_s (kg/m ³)	Q_p	Q_s
Soft sediment	1,550	200	1,400	50	30
Medium sediment	1,800	400	1,800	100	60
Hard bottom	3,000	800	2,500	300	150

Notes: Classification follows Hamilton.¹⁰ Q_p and Q_s : quality factors for compressional and shear waves.

3.5.2. Results and analysis

Figure 6 compares the coupled and acoustic-only model TL at approximately 101 m receiver depth for the three seabed types.

The spatial average of $|\Delta TL|$ for the soft sediment ($V_s = 200$ m/s) is approximately 0.8 dB, for the medium sediment ($V_s = 400$ m/s) approximately 1.8 dB, and for the hard bottom ($V_s = 800$ m/s) approximately 5.1 dB. Note that these $|\Delta TL|$ values differ from the medium-sediment result in Section 3.3 (mean $|\Delta TL|$ of approximately 5.97 dB at 101 m depth). This difference arises primarily because Sections 3.1–3.4 use a lossless seabed ($Q_p = Q_s = \infty$) to isolate the coupling effect, whereas Section 3.5 introduces intrinsic attenuation ($Q_p = 100$, $Q_s = 60$ for medium sediment; Table 4), which damps the shear waves in the seabed and reduces the net $|\Delta TL|$. Furthermore, all geoacoustic parameters (V_p , ρ_s , Q_p , Q_s) vary simultaneously across the three seabed types (Table 4), so the reported $|\Delta TL|$ values reflect the combined influence of all material properties, not V_s alone.

These results quantitatively confirm that geoacoustic seabed properties are decisive for accurate TL prediction. The increasing $|\Delta TL|$ from soft to hard seabed is driven by the growing impedance contrast (Section 4). For the soft sediment ($Z_s/Z_f \approx 1.4$), the impedance contrast is small, yielding modest coupling. For the hard bottom ($Z_s/Z_f \approx 5.0$), the large impedance contrast introduces a pronounced phase shift in the reflection coefficient—particularly near the P-wave critical angle $\theta_{cp} = \sin^{-1}(c_f/V_p) \approx 30^\circ$ —producing the largest $|\Delta TL|$.^{1,9}

The variation in $|\Delta TL|$ across the three seabed types demonstrates the strong sensitivity of the coupled model to seabed properties, with practical implications for model selection. The coupled model's sensitivity to V_s confirms that it possesses the parametric sensitivity required to serve as a forward model for geoacoustic inversion of seabed shear-wave velocity—analogue to the role of the elastic wave equation in seismic FWI.¹³ These results provide a practical guideline for selecting model complexity: for environments where errors of 1–2 dB are intolerable, or

where V_s estimation is an inversion objective, an acoustic-elastic coupled formulation is indispensable.

3.6. Case study: Staircase vs. curvilinear mesh

3.6.1. Model configuration

Three representative irregular bathymetric profiles are considered to compare the numerical accuracy of staircase and curvilinear meshes:

- **Downslope:** water depth increases linearly from 790 m to approximately 980 m over 0–3.98 km (slope gradient 5%).
- **Anticline:** a Gaussian bathymetric high centered at approximately 2 km, with depth rising from a mean of 790 m to 690 m at the apex (amplitude 100 m).
- **Syncline:** a Gaussian bathymetric low centered at approximately 2 km, with depth dropping from a mean of 790 m to 890 m at the trough (amplitude 100 m).

The domain is discretized on a 399×399 grid with spacing $h = 10$ m. This spacing corresponds to approximately 3 elements per acoustic wavelength at the highest frequency (200 Hz, $\lambda_f = 7.5$ m), which is coarser than the $h \leq \lambda/8$ criterion used in the TL experiments of Sections 3.1–3.5. However, since the objective of this case study is to compare the *relative* difference between staircase and curvilinear mesh solutions; the identical grid ensures observed differences are attributable solely to boundary representation. The analysis covers 50–200 Hz with medium sediment properties (Table 4).

3.6.2. Results and analysis

Figure 7 presents the velocity model and broadband seismograms for each of the three bathymetric profiles. For each case, the left panel shows the velocity model with the true seabed boundary (solid black line) and the staircase approximation (dashed red line) overlaid. The remaining columns display three wavefield components—horizontal displacement u_x , vertical displacement u_z , and pressure p —obtained by inverse Fourier transform of the frequency-domain elastic solutions over 50–200 Hz.

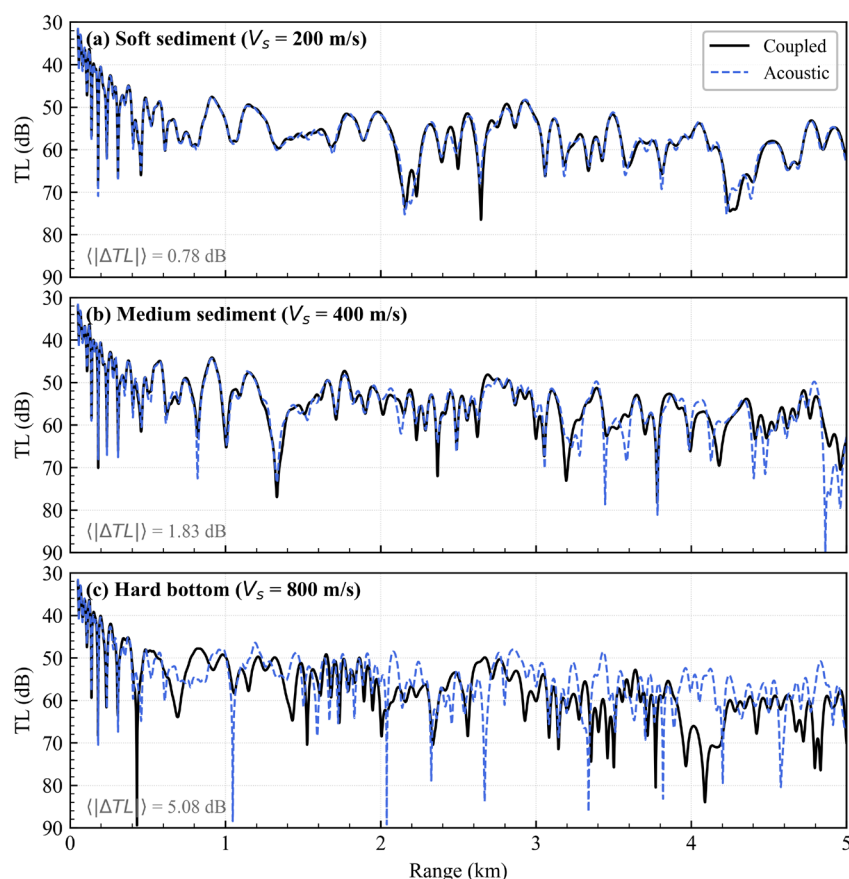


Figure 6. Shear-wave velocity sensitivity: Coupled (solid) vs. acoustic-only (dashed) FEM models for (a) soft sediment ($V_s = 200$ m/s), (b) medium sediment ($V_s = 400$ m/s) and (c) hard bottom ($V_s = 800$ m/s) at approximately 101 m receiver depth (100 Hz). Abbreviations: FEM: Finite element method; TL: Transmission loss.

For each component, the staircase mesh result (top) is compared against the curvilinear mesh result (bottom).

Table 5 summarizes the relative difference between the staircase and curvilinear solutions for all three wavefield components, measured by the L_2 norm $\|\mathbf{u}_{\text{stair}} - \mathbf{u}_{\text{curv}}\|_2 / \|\mathbf{u}_{\text{curv}}\|_2$. The displacement components (u_x , u_z) show substantially larger differences (53–116%) than pressure (23–24%), as the displacement field depends directly on traction boundary accuracy. Among the three profiles, the anticline and syncline exhibit the largest u_z discrepancies (98% and 116%), consistent with their stronger seabed curvature.

The physical origin of the staircase mesh error is twofold, as discussed in Section 2: artificial diffraction at step corners generates numerically spurious diffracted waves, and incorrect local normal directions at step faces introduce errors in the traction boundary conditions. Because the displacement field depends directly on the tangential traction continuity at the fluid-solid interface,

the displacement components u_x and u_z are affected more severely than the scalar pressure. The approximately 2–5 times greater sensitivity of the displacement components (Table 5) confirms that terrain-following meshing is essential for accurate elastic wavefield prediction at irregular seabeds.

Table 5. Relative L_2 difference (%) between staircase and curvilinear mesh solutions for three wavefield components across the three bathymetric profiles.

Profile	u_x (%)	u_z (%)	p (%)
Downslope	78.7	84.8	24.4
Anticline	53.0	98.0	23.4
Syncline	63.4	115.5	24.2

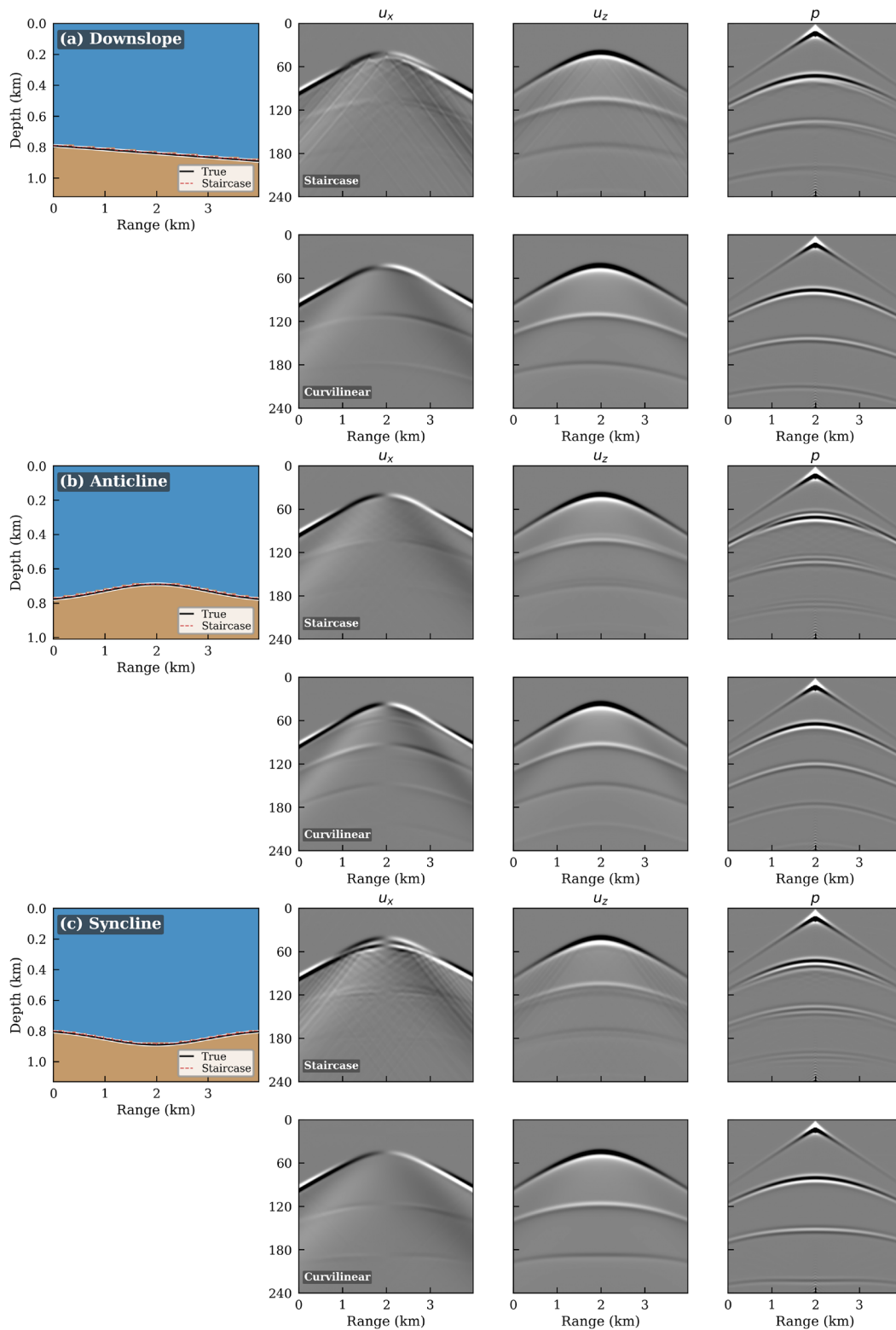


Figure 7. Staircase vs. curvilinear mesh comparison for three bathymetric profiles: (a) Downslope, (b) anticline, and (c) syncline. Left column: Velocity model with true seabed (solid black) and staircase approximation (dashed red). Remaining columns: Broadband seismograms of u_x , u_z , and p components (50–200 Hz), with staircase mesh (top) and curvilinear mesh (bottom) for each profile.

4. Discussion

The following subsections interpret the numerical results in terms of the underlying wave physics and assess the practical applicability of the proposed FEM model.

4.1. Physical interpretation of acoustic-elastic coupling effects

The numerical examples demonstrate that the acoustic-elastic coupled model differs from the acoustic-only model by a spatially averaged $|\Delta TL|$ that varies systematically with seabed type, reaching approximately 5.1 dB for the hard bottom ($V_s = 800$ m/s), 1.8 dB for the medium sediment ($V_s = 400$ m/s), and 0.8 dB for the soft sediment ($V_s = 200$ m/s), with local maxima exceeding 10 dB at interference nulls. These differences originate from two distinct physical mechanisms at the water-seabed interface.

The first mechanism is direct energy loss via P-to-S mode conversion. When a compressional wave impinges on the elastic seabed, a fraction of the incident energy is converted into shear waves in the sediment, which is captured by the coupled model but entirely absent from the acoustic-only model. The second, and often dominant, mechanism is the modification of the plane-wave reflection coefficient phase by the presence of shear rigidity ($\mu = \rho_s V_s^2$). Even when shear-wave energy conversion is modest, the phase of the reflected wave changes relative to the fluid-only case, shifting the positions and amplitudes of the multipath interference pattern in the water column and thereby producing large $|\Delta TL|$ at interference nulls.^{1,9}

The observed trend— $|\Delta TL|$ increasing from soft to hard seabed—is explained by this phase-shift mechanism rather than by mode-conversion energy loss alone. It should be noted that V_p , ρ_s , and Q also vary across the three seabed types (Table 4), so the reported $|\Delta TL|$ values reflect the combined effect of all geoacoustic parameters, not V_s in isolation. For the soft sediment ($V_p = 1,550$ m/s, $V_s = 200$ m/s), the compressional impedance contrast with water is small ($Z_s / Z_f \approx 1.4$), so the reflection coefficient differs only modestly between the elastic and fluid models, and $|\Delta TL|$ remains below 1 dB. For the hard bottom ($V_p = 3,000$ m/s, $V_s = 800$ m/s), the large impedance contrast ($Z_s / Z_f \approx 5.0$) and the emergence of a P-wave critical angle at $\theta_{cp} = \sin^{-1}(c_f / V_p) \approx 30^\circ$ cause a pronounced phase jump in the reflection coefficient near the critical angle. This phase jump shifts the positions of constructive and destructive interference, producing the largest $|\Delta TL|$ of 5.1 dB. The systematic dependence of $|\Delta TL|$ on seabed type observed in Figure 6 is thus primarily governed by the impedance contrast and the associated reflection-coefficient phase behavior, rather than by the absolute magnitude of shear-wave energy conversion.

The frequency sensitivity analysis (Section 3.4) reveals that the range-averaged $|\Delta TL|$ remains approximately constant across frequencies: 5.6 dB at 50 Hz, 6.0 dB at 100 Hz, and 6.0 dB at 200 Hz. This near-frequency-independence is primarily governed by the plane-wave reflection coefficient at the fluid–solid interface. The Zoeppritz equations yield reflection and transmission coefficients that depend on incidence angle and material properties, but not on frequency;¹ consequently, the energy partition per bottom reflection is the same at all frequencies for a given seabed type and grazing angle. In a modal waveguide, although the number of propagating modes and their angular distribution are frequency-dependent, the range-averaged $|\Delta TL|$ is approximately preserved across the 50–200 Hz band. The residual frequency dependence appears in range-resolved results: at 50 Hz, fewer propagating modes at steeper grazing angles produce $|\Delta TL|$ of 14.3 dB at 10 km; at 200 Hz, the broader modal distribution yields 8.7 dB, averaging out over the full propagation range. The operating frequency therefore does not strongly affect the mean coupling magnitude, though the spatial distribution of ΔTL varies with frequency.

Taken together, these results directly address the second limitation identified in the introduction section—the neglect of shear-wave effects in acoustic-only FEM models. The practical implications for model selection can be summarized as follows. In shallow-water continental shelf environments where V_s exceeds approximately 400 m/s, acoustic-elastic coupled modeling is essential; for medium sediments ($V_s \approx 400$ m/s), the mean $|\Delta TL|$ of 1.8 dB makes coupled modeling recommended for quantitative TL prediction; only for soft sediments ($V_s \lesssim 200$ m/s) can a simpler acoustic-only formulation be considered, and even then only when the associated errors can be tolerated. For geoacoustic inversion aimed at estimating V_s , a coupled forward model is indispensable regardless of substrate type, since even the soft-sediment $|\Delta TL|$ of 0.8 dB exceeds typical inversion sensitivity thresholds.^{9,10,15,16}

4.2. Performance and applicability of the curvilinear mesh

The case study in Section 3.6 demonstrates that the staircase mesh introduces pressure errors of 23–24% and displacement errors of 53–116% relative to the curvilinear mesh (Table 5), confirming that terrain-following meshing substantially improves waveform prediction accuracy in irregular-bathymetry environments.

Two distinct mechanisms contribute to staircase mesh errors. First, artificial diffraction at step corners generates spurious scattered waves absent from the true solution at a smooth seabed, perturbing the pressure waveform and

shifting peak and trough positions. Second, the normal direction is forced to be vertical or horizontal at each step face, yielding incorrect reflection coefficients at the wrong incidence angle.²¹

The TFI-based curvilinear mesh²⁰ eliminates staircase errors by aligning element boundaries with the physical seabed. The linear TFI mapping adopted here generates the interior mesh by algebraic interpolation between the sea surface and the seabed profile. The mapping is effective for bathymetric slopes below approximately 30°; at greater slopes, the Jacobian determinant of the isoparametric mapping can approach zero at integration points, leading to ill-conditioned stiffness matrices—an important constraint for environments with steep submarine canyons or escarpments. For such cases, elliptic PDE-based mesh smoothing²¹ or fully unstructured triangular meshes provide more robust alternatives.

4.3. Comparison with RAMGEO and methodological differences

The FEM methodology adopted here draws on established techniques from seismic wave modeling. The frequency-domain approach, the use of sparse direct solvers via LU factorization, and the coupling of acoustic and elastic domains all derive from the computational seismology tradition.^{13,23} The present work demonstrates that these tools transfer directly and effectively to the underwater acoustic problem.

The validation in Section 3.1 shows FEM-RAMGEO agreement with a running-average MAE of 1.8 dB (mean bias −0.17 dB) in a range-independent horizontally stratified environment. The residual discrepancy arises primarily from three methodological differences. First, PE is inherently a one-way propagation method that neglects backscattering, whereas FEM fully resolves two-way wave propagation—a capability well established in computational seismology, where backscattering from impedance contrasts is routinely handled.¹³ In horizontal stratification, backscattering is weak and the effect of this difference is small; however, in range-dependent environments with strong impedance contrasts or irregular topography, backscattering contributions may become significant, and FEM's two-way treatment would then provide a meaningful accuracy advantage. Second, RAMGEO solves the elastic wave equation within the one-way PE framework,⁸ which inherently restricts backscattering and wide-angle propagation, while the present FEM solves the full Navier equation without directional constraints and therefore captures all elastic wave phenomena without approximation. Third, the finite-order Padé approximation underlying the PE formulation

introduces phase errors at wide propagation angles, and these errors accumulate at higher frequencies.

PE-based methods retain superior computational efficiency—roughly 20–30 times shorter runtime than the present FEM for equivalent geometries—making them preferred for long-range (tens to hundreds of kilometers) and real-time operational applications. The present FEM model is particularly suited to short-to-medium range propagation (1–10 km), complex irregular seabed environments, and geoacoustic forward modeling, where two-way propagation, complete elastic treatment, and bathymetric conformity provide the most decisive accuracy benefits.

4.4. Connection to geoacoustic full-waveform inversion

The sensitivity of the coupled FEM forward model to seabed parameters (V_p , V_s , ρ_s , Q) demonstrated in Sections 3.3–3.5 has direct implications for geoacoustic inversion¹⁷—the underwater acoustic analogue of seismic full-waveform inversion (FWI),¹³ where an accurate forward model directly constrains inversion accuracy and convergence.^{15,16}

Both problems share a common mathematical structure in the frequency domain: the sparse system matrix is factorized once per frequency and reused for multiple source positions via back-substitution, a strategy central to the efficiency of frequency-domain seismic FWI.¹³ The present results confirm that the FEM model possesses sufficient parametric sensitivity to V_s (mean $|\Delta TL|$ ranging from 0.8 to 5.1 dB across seabed types) to serve as such a forward engine for geoacoustic inversion.

4.5. Computational efficiency and practical considerations

The FEM model developed in this study employs the actual grid dimensions of $n_x = 11114$ and $n_z = 557$ nodes at grid spacing $h = 0.9$ m for the 200 m water depth configuration at 100 Hz. Using the PARDISO direct solver on an Intel Core i9-13900K workstation (8 threads, 64 GB RAM), the total computation time per frequency—including matrix assembly, LU factorization, and back-substitution—is approximately 10 seconds per frequency (Table 3), which is roughly 20–30 times slower than RAMGEO under equivalent conditions. However, the frequency-domain FEM formulation offers important practical advantages in multi-source and multi-receiver scenarios: the LU factorization of the system matrix is computed only once per frequency and reused for all source positions via back-substitution, and the responses at all receiver locations are obtained simultaneously from a single matrix solve.¹³ Multi-

frequency analyses can be parallelized straightforwardly across frequency channels.

The cost difference reflects fundamentally different algorithmic structures: RAMGEO uses range-marching at $O(N_z z \times N_r r)$ cost, while FEM factorizes a global sparse matrix at $O(N^{3/2})$ cost via PARDISO, with 2–4 GB LU factor storage as the primary practical constraint. Scalability to larger problems can be addressed through iterative Krylov-subspace solvers (e.g., the generalized minimal residual method, GMRES, with domain-decomposition preconditioners), distributed-memory parallelization via the finite element tearing and interconnecting (FETI) method, or GPU-accelerated sparse matrix-vector products. Alternative high-order schemes such as the discontinuous Galerkin method on unstructured meshes⁴⁴ offer comparable geometrical flexibility but at higher computational cost per degree of freedom.

4.6. Modeling assumptions, limitations, and future works

The present model relies on two simplifying constitutive assumptions. First, the water column is modeled as an inviscid fluid, neglecting volume attenuation. At the frequencies considered here (50–200 Hz), the absorption coefficient of seawater is on the order of 10^{-3} dB/m,⁴⁵ corresponding to a cumulative loss of less than 10 dB over the 10 km propagation range—small compared to spreading and bottom-interaction losses. The inviscid approximation is therefore well justified for the present frequency range and propagation distances. At frequencies above 1 kHz, where seawater absorption increases quadratically, a complex bulk modulus would be required to represent volume attenuation.

Second, the seabed is modeled as an isotropic, linearly elastic solid. Real marine sediments are often transversely isotropic (VTI) due to horizontal layering and compaction-induced fabric, and exhibit viscoelastic attenuation that is poorly described by the Kelvin–Voigt model implicit in many FEM formulations. The isotropic assumption is adequate for demonstrating the methodology and quantifying the first-order effect of shear-wave conversion, but extension to VTI elasticity or constant-Q viscoelasticity⁴⁶ would be necessary for quantitative geoacoustic inversion in environments with strong anisotropy or high intrinsic attenuation.

The present study is restricted to 2D; real ocean environments include azimuth-dependent 3D effects, as demonstrated by recent 3D finite-difference models for ocean acoustic propagation.¹⁸ Horizontal refraction and 3D scattering by complex seabed topography can play significant roles in certain environments, and 2D

models cannot fully reproduce these phenomena. The present model also assumes a single seabed layer, whereas real seabeds typically consist of multiple sediment layers with distinct impedance contrasts, requiring additional interfacial coupling conditions. Sound-speed profile gradients in the water column, sea-surface roughness, and seabed roughness are additional physical effects that should be progressively incorporated in future model extensions.

The demonstrated sensitivity of the forward model to seabed parameters (Section 3.5) also suggests potential for future application as a forward engine in geoacoustic parameter estimation. Future work will extend the present FEM model to implement acoustic-elastic coupled full-waveform inversion (FWI), enabling simultaneous recovery of seabed geoacoustic parameters (V_p , V_s , ρ , Q) from field-measured acoustic propagation data. Application of the model to field data from the seas surrounding the Korean Peninsula, including the East Sea and the Yellow Sea, where complex bathymetry and variable sediment cover coexist, will provide an important test of the method's practical applicability.⁴⁷

5. Conclusion

A two-dimensional frequency-domain acoustic-elastic coupled FEM model was developed and validated through six numerical examples. The main findings are as follows.

First, in a range-independent 200 m deep Pekeris waveguide environment, the proposed FEM acoustic-elastic coupled model agrees with RAMGEO with a 100 m running-average MAE of 1.8 dB and a mean bias of -0.17 dB, confirming that the amplitude envelope of multipath interference is accurately reproduced. The approach of coupling the Helmholtz equation for the acoustic domain with the Navier equation for the elastic domain within a single weak-form framework, and imposing fluid-solid interface conditions as natural boundary conditions, is shown to be numerically effective and consistent.

Second, a mesh convergence study confirms that the $\lambda/8$ resolution rule provides adequate numerical accuracy, with the RMS TL error decreasing monotonically from 12.3 dB at $\lambda/4$ to 3.9 dB at $\lambda/12$ relative to a $\lambda/16$ reference solution.

Third, quantitative comparison of the coupled and acoustic-only models reveals that the mean $|\Delta TL|$ at approximately 101 m receiver depth is approximately 0.8 dB for soft sediment ($V_s = 200$ m/s), 1.8 dB for medium sediment ($V_s = 400$ m/s), and 5.1 dB for hard bottom ($V_s = 800$ m/s), with local maxima exceeding 10 dB at interference nulls. The strong dependence on seabed type confirms that acoustic-elastic coupled modeling is essential

for accurate TL prediction in environments with variable seabed stiffness, and that the model possesses sufficient parametric sensitivity to serve as a forward engine for geoacoustic inversion of V_S .

Fourth, frequency sensitivity analysis at 50, 100, and 200 Hz yields mean $|\Delta TL|$ values of approximately 5.6 dB, 6.0 dB, and 6.0 dB respectively between the coupled and acoustic-only FEM models, confirming that the elastic coupling effect persists consistently across the tested frequency range for the 200 m waveguide configuration.

Fifth, across three irregular seabed profiles—downslope, anticline, and syncline—the staircase mesh introduces pressure differences of 23–24% and displacement errors of 53–116% compared to the curvilinear mesh, confirming that terrain-following meshing substantially improves waveform prediction accuracy in irregular-bathymetry environments.

The proposed FEM framework overcomes three fundamental limitations of existing PE and FDM methods—the one-way propagation approximation, staircase bathymetric representation, and neglect of shear-wave effects—within a single unified formulation. Future work will extend to 3D configurations, multilayer seabeds, and geoacoustic full-waveform inversion validated against field data.

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Conflict of interest

The authors declare they have no competing interests.

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Availability of data

The data and FEM code supporting the findings of this study are available from the corresponding author (Ho Seuk Bae) upon reasonable request.

Further disclosure

AI writing tools (Claude, Anthropic) were used for literature search assistance and language polishing. All scientific content, experimental design, numerical implementation, and conclusions were conceived, directed, and verified by the authors.

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