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Topological Audit of Curriculum Architectures: Quantifying Competency Flow in Engineering Education via the Fluid Dynamics Network Model

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Abstract This study proposes the Fluid Dynamics Network (FDN) model to transition curriculum evaluation from static administrative checklists to rigorous network flow diagnostics, addressing critical issues such as excessive curricular load concentration and hidden prerequisite barriers. By mathematically formulating competency acquisition as a capacity-constrained directed flow, we combine explicit topological mapping and latent dependency inference to computationally audit three undergraduate tracks within the *Mechanical Design, Manufacturing, and Automation* discipline. Our analysis reveals three topological archetypes with distinct pedagogical implications: the sequential *Deep Chain*, where singular topological bottlenecks necessitate targeted faculty intervention; the diffuse *Implicit Grid*, requiring the structural mandate of integrative capstones; and the dual-stream *Research Spine*, balancing theoretical depth and cumulative innovation at a higher instructional resource demand. By quantifying metrics like Betweenness Centrality (C_B) to computationally identify potential structural vulnerabilities, this research translates topological properties into actionable decision-support strategies, offering a data-driven framework to optimize curriculum architectures for network resilience, curricular progression, and the coherent transmission of competency.

Keywords evaluation methodologies, engineering education, curriculum topology, Fluid Dynamics Network, competency flow

1 Introduction

The digital transformation of education is reshaping the landscape of engineering curricula, demanding a shift from static content delivery to dynamic competency modeling (Schleicher, 2024). In the context of the rapidly evolving industrial landscape, the competency demands placed on engineering graduates have shifted from the mastery of isolated technical skills to the capability for systemic integration and interdisciplinary innovation (Ma et al., 2025). Consequently, leading institutions are exploring “AI+” professional knowledge graphs to span the teaching-management-evaluation chain (Wu

et al., 2025), aiming to navigate the competing imperatives of theoretical depth, disciplinary breadth, and practical resilience. However, as interdisciplinary modules are continuously appended to traditional engineering programs to meet these modern demands, curricula experience increased structural complexity and credit inflation. The methodology used to design and evaluate these increasingly complex curriculum structures remains predominantly heuristic, lacking the quantitative rigor necessary to balance pedagogical ambition with curricular credit and workload constraints.

For decades, curriculum design has relied heavily on the experience of pedagogical experts. While accreditation frameworks such as the Washington Accord have standardized outcome-based education principles (Qadir et al., 2020), maintaining alignment across program-level outcomes and course-specific modules presents significant ontological challenges (Aminah et al., 2025). Currently, the structural auditing of curricula often operates on a "checklist" paradigm. Administrators verify whether specific courses map to required graduation attributes, generating a static compliance matrix (Wang et al., 2025). This static administrative view, however, disconnects curriculum design from the actual student learning experience. By merely confirming that content is offered, it fails to capture the *dynamics* of competency progression—how knowledge flows, accumulates, and creates pressure points across semesters (Yang et al., 2025). When these structural dependencies are not dynamically quantified, curricula often develop hidden prerequisite barriers. Such topological bottlenecks concentrate prerequisite integration demands at isolated junctures, potentially increasing the risk of progression delays and temporal desynchronization. Identifying these potential structural vulnerabilities during curriculum design is a critical challenge for modern educational administrators.

To address this methodological gap, the emerging field of Curricular Analytics has begun to apply graph theory to educational systems (Heileman et al., 2018). By conceptualizing courses as nodes and prerequisites as edges, researchers can mathematically analyze connectivity. Recent studies have utilized course-prerequisite networks to visualize curriculum structures (Aldrich, 2014; Stavrinides & Zuev, 2023) and define global measures such as breadth and flux (Zuev & Stavrinides, 2025). However, existing network models typically suffer from a binary limitation: they treat dependency as a simple unweighted link. As formalized in recent quantitative curriculum research (Liu et al., 2026), this binary abstraction is constrained by the *Equifinality Paradox*, wherein disparate learning modules appear structurally identical despite transferring orthogonal competencies. An unweighted link assumes that all prerequisites impose an equivalent prerequisite demand, ignoring the reality that specialized courses (e.g., advanced mechatronics) rely on complex, multi-layered semantic couplings from their foundational precursors (Tang et al., 2025). Treating all curricular connections as uniform obscures the true pedagogical hierarchy, limiting the data available to administrators for allocating high-value teaching resources or designing effective academic interventions (Simon de Blas et al., 2021).

This study proposes a methodological shift from static structural analysis to dynamic flow auditing. Within the scope of this research, the term *dynamic* pertains specifically to the mechanics of network flow theory—computing the steady-state load distribution across the curriculum as an *a priori* structural diagnostic—rather than evaluating time-variant student outcomes. We introduce the *Fluid Dynamics Network (FDN)* model, a computational framework that treats competency acquisition as a directed flow process governed by topological constraints. Drawing upon formal network flow theory (Borgatti, 2005)

and principles of transport phenomena (Vaidya, 2020), we formulate knowledge transmission as a system regulated by physical variables: steady-state flow load representing structural demand, topological impedance quantifying dependency resistance, and channel capacity determined by semantic alignment. By defining a fuzzy vectorization method to compute the competency bandwidth between courses, we construct weighted directed graphs that delineate the load-bearing conduits of the curriculum. Within this framework, fluid-transport terms—such as flow load, channel capacity, and impedance—serve as interpretive analogies for exact graph-theoretic quantities (edge weights, reciprocal distances, and path centralities) rather than literal models of student cognition. Consequently, this model translates abstract mathematical graphs into structural diagnostics, enabling educators to computationally isolate specific nodes where theoretical demands converge to create potential bottlenecks.

To demonstrate the diagnostic power of this model, we perform a comparative topological audit on three distinct undergraduate tracks within the *Mechanical Design, Manufacturing, and Automation* discipline at a single university: a specialized "New Engineering" track, an elite research-oriented experimental class, and a standard generalist track. Within this institutional context, these three cases serve as illustrative topological archetypes, representing typical structural choices available to modern educators—linear efficiency, parallel innovation, and distributed flexibility. Through the calculation of metrics such as Betweenness Centrality (C_{B}) and the visualization of latent dependency flows, this research aims to bridge the gap between algorithmic network analysis and practical curriculum administration. By quantifying the trade-offs between transmission efficiency and structural resilience, we provide educational policymakers with a decision-support framework to transform curriculum reform from a heuristic, ad hoc practice into a systematic topological engineering discipline.

2 Methodology: The FDN Framework

To quantify the structural efficiency of engineering curricula, this study introduces a computational framework termed the FDN. Diverging from traditional accreditation audits that treat curriculum matrices as static checklists, the FDN protocol reconceptualizes the curriculum as a directed, weighted complex network, denoted as the tuple $G=(V,E,W)$. To bridge abstract network theory with practical curriculum administration, a formal mapping of these topological constructs is established. Within this framework, the vertex set $V=\{v_1,v_2,\ldots,v_n\}$ represents the *nodes*, each designating a discrete learning module sequenced across eight semesters. The edge set $E\subseteq V\times V$ represents the *edges*, denoting the directional prerequisite dependencies that logically link a foundational precursor to its subsequent application. Crucially, $W\in[0,1]^{n\times n}$ serves as the adjacency weight matrix, where each assigned *weight* w_{ij} signifies the structural channel capacity, quantifying the effective competency bandwidth required to transition from course v_i to v_j. Rather than treating a prerequisite simply as a binary administrative hurdle, the FDN models it as a capacity-constrained knowledge-transmission channel. This conceptual transformation enables a shift in the evaluation paradigm from a static analysis of content coverage to a dynamic audit of flow efficiency. The computational workflow is illustrated in Figure 1.

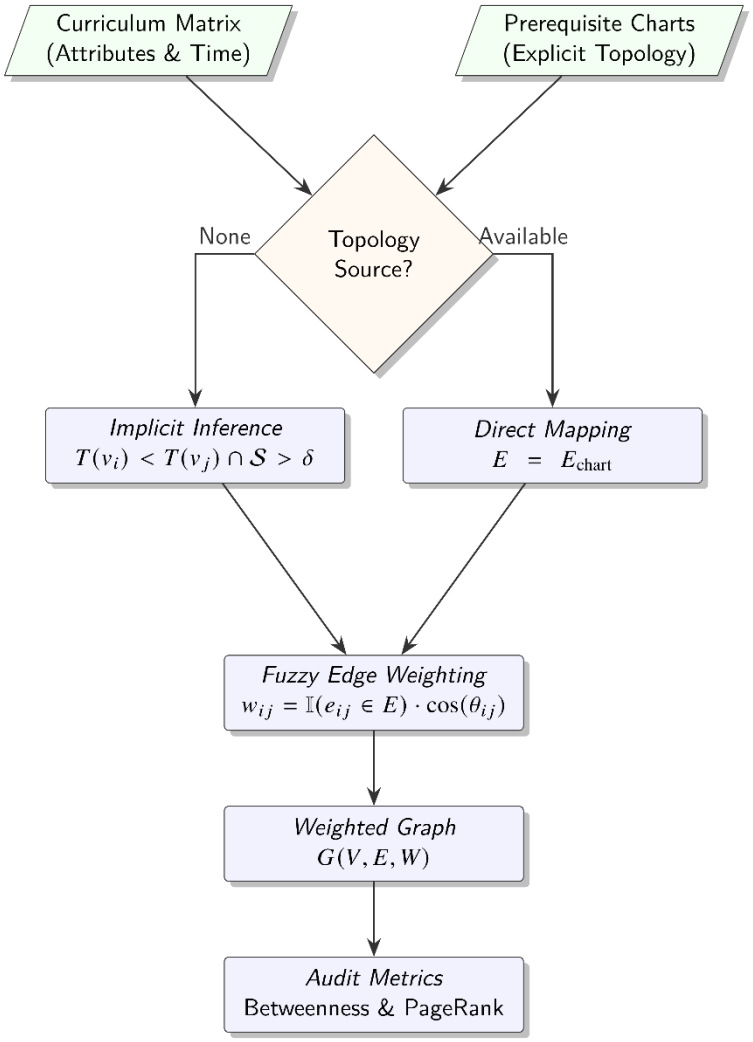


Figure 1 The computational workflow of the FDN Audit Protocol. The framework synthesizes explicit charts and inferred dependencies into a unified weighted graph for topological analysis.

2.1 Construction of the Topological Skeleton

The primary challenge in establishing a unified audit framework lies in the heterogeneity of curriculum documentation across different engineering tracks. To construct the topological skeleton $G(V, E)$, we employ a dual-path strategy that adapts to the granularity of available data, as illustrated in the methodology framework (Figure 1).

For specialized tracks such as the *Advanced Electronic Manufacturing* and the *Elite Qiming Class*, the curriculum syllabi include explicit prerequisite charts codified within institutional training schemes. In these instances, we perform a *Direct Topology Mapping*, where the edge set E_{explicit} is constructed to be isomorphic to the official flowcharts. This approach ensures that the resulting graph preserves the intended pedagogical logic without introducing algorithmic artifacts, aligning with established practices in course-prerequisite network analysis (Slim et al., 2014; Molontay et al., 2020).

Conversely, for standard tracks lacking explicit prerequisite diagrams, such as the *Standard Intelligent Manufacturing* track, we introduce a *Latent Dependency Inference* algorithm. Grounded in the constructivist principle of cumulative learning, this algorithm postulates a latent structural dependency between courses. This approach draws inspiration from both classical data-driven

prerequisite mining (Vuong et al., 2011) and recent advances in context-aware relation learning using graph neural networks (Chu et al., 2017; Sun et al., 2022). To establish the analytical variables prior to formal algebraic derivations, each learning module is conceptualized as a multidimensional competency vector, denoted as c_i, which maps its pedagogical contribution across formalized graduation attributes. Concurrently, the semantic similarity function, denoted as S, quantifies the geometric alignment between any two such competency vectors. Formally, we define a latent edge set E_{implicit} where a directed link (v_i, v_j) exists if and only if two conditions are simultaneously satisfied:

$$T(v_i) < T(v_j) \quad \text{and} \quad \mathrm{S}(c_i, c_j) > \delta \tag{1}$$

Here, $T(\cdot)$ denotes the semester index, ensuring temporal causality. As established above, the function $\mathrm{S}(\cdot,\cdot)$ computes the semantic intersection between the prerequisite and the target module. By filtering for substantive intersections—similar to multi-criteria approaches in educational knowledge graphs (Roy et al., 2019; Alatrash et al., 2025)—this inference mechanism reconstructs a structural backbone. Pedagogically, this reconstructed backbone uncovers the *hidden curriculum*—the implicit prerequisite dependencies where advanced coursework assumes prior mastery of specific foundational skills. By formalizing these latent links, administrators can pinpoint structural learning gaps that are not explicitly documented in official syllabi, thereby providing a quantitative basis for targeted curriculum integration.

2.2 Quantification of Competency Flow via Fuzzy Vectorization

With the topological skeleton E established, the subsequent phase involves quantifying the interaction bandwidth between connected learning modules. This requires transforming the qualitative syllabi data into a computable algebraic structure. We define a mapping from the linguistic evaluation space to a high-dimensional Euclidean space, denoted as the *Fuzzy Vectorization* process.

In standard engineering accreditation documents, the support intensity of a course for a graduation attribute is typically categorized as High (H), Medium (M), or Low (L). Recognizing that such linguistic variables inherently contain ambiguity and are best modeled using fuzzy set theory (Zadeh, 1965), we map these labels to triangular fuzzy numbers in the space Γ. Following established fuzzy assessment protocols in engineering education (Baba et al., 2012; Aziz et al., 2025), these fuzzy numbers are defuzzified into the crisp scalar weights μ_{H}, μ_{M}, and μ_{L} using the Graded Mean Integration Representation (GMIR) method (Kusumadewi et al., 2024). To render this pre-processing mechanism transparent, the linguistic-to-numeric mapping utilizes a parameterization consistent with the domain-aware fuzzy-vector framework established by Liu et al. (2026) and supported by fuzzy evaluation protocols (Kusumadewi et al., 2024; Caiado et al., 2025). Formally, if a linguistic variable is modeled as a triangular fuzzy number $\tilde{A} = (l, m, u)$, its GMIR defuzzified crisp scalar x is calculated as $x = \dfrac{l + 4m + u}{6}$. For instance, consider an operational example involving two hypothetical modules: Course A (*Theoretical Mechanics*) and Course B (*Robotics*). If Course A provides *High* support, parameterized as $\tilde{A}_{\mathrm{H}} = (0.7, 0.9, 1.0)$, it is scalarized into a crisp value of $(0.7 + 3.6 + 1.0)/6 \approx 0.88$. Similarly, *Medium* $\tilde{A}_{\mathrm{M}} = (0.3, 0.5, 0.7)$ maps to 0.50, and *Low* $\tilde{A}_{\mathrm{L}} = (0.0, 0.1, 0.3)$ yields 0.12.

Consequently, for a curriculum evaluated across d high-resolution, domain-specific sub-indicators rather than broad macro-level attributes, each course v_i is encapsulated as a multidimensional competency feature vector $c_i \in {}^{d}$. This high-dimensional granularity mathematically preserves the orthogonality between disparate knowledge domains, ensuring that this initial GMIR defuzzification remains decoupled from subsequent topological analytics. The k-th component of c_i represents the scalarized intensity of the course's support for the k-th attribute.

The adjacency weight matrix W, which governs the flow dynamics of the network, is mathematically constructed as the fusion of the topological constraints and the semantic similarity. To resolve potential ambiguities regarding algorithmic double-counting, this formulation operates via a two-stage sequential mechanism. Initially, the spatial connectivity of the conduits is defined, either via explicit administrative charts or via threshold inference where semantic similarity acts as a topological gate. Subsequently, a multiplicative scaling operation is applied to compute the transmission capacity accommodated within those established conduits. We formalize this as the Hadamard product of the structural connectivity and the content alignment. Specifically, the weight w_{ij} corresponding to the edge from v_i to v_j is defined by:

$$w_{ij} = \mathbb{I}\left((v_i, v_j) \in E\right) \cdot \frac{c_i \cdot c_j}{\|c_i\| \|c_j\|} \tag{2}$$

Here, $\mathbb{I}(\cdot)$ serves as the topological indicator function, taking the value of 1 if the edge (v_i, v_j) exists in the skeleton E derived in the previous step, and 0 otherwise. The second term represents the cosine similarity between the competency vectors, quantifying the *semantic alignment* between the precursor and the successor. Continuing the previous operational example, if the scalarized competency vectors for Course A and Course B across an 11-dimensional attribute space yield c_{A} and c_{B}, their semantic similarity S is algebraically computed via the dot product normalized by their vector magnitudes. If the computed cosine similarity registers at 0.87, it surpasses the established semantic threshold ($\delta = 0.60$), thereby validating the substantive competency transfer and assigning $w_{\mathrm{AB}} = 0.87$. Conversely, computing the cosine similarity between two geometrically divergent vectors yields a sub-critical scalar value that attenuates the transmission capacity in explicit networks or fails to satisfy the threshold for a structural connection in inferred networks, thereby preventing the algorithmic conflation of distinct engineering disciplines. In practical pedagogical terms, this scaling operation ensures that a high-bandwidth connection is recognized if the foundational course provides the specific competencies required by the advanced course.

Furthermore, the topological properties of this weight matrix require specific mathematical clarification. The curriculum architecture constitutes a directed acyclic graph governed by temporal causality, ensuring that knowledge transmission exhibits a unidirectional gradient. Consequently, the matrix W is asymmetric ($W \neq W^{\mathrm{T}}$). If a foundational module v_i serves as a prerequisite for an advanced module v_j, competency flows directionally ($w_{ij} > 0$). Because temporal causality prevents the advanced module from chronologically preceding its foundation, the reverse flow is prohibited ($w_{ji} = 0$). The resulting matrix W thus creates a sparse, weighted representation of the steady-state flow load,

where flow is permitted only through structurally valid and semantically compatible conduits, as visualized in Figure 2.

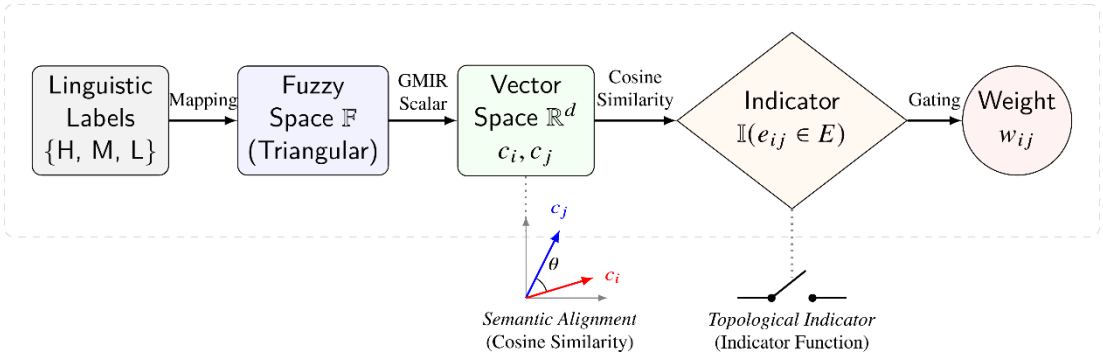


Figure 2 The computational mechanism of Fuzzy Edge Weighting.

The workflow illustrates the sequential transformation of linguistic syllabus evaluations into high-dimensional competency vectors via GMIR. The lower panels detail the two-stage logic: *Semantic Alignment* (left) calculates content similarity via vector angles, while the *Topological Indicator* (right) functions as a binary discriminator, ensuring that continuous transmission capacities are exclusively allocated to structurally valid conduits.

2.3 Network Metrics for Structural Diagnosis

With the curriculum mapped as a weighted directed graph G, the final phase of the protocol involves a computational audit to diagnose the structural health of the knowledge pipeline. We deploy two complementary spectral graph metrics to capture distinct dimensions of topological efficiency: flow congestion and structural authority.

To identify vulnerability points where the knowledge flow is most susceptible to congestion, we first calculate the *Betweenness Centrality* (C_B). In the context of a curriculum network, a course with high Betweenness Centrality functions as a *critical gateway* or bridge if it lies on the shortest dependency paths connecting foundational sciences to engineering capstones. Pedagogically, this metric flags a potential structural vulnerability: if prerequisite mastery is not adequately consolidated at this node, downstream progression toward advanced senior applications may be disproportionately impeded. Since our edge weights w_{ij} represent competency bandwidth, we formally define the topological distance—or *impedance*—between connected nodes as the reciprocal of their weight, $d_{ij} = 1/w_{ij}$. To ensure comparability across curriculum networks of varying sizes (N), we employ the normalized definition:

$$C_B(v_i) = \frac{1}{(N-1)(N-2)} \sum_{s \neq v_i \neq t} \frac{\sigma_{st}(v_i)}{\sigma_{st}} \tag{3}$$

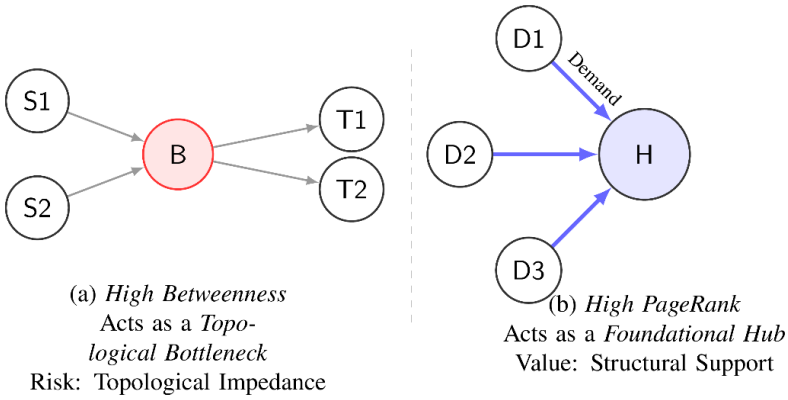
Here, a shortest impedance path is defined as the trajectory of least cumulative resistance from a foundational source module v_s to a specialized target module v_t, computationally resolved utilizing optimization algorithms for directed weighted graphs. Within this context, σ_{st} denotes the total count of these optimal pathways across the network, and $\sigma_{st}(v_i)$ enumerates the subset of these paths traversing node v_i. The normalization factor allows us to objectively compare the structural stress between the standardized tracks ($N = 55$) and specialized tracks ($N = 51$). Nodes exhibiting high

normalized C_{B} values function as topological load concentrators within the network. A pedagogical failure at these loci indicates that the steady-state competency demand exceeds localized transmission capacity, creating structural congestion that disrupts the connectivity between upstream prerequisites and downstream applications.

Complementarily, to assess the recursive importance of each learning module, we employ a *Weighted Fuzzy PageRank* algorithm. While standard degree centrality merely counts direct connections, PageRank posits that a course's authority is derived from the importance of the nodes connected to it. Pedagogically, this metric identifies *hidden foundational hubs*—courses that carry a relatively low credit weight but possess high systemic importance because they are heavily demanded by advanced specialized modules. Identifying these hubs signals to administrators where instructional quality and teaching faculty must be prioritized. To compute this reverse demand flow, we operate on the transpose of the weight matrix, W^{T}. Because the initial adjacency matrix W is asymmetric due to the unidirectional temporal gradient of prerequisite learning, calculating its transpose functions as an algebraic transformation to invert the directional flow vectors. This topological inversion permits the algorithm to simulate a reverse demand propagation, computing how structural dependencies accumulate backward from terminal capstones to foundational nodes. The rank $PR(v_i)$ is computed iteratively via the power method:

$$PR(v_i) = \frac{1-\alpha}{N} + \alpha \sum_{v_j \in \mathrm{N}_{\mathrm{in}}(v_i)} \frac{w_{ij} \cdot PR(v_j)}{\sum_{v_k \in \mathrm{N}_{\mathrm{out}}(v_j)} w_{kj}} \tag{4}$$

where N is the total number of courses and α is the damping factor set to 0.85. To formalize the directionality of the recursive calculations, the neighborhood variables are defined as follows: $\mathrm{N}_{\mathrm{in}}(v_i)$ denotes the set of advanced courses that structurally depend on v_i in the original curriculum (thus functioning as incoming demand sources in this reverse topology), whereas $\mathrm{N}_{\mathrm{out}}(v_j)$ represents the subset of foundational prerequisites demanded by course v_j. This reverse-demand metric thus structurally isolates the courses that sustain the core curriculum. The conceptual distinction between these two topological roles (C_{B} and PageRank) is visualized in Figure 3. These two metrics capture distinct structural roles and therefore suggest different priorities for curriculum review. Specifically, C_{B} measures midstream pathway concentration, identifying bridge courses where multiple prerequisite chains converge before diverging into specialized modules; for such nodes, curriculum review may focus on pathway decompression, such as aligning prerequisite syllabi, modularizing dense course content, or providing supplementary bridge tutorials. Conversely, Weighted Fuzzy PageRank (PR) measures upstream foundational authority via reverse demand propagation, identifying root courses whose concepts are repeatedly required by downstream modules even if they do not lie on a narrow midstream constriction; for high-PR nodes, administrative attention may prioritize foundational quality assurance, such as experienced faculty assignment and standardized core assessments.



(a) *High Betweenness* Acts as a *Topological Bottleneck* Risk: Topological Impedance

(b) *High PageRank* Acts as a *Foundational Hub* Value: Structural Support

Figure 3 Conceptual visualization of the topological metrics. (a) A node with high *Betweenness Centrality* (red) lies on critical paths, structurally concentrating the steady-state flow load. (b) A node with high *PageRank* (blue) is identified by the reverse accumulation of structural demand from multiple downstream courses, signifying its status as a foundational hub.

The definitions of the 11 graduation-attribute sub-indicators, the complete course competency vectors, the prerequisite link inventories, step-by-step numerical calculations—including the distinction between graph-based shortest impedance paths and jointly compulsory prerequisites—and the step-by-step algorithmic procedure of the FDN framework (Algorithm S1) are provided in the Electronic Supplementary Material (Sections S1–S4).

3 Data Sources and Experimental Setup

To empirically evaluate the FDN audit protocol, this study utilizes a dataset derived from the *2025 Undergraduate Talent Training Scheme* of the School of Mechanical Science and Engineering at the evaluated institution. *Mechanical Design, Manufacturing, and Automation* constitutes the overarching disciplinary major. Within this unified discipline, the institution has established three cultivation pathways to address varying industrial demands. Rather than using random sampling, we purposively selected these three distinct undergraduate tracks within a single institution to serve as comparative case studies. While this single-institution, single-discipline design limits direct statistical generalization, these programs function as *topological archetypes*, spanning the structural spectrum of contemporary engineering education reform—ranging from targeted specialization to research-driven architectures. By selecting these specific tracks under a unified institutional credit framework, the computational audit evaluates implemented pedagogical policies rather than theoretical network constructs.

3.1 Selection of Curriculum Archetypes

The comparative analysis is structured around three representative curriculum models, chosen to reflect the divergent pedagogical frameworks currently utilized in the engineering domain. The structural characteristics and data modalities of these archetypes are summarized in Table 1.

Table 1 Summary of Comparative Topological Archetypes and Structural Logics

Case	***Program track***	***Topological archetype***	***Structural logic***	***Data source***
A	Advanced Electronic Manufacturing	*The Deep Chain*	Concentrated Sequential	Explicit Chart
B	Qiming Elite Class	*The Research Spine*	Dual-Stream Integration	Explicit Chart
C	Standard Intelligent Manufacturing	*The Implicit Grid*	Diffuse, Decentralized	Inferred ($\delta = 0.60$)

Archetype I: The Deep Chain (Case A). Represented by the *Advanced Electronic Manufacturing* track, this program reflects a response to specific industrial needs, such as semiconductor manufacturing. This track is structured to produce specialized engineers for the equipment industry, requiring a graduate profile built upon a sequential theoretical foundation in physics and precision engineering. The curriculum is characterized by a density of specialized courses arranged in a sequential order. Its topology is explicitly defined by official prerequisite charts, creating a *Deep Chain* structure designed to guide competency accumulation for a specific domain. From an administrative standpoint, this archetype embodies a pedagogical strategy where student progression follows a defined sequence.

Archetype II: The Research Spine (Case B). Represented by the *Intelligent Manufacturing Qiming Elite Class*, this program serves as a research-oriented cohort. It aims to cultivate future academic researchers and development leaders, featuring a graduate profile that emphasizes project-based learning running parallel to standard didactics. While sharing the same disciplinary root as the standard track, it incorporates a series of project-based research courses parallel to theoretical instruction. Its topology is explicitly mapped, featuring a *Research Spine* intended to integrate continuous research training. This archetype tests the structural viability of running resource-intensive, student-directed research pathways alongside standard instruction.

Archetype III: The Implicit Grid (Case C). Represented by the *Standard Intelligent Manufacturing* track, this program serves as the baseline model, reflecting the broad-spectrum engineering curriculum aiming to prepare versatile generalists adaptable to diverse roles within modern smart factories. Unlike the specialized or elite tracks, it lacks a designated prerequisite flowchart in the official documentation. Consequently, its topology represents an *Implicit Grid*, which requires reconstruction via the Latent Dependency Inference algorithm to determine its structural properties. This model serves as a baseline for decentralized curriculum management, prioritizing elective flexibility over sequential competency integration.

3.2 Calibration of Model Parameters

To ensure the reproducibility of the computational audit and to maintain consistency across the heterogeneous datasets, specific algorithmic parameters were systematically calibrated prior to the network construction.

First, in the *Fuzzy Vectorization* phase, we addressed the quantification of the qualitative syllabus identifiers—*High*, *Medium*, and *Low*. To preserve the gradient of pedagogical support while enabling vector computation, these linguistic variables were mapped to triangular fuzzy numbers and subsequently defuzzified using the GMIR method. To ensure empirical validity and circumvent arbitrary assignments, we adopted the defuzzified calibration values of $\mu_{\mathrm{H}} = 0.88$, $\mu_{\mathrm{M}} = 0.50$, and $\mu_{\mathrm{L}} = 0.12$. These specific parameters are explicitly anchored to the domain-aware fuzzy-vector representation formalized by Liu et al. (2026) and the GMIR defuzzification protocol established by Kusumadewi et al. (2024), thereby establishing a validated theoretical baseline for quantifying pedagogical semantics. This non-linear spacing serves as a soft-thresholding mechanism, intentionally penalizing weak correlations to ensure that only courses with substantial competency contributions exert significant influence on the network topology.

Second, for the *Latent Dependency Inference* applied to Case C, the similarity threshold was set to $\delta = 0.60$. This critical value serves as a semantic noise filter. Mathematically, it implies that a latent prerequisite relationship is inferred only if the two courses share a competency intersection equivalent to at least two *Medium* attributes or one *High* attribute plus minor supports. This constraint effectively prunes spurious connections that might arise from generic description overlaps (e.g., two fundamentally unrelated courses both claiming minor support for generic *communication* or *teamwork* outcomes), thereby reconstructing a topological backbone that reflects genuine domain-specific knowledge dependency. In practice, the selection of δ follows a reproducible two-step principle: first, establishing an algebraic lower bound based on the GMIR scalar encoding so that edges require substantive multi-attribute or primary-attribute alignment rather than isolated minor overlaps; and second, performing sensitivity bracketing across the candidate interval ($\delta \in [0.50, 0.70]$, as reported in Section 4.4) to avoid both over-connected generic links at lower thresholds ($\delta = 0.5$) and premature network fragmentation at overly strict thresholds ($\delta = 0.7$). Because δ is an analyst-specified hyperparameter that directly affects the edge density of inferred graphs, administrators should account for this parameter dependence and conduct local sensitivity checks when applying latent inference to curricula without explicit prerequisite charts.

Finally, for the *Weighted Fuzzy PageRank* calculation, the damping factor was standardized at $\alpha = 0.85$. In the context of curriculum dynamics, this parameter models the probability of *pedagogical continuity*—positing that 85% of a student's competency accumulation flows strictly along the designed prerequisite chain, while allowing a 15% margin for non-linear knowledge synthesis equivalent to a network random walk. While these baseline parameters provide a robust theoretical foundation, a dedicated sensitivity analysis is integrated in the subsequent sections to empirically evaluate the structural stability of the network across varying computational configurations. The summary of these experimental settings is presented in Table 2.

Table 2: Configuration of Computational Parameters for the FDN Audit

Parameter Domain	***Symbol***	***Calibrated value & Mathematical rationale***
Fuzzy Weights	μ_{H}	0.88 (Primary driver, GMIR scalar)
	μ_{M}	0.50 (Auxiliary driver, GMIR scalar)
	μ_{L}	0.12 (Sub-critical support, GMIR scalar)
Inference Logic	δ	0.60 (Threshold for semantic validity)
Network Dynamics	α	0.85 (Damping factor for reverse demand)

4 Comparative Analysis of Topological Archetypes

To validate the diagnostic capability of the FDN model, this study applies the topological audit protocol to three distinct undergraduate tracks within the *Mechanical Design, Manufacturing, and Automation* discipline. Although these tracks share a common disciplinary root, the computational analysis reveals that they have evolved into fundamentally distinct structural archetypes.

By mapping the competency flow (F_{ij}) and calculating node centrality (C_{B}), we identify a distinct spectrum of topological complexity. Specifically, the topological skeleton defining the valid edges is derived either from explicit administrative mandates or inferred via a semantic gating). Once these

structural conduits are established, the visual properties of the directed edges—namely, their thickness and color-coding—are scaled to mathematically reflect the calculated magnitude of the continuous competency bandwidth (w_{ij}). The analysis progresses from the highly compressed, linear structure of the *Advanced Electronic Manufacturing* track (Case A: *The Deep Chain*), through the parallel, dual-drive architecture of the *Qiming Experimental Class* (Case B: *The Research Spine*), to the decentralized, inferred network of the *Standard Intelligent Manufacturing* track (Case C: *The Implicit Grid*). This comparative framework demonstrates how curriculum topology directly governs the efficiency, resilience, and specialization modes of competency transmission, thereby providing a structural reference for evaluating potential progression bottlenecks and instructional resource allocation. Furthermore, to ensure the empirical robustness of these comparative findings, this section concludes with a rigorous sensitivity analysis, validating that the identified topological characteristics remain consistently stable across varying computational parameters.

4.1 Case A: The Deep Chain and Serial Compression

The topological audit of the *Advanced Electronic Manufacturing* track reveals a structure defined by pronounced seriality and concentrated load transmission. As visualized in the *Layered Flow Topology* (Figure 4), the curriculum is organized into strict temporal layers ($S_1 - S_8$), functioning effectively as a capacity-constrained flow system characterized by topological convergence and divergence where competency is compressed, processed, and reconstructed. To interpret the underlying physics of this system, the visual mapping functions as a topological rendering of the steady-state flow load. The directed edges do not merely illustrate nominal administrative prerequisites; rather, their varying thicknesses algebraically reflect the computed magnitude of the competency bandwidth (w_{ij}). Concurrently, a specific color-coding protocol is employed: red indicates topological load concentrators characterized by peak Betweenness Centrality (C_{B}), orange represents functional hubs, while cyan denotes the foundational support layer.

The computational audit identifies two distinct primary conduits of competency flow. The first is the *Theoretical Compression Pathway*, which manifests as the network's critical path. As shown by the converging cyan streamlines, the node *Mechanical Design Theory II* (ID 32) emerges as the network's primary topological bottleneck ($C_{\mathrm{B}} \approx 0.034$). It bears the structural load of integrating all upstream mechanics and mathematics modules. Visually represented as a large red node, ID 32 functions as a critical structural bottleneck—a high-impedance node where steady-state competency demand approaches or exceeds localized transmission capacity. This dictates that foundational theory must be successfully integrated before passing to the application stage. From an educational management perspective, quantitatively identifying this high-impedance node provides administrators with a data-driven focal point for deploying supplemental instruction and early-warning academic monitoring.

Parallel to this theoretical core, the data reveal a sustained *Practice Chain*, highlighted by high-capacity orange streamlines. This stream originates from *Mechanical Design I* (ID 12), flows through *Engineering Training I (ID 23)*, evolves into *Intelligent Manufacturing Equipment I (ID 35)* and *Professional Social Practice (ID 36)*, and finally converges in the *Production Internship* (ID 44). This

distinct pathway ensures that practical skills are built continuously alongside theoretical depth, creating a dual-stream architecture that balances abstraction with application.

Following the compression at Semester 4, the flow channels into the *Divergence Gateway—Micro-Electronic Manufacturing Process (ID 40)*. Unlike the upstream bottleneck, this node is responsible for divergence, distributing the accumulated steady-state flow load into the parallel specialized clusters of Semester 6. In conclusion, Case A exhibits a highly concentrated, low-redundancy topology. The system relies heavily on the critical path ($32 \rightarrow 40$) to bridge the gap between basic sciences and advanced applications. While efficiently structured, this *Deep Chain* architecture requires rigorous quality control at ID 32, as insufficient prerequisite consolidation at this juncture may create downstream structural bottlenecks along the sequential chain.

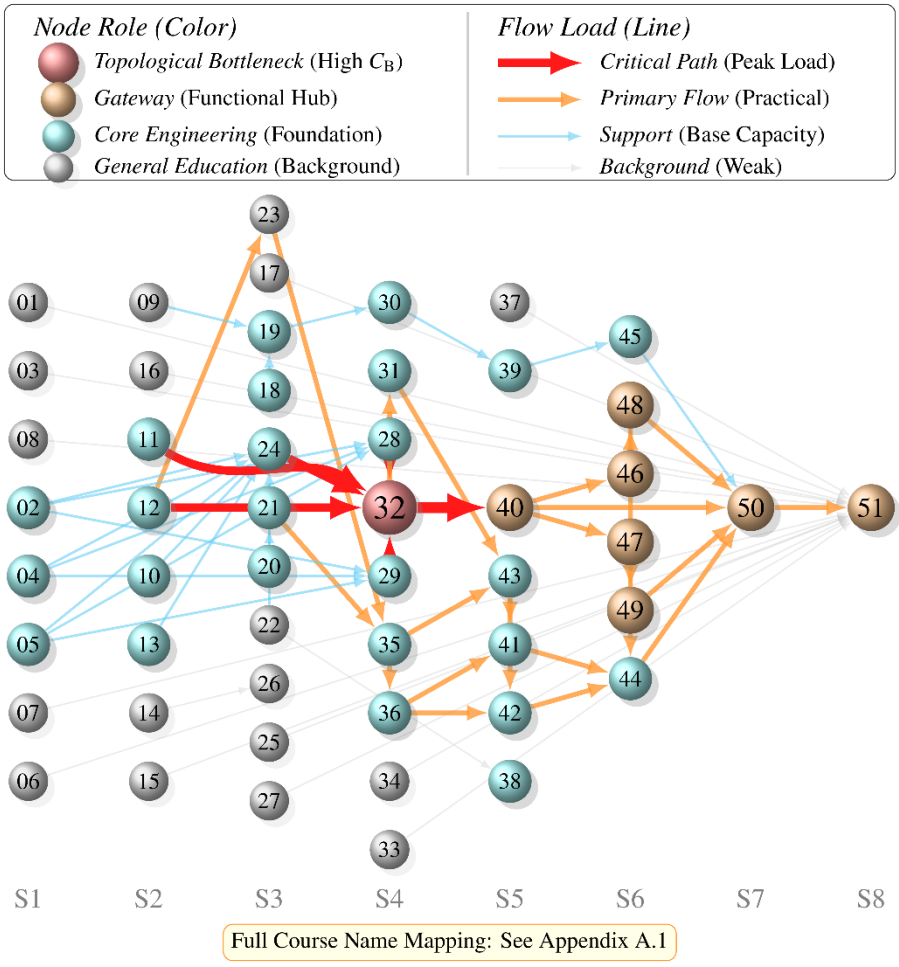


Figure 4 Layered Flow Topology of Case A (Advanced Electronic Manufacturing).

The visualization reveals the "Deep Chain" architecture. Nodes are categorized by topological role: Red indicates structural bottlenecks with peak Betweenness Centrality (C_B), Orange denotes functional gateways/hubs, Cyan represents core engineering foundations, and Gray indicates general education. Directed edges denote computed steady-state flow load, with thickness reflecting the magnitude of competency bandwidth: Red marks the primary critical path, Orange highlights the parallel practical conduit, and Cyan shows foundational support flows. Background dependencies are rendered in Gray. Note the distinct parallel "Practice Chain" ($12 \rightarrow 23 \rightarrow 35 \rightarrow 36 \rightarrow 44$). Full course mapping is provided in Appendix A.1.

4.2 Case B: The Research Spine and Parallel Integration

The topological audit of the *Intelligent Manufacturing* track (Qiming Class) reveals a fundamentally different structural logic compared to the standard track, characterized here as the *Research Spine* architecture. As illustrated in the *Layered Flow Topology* (Figure 5), the curriculum diverges from the highly concentrated load model in favor of a continuous, parallel competency distribution network. Consistent with the established visual protocol, the mapped edges denote computed steady-state flow distributions rather than static administrative prerequisites. The visualization uses a distinct color palette to highlight this functional shift: purple represents the central research hub and training sequence, orange denotes the mechatronics and system control stream, while cyan remains the foundational support layer.

The defining topological feature of this system is the central *Research Spine*. Unlike traditional curricula where research is often a terminal activity, this structure establishes a continuous longitudinal conduit spanning from Semester 3 to Semester 6 ($S_3 - S_6$). The node *Research Practice* (ID 23) acts as the primary topological hub ($C_B \approx 0.028$). It functions as a high-capacity load distributor, integrating inputs from basic engineering sciences and distributing the steady-state demand into a sequential training series consisting of *Research Training I, II, and III* (IDs 34, 40, 49). Visually depicted by the prominent purple streamlines representing substantial competency bandwidth, this pathway ensures that research capability is treated as a cumulative variable rather than a discrete event. Pedagogically, this longitudinal integration distributes research-training demands incrementally across four semesters rather than concentrating them solely within terminal capstone projects.

Complementing this research core is the *System Integration Stream*. Highlighted by orange streamlines, this pathway builds a parallel competency in control and systems engineering. Contrary to the singular bottleneck observed in Case A, this stream exhibits a gradient centrality profile. As the structural load diverges toward specialized applications, the topological impedance attenuates from the foundational *Circuit Theory* (ID 20, $C_B \approx 0.022$) through the intermediate system integration hub *Decision & Design* (ID 37, $C_B \approx 0.017$), ultimately distributing the demand down to the Mechatronics modules (e.g., IDs 31 and 50). This distributed load profile indicates a higher structural resilience, as the competency transmission does not rely on a singular topological bottleneck.

The topological interaction between the purple research spine and the orange system stream suggests a *Dual-Drive* mechanism. The research spine provides the conceptual framework for innovation, while the system stream provides the implementation capability. By distributing competency load across these parallel high-bandwidth channels, the Qiming Class architecture effectively mitigates the risk of structural congestion (0.028 vs 0.034 in Case A), achieving a higher network resilience. Administratively, while this hybrid topology reduces the vulnerability of strict bottlenecks, it necessitates a substantially higher investment of instructional resources and faculty coordination to maintain these parallel knowledge streams.

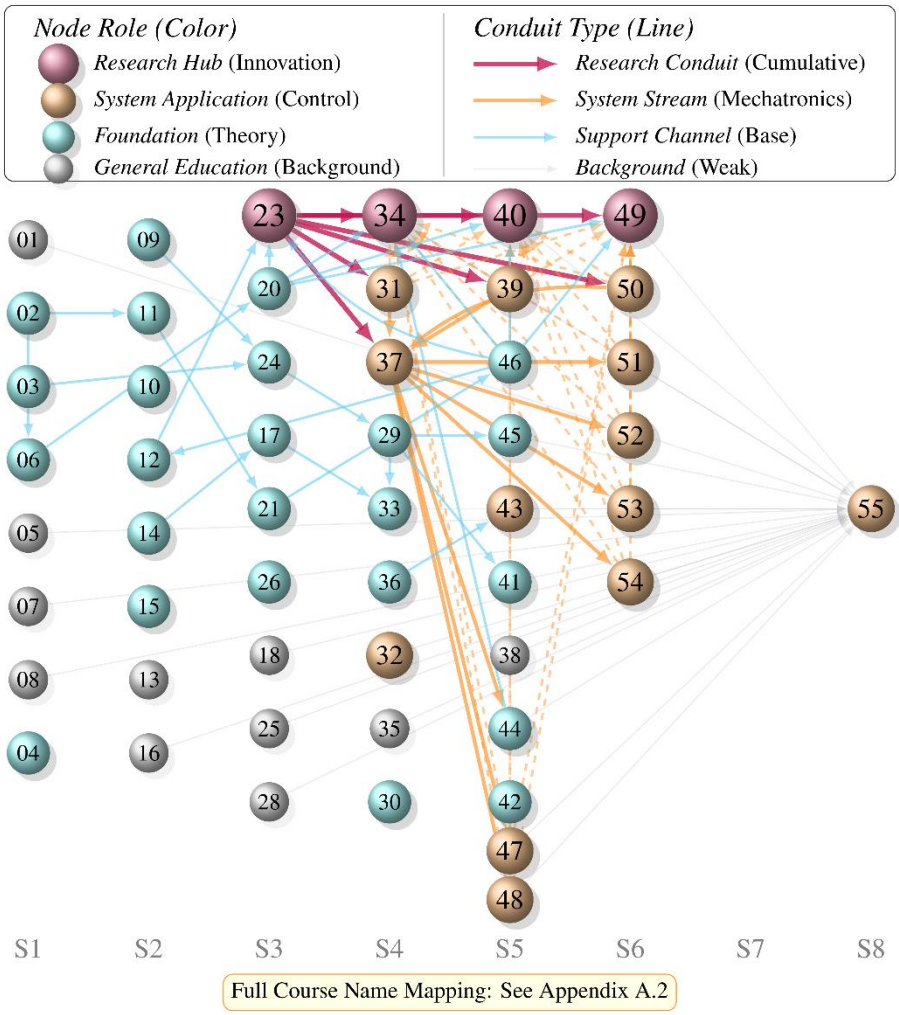


Figure 5 Layered Flow Topology of Case B (Intelligent Manufacturing Qiming Class).

The visualization highlights the "Research Spine" architecture. Nodes are categorized by functional role: Purple signifies high-capacity research hubs, Orange denotes system control applications, Cyan represents theoretical foundations, and Gray indicates general education. Directed edges denote computed steady-state flow load: Purple identifies the longitudinal research conduit, Orange highlights the parallel mechatronics stream, and Cyan maps foundational support pathways. The topological signature is defined by the central, continuous "Research Spine" ($23 \rightarrow 34 \rightarrow 40 \rightarrow 49$), functioning as a parallel load-distribution mechanism unique to this elite track. Full mapping is provided in Appendix A.2.

4.3 Case C: The Implicit Grid and Decentralized Connectivity

In stark contrast to the structured hierarchies observed in the Advanced Electronic Manufacturing and Qiming tracks, the topological audit of the *Standard Intelligent Manufacturing* track reveals a distinct architecture termed the *Implicit Grid*. As visualized in Figure 6, this curriculum is characterized by the absence of explicit administrative prerequisite chains in the raw data. Consequently, the connectivity presented here is derived from the Latent Dependency Inference algorithm, linking courses that share high attribute similarity (cosine similarity surpassing the semantic threshold $\delta = 0.60$) across temporal sequences rather than through enforced administrative mandates.

Visually, the topology appears diffuse and decentralized. Unlike the highly concentrated structural bottleneck of Case A or the longitudinal research conduit of Case B, Case C is dominated by cyan and gray nodes distributed isotropically across the semester grid ($S_1 - S_8$). The network lacks a dominant topological load concentrator; instead, competency transmission relies on a diffuse mesh of inferred flow capacities. While local clusters emerge around *Mechatronics* and *Mechanical Design* modules (shown as orange local hubs), they do not coalesce into a continuous, end-to-end critical path.

This *Grid* structure implies a generalist educational model that prioritizes breadth and flexibility. However, the computational audit warns of a potential *integration deficit*. Quantitatively, the peak normalized Betweenness Centrality in Case C registers at only $C_{\mathrm{B}} \approx 0.011$, concentrated at node *Intelligent Equipment I* (ID 35), significantly lower than the critical bottlenecks observed in Case A (0.034) and Case B (0.028). This low-centrality profile quantitatively indicates the absence of structural enforcement, suggesting that the synthesis of knowledge relies heavily on the student's individual initiative to bridge disparate domains. Without explicit topological guidance, the curriculum exhibits a higher risk of conceptual fragmentation, where engineering topics are delivered as isolated modules with limited interdisciplinary linkage. This topology offers substantial resilience against localized topological blockages but risks creating fragmented competency islands if learners do not self-integrate the loosely connected curriculum modules. To address this potential structural vulnerability, educational policymakers may consider introducing explicit binding mechanisms, such as multidisciplinary synthesis workshops, to supply the structural integration that the curriculum topology currently lacks.

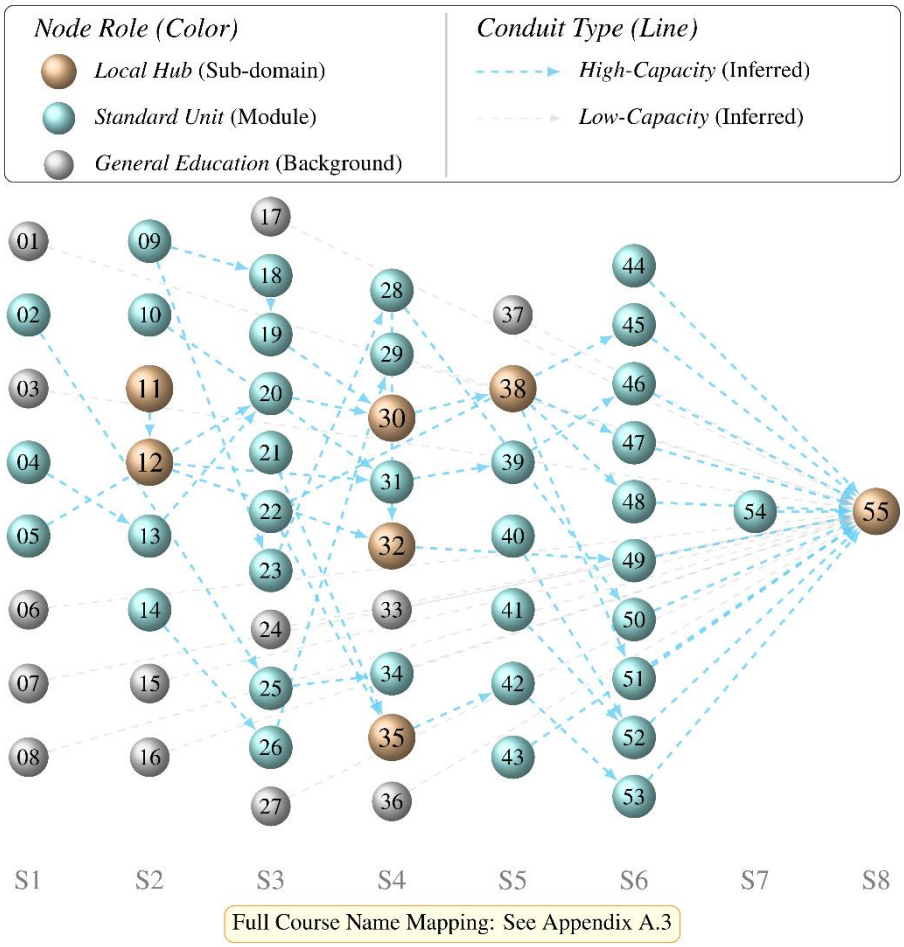


Figure 6 Layered Flow Topology of Case C (Standard Intelligent Manufacturing).

The visualization illustrates the "Implicit Grid" architecture derived via Latent Dependency Inference. Unlike Cases A and B, directed edges are dashed to indicate inferred competency flows rather than explicit administrative mandates, scaled by a semantic threshold ($\delta = 0.60$). Nodes are colored by functional role: Orange indicates inferred local hubs bridging sub-domains, Cyan represents standard modules, and Gray signifies general education. The topological signature is defined by its isotropic, diffuse mesh lacking a singular critical path, maximizing theoretical resilience but creating potential integration deficits. Full mapping is provided in Appendix A.3.

4.4 Sensitivity Analysis of Topological Parameters

To empirically evaluate the structural stability of the FDN model and ensure that the identified architectural characteristics are not transient artifacts of specific scalar calibrations, a systematic sensitivity analysis was conducted. This evaluation systematically perturbs the two primary computational boundaries of the framework: the semantic gating threshold (δ) utilized during topology reconstruction, and the network damping factor (α) governing the recursive demand propagation. The analysis tracks the computational responses of the primary topological load concentrators identified in the previous sections, specifically isolating *Mechanical Design Theory II* (ID 32) in Case A, *Research Practice* (ID 23) in Case B, and the inferred local hub *Intelligent Equipment I* (ID 35) in Case C.

As detailed in Table 3, adjusting the semantic threshold δ from 0.50 to 0.70 inherently alters the topological sparsity of the inferred network (Case C). A higher threshold imposes a stricter semantic

filter, eliminating marginal competency channels and thereby inducing sparsity within the diffuse competency network. However, because the explicit networks (Cases A and B) are defined by administrative mandates, they remain structurally invariant relative to the semantic threshold δ. Consequently, the absolute scalar value of the Betweenness Centrality (C_B) for node ID 32 remains mathematically constant at 0.034, reflecting its structural rigidity as an explicitly mandated, non-bypassable topological bottleneck. More fundamentally, C_B is a structural metric that depends exclusively on the set of shortest weighted paths in the network; since varying δ or α does not alter the underlying adjacency matrix W for these explicit tracks, the shortest-path topology and hence C_B are fully preserved. Likewise, the damping factor α appears nowhere in the formulation of C_B, which further guarantees its mathematical independence from α across all cases.

Table 3 Sensitivity analysis of centrality metrics under parameter perturbations

Parameter configuration		*Case A (ID 32)*		*Case B (ID 23)*		*Case C (ID 35)*	
Threshold (δ)	Damping (α)	C_B	Rank	*PR*	Rank	C_B	Rank
0.50	0.85	0.034	1	0.108	1	0.014	1
0.60	0.85	0.034	1	0.108	1	0.011	1
0.70	0.85	0.034	1	0.108	1	0.003	1
0.60	0.80	0.034	1	0.104	1	0.011	1
0.60	0.90	0.034	1	0.112	1	0.011	1

Notes.While explicit architectures (Cases A and B) exhibit strong structural invariance, the inferred network (Case C) demonstrates structural vulnerability under strict semantic filtering. Italicized values indicate the baseline configuration.

Conversely, the inferred topology of Case C exhibits pronounced sensitivity to the semantic threshold. When δ is elevated from 0.60 to 0.70, the peak Betweenness Centrality of its local hub (ID 35) attenuates notably from 0.011 to 0.003. Far from being a data anomaly, this topological phase transition is consistent with the mathematical properties of threshold networks. Because the *Implicit Grid* lacks rigid administrative mandates, its bridging conduits rely heavily on medium-strength semantic overlaps. Subjecting this structure to a strict threshold ($\delta = 0.70$) truncates these marginal connections, severing the shortest-path bridges that sustain the hub's centrality. This structural fragmentation provides quantitative evidence of the aforementioned *integration deficit*, indicating that without enforced structural binding, the learning pathways in generalist curricula are susceptible to network disconnection under stricter semantic filtering criteria.

The behavior of the PageRank metric is more nuanced, yet consistent with its mathematical definition. For Case B, the PR value of the research hub (ID 23) remains constant at 0.108 when δ is varied (with α fixed at 0.85). This invariance is a direct consequence of the same structural property: δ cannot modify the prerequisite skeleton of an explicitly documented curriculum, leaving the transpose weight matrix W^T and thus the iterative PageRank computation unaffected. Concurrently, modulating the damping factor α between 0.80 and 0.90 shifts the depth of the reverse demand propagation. A higher α allows downstream application requirements to accumulate further backward through the topological skeleton, which proportionally elevates the absolute PageRank (PR) magnitude of

foundational hubs such as node ID 23 (from 0.104 to 0.112). This sensitivity is expected because α directly enters the iterative formula as the weight of the network-derived contribution; however, the relative importance of the nodes remains strictly conserved, as reflected by the stable ordinal rank (Rank 1).

Despite these induced fluctuations in the absolute scalar magnitudes of the network metrics, the empirical data provide quantitative evidence of structural invariance. Across all perturbed computational configurations, the ordinal ranking of the primary functional nodes remains consistently stable. Node ID 32 retains its status as the primary load concentrator (Rank 1) within the *Deep Chain* topology, and Node ID 23 consistently emerges as the primary innovation hub (Rank 1) within the *Research Spine* architecture. Notably, despite its significant metric attenuation at $\delta = 0.70$, Node ID 35 maintains its relative status as the peak load-bearer (Rank 1) in the *Implicit Grid*. This relative stability validates the diagnostic reliability of the FDN protocol, demonstrating that the observed topological vulnerabilities and pedagogical pathways are inherent structural properties of the curriculum designs rather than mathematical anomalies derived from arbitrary parameterization.

5 Discussion: The Topological Trilemma in Curriculum Design

The comparative audit of the three mechanical engineering tracks validates the FDN model not merely as a visualization tool, but as a diagnostic instrument for structural vulnerabilities. By quantifying the competency flow through the variables of Betweenness Centrality (C_B) and topological connectivity, we observe that curriculum design is governed by a fundamental trade-off mechanism. We define this as the *Topological Trilemma*, where a curriculum architecture must navigate the competing constraints of transmission efficiency, network resilience, and structural redundancy. For educational administrators, this trilemma mathematically formalizes the institutional challenge of balancing rigorous academic standardization with pathway flexibility and finite faculty resources.

5.1 Trade-offs in Structural Rigidity and Efficiency

The three curriculum architectures analyzed in this study are not isolated archetypes but rather distinct coordinates along a continuous spectrum of structural rigidity. At the pole of elevated constraint lies the *Advanced Electronic Manufacturing* track (Case A), representing the *Deep Chain* model. This topology prioritizes transmission efficiency through concentrated sequential load transmission. By channeling competency flow through a single critical path and high-stress nodes like *Mechanical Design Theory*, the system ensures that every student undergoes a highly uniform and rigorous standardization. However, this efficiency incurs a penalty in topological resilience. The high peak centrality ($C_B \approx 0.034$) indicates pronounced structural concentration, where inadequate prerequisite mastery at a single bottleneck node may impede readiness for multiple downstream courses. As noted in recent studies on the *topology of hardship* (Paz, 2025), such bottleneck-heavy curricula are often associated with higher dropout rates and temporal desynchronization.

Diametrically opposed to this model is the *Standard Intelligent Manufacturing* track (Case C), representing the *Implicit Grid* architecture. Here, the topology is characterized by a diffuse connectivity profile and significantly lower centrality peaks ($C_B \approx 0.011$). This structure inherently enhances resilience and flexibility, as the failure of any single node does not topologically obstruct the overall flow. Yet, this looseness comes at the cost of structural guidance. The absence of a dominant spine results in

an *integration deficit*, where the synthesis of discrete knowledge modules relies heavily on students' independent cross-course integration rather than on the curriculum's structural enforcement. Consequently, institutions utilizing this topology may face greater variability in how effectively learners bridge these unmapped conceptual gaps without formal prerequisite guidance.

Occupying the strategic middle ground is the *Qiming Experimental Class* (Case B), which employs the *Research Spine* architecture. This model appears to resolve the trade-off between the rigidity of Case A and the diffusion of Case C by introducing a parallel load-distribution mechanism. The topological arrangement creates a dual-stream competency flow: the research chain provides the primary conduit for cumulative innovation ($C_B \approx 0.028$), while the mechatronics stream ($C_B \approx 0.017$) ensures technical breadth. This hybrid topology achieves a high-bandwidth transmission of competency without creating the singular topological bottleneck observed in the Deep Chain model, effectively enhancing the system's adaptive capacity (Molyneaux et al., 2016).

5.2 Morphological Comparison and Educational Implications

To systematically evaluate the trade-offs inherent in these architectures, we synthesize their topological signatures into a morphological matrix (Table 4). This comparative framework reveals that there is no universally optimal topology; rather, the appropriate structural choice depends on the alignment between the topological constraint and the specific pedagogical objective.

Table 4 Morphological matrix of curriculum architectures

Attribute	*Case A: Deep Chain*	*Case B: Research Spine*	*Case C: Implicit Grid*
Primary Topology	Concentrated Sequential	Dual-Stream Integration	Diffuse, Decentralized
Flow Dynamics	Concentrated Load, High Impedance	Parallel Distributed Load	Distributed Capacity, Isotropic
Peak Centrality (C_B)	High (≈ 0.034)	Medium (≈ 0.028)	Low (≈ 0.011)
Structural Risk	Systemic Flow Blockage	High Institutional Resource Demand	Integration Deficit (Conceptual Fragmentation)
Pedagogical Goal	Standardization & Depth	Innovation & Capability	Breadth & Flexibility

Notes. Comparing topological signatures, structural risk profiles, and pedagogical alignment across the three cases.

The implications for educational administration are significant and distinct for each archetype. For programs aiming to cultivate specialized experts in narrow technical domains, the *Deep Chain* topology of Case A proves effective, provided that academic support resources are concentrated at the identified bottleneck nodes (e.g., ID 32). From an administrative perspective, quantifying this bottleneck ($C_B \approx 0.034$) provides a data-driven justification for targeted educational interventions, such as assigning senior faculty to these high-impedance courses, implementing mandatory supplemental instruction, or expanding peer-tutoring programs. In this model, the network's efficiency relies on the concentrated steady-state flow load along the critical path to structurally enforce depth of learning.

Conversely, for programs targeting broad-spectrum engineering literacy, the *Implicit Grid* architecture of Case C is applicable. However, due to its low centrality profile, this diffuse topology requires the introduction of external *binding mechanisms*—such as mandatory capstone projects—to structurally mandate the knowledge integration that the curriculum network does not naturally induce. Institutionally, this necessitates a strategic shift in faculty workload, reallocating resources from

delivering standardized discrete lectures toward providing intensive, individualized mentoring in these integrative capstone modules to address the integration deficits identified by the FDN audit.

The *Research Spine* of Case B offers a structural blueprint for elite research-oriented education, balancing the sequential depth of Case A with the technical breadth of Case C. By treating research training not as a discrete terminal event but as a continuous topological backbone, institutions can foster innovation capabilities cumulatively. Yet, this topology incurs the highest institutional resource demand. The maintenance of parallel, high-bandwidth capacity streams requires a significantly higher faculty-to-student ratio, rendering this architecture less scalable than the single-stream or diffuse models. Therefore, educational policymakers must weigh the pedagogical benefits of parallel innovation tracks against the pragmatic constraints of departmental teaching budgets. Across all three archetypes, combining Betweenness Centrality (C_{B}) and Weighted Fuzzy PageRank (PR) enables differentiated resource allocation: whereas high- C_{B} bridge modules (e.g., ID 32 in Case A) warrant structural decompression and prerequisite-alignment support, high- PR foundational hubs (e.g., ID 23 in Case B, $PR = 0.108$) warrant priority investment in core instructional staffing and baseline quality assurance.

5.3 Strategies for Topological Optimization

Beyond structural classification, the FDN audit serves as a functional guide for topological optimization. By quantitatively isolating the specific structural bottlenecks of a curriculum, administrators can intervene with data-driven, localized reforms rather than broad, disruptive reorganizations.

For an existing curriculum exhibiting the structural vulnerability characteristic of the *Deep Chain* model (Case A), the optimization strategy focuses on load redistribution. The introduction of parallel topological conduits—such as elective modules that offer alternative pathways bypassing critical bottlenecks like *Mechanical Design Theory*—can distribute the steady-state flow load more evenly across the network. By formalizing these alternative elective pathways, curriculum managers can effectively attenuate localized prerequisite load concentration without compromising the overarching pedagogical objectives. This intervention reduces the network's peak Betweenness Centrality (C_{B}), thereby enhancing the system's resilience against structural congestion.

Conversely, for a curriculum exhibiting the structural fragmentation typical of the *Implicit Grid* (Case C), the remedial strategy necessitates connectivity enhancement. In this scenario, structural integration nodes can be introduced through project-based learning courses that explicitly bridge previously disconnected knowledge domains. Practically, this approach might involve cross-disciplinary co-teaching initiatives, where faculty from disparate departments collaborate to design high-capacity, integration-focused coursework. Furthermore, combining these topological insights with personalized learning trajectories (Li et al., 2025) could facilitate dynamic curriculum adjustments, allowing the diffuse network to adapt to individual competency profiles.

Ultimately, the application of the FDN model transforms curriculum evaluation from a qualitative debate based on heuristic intuition into a quantifiable structural engineering problem. It allows educators to regulate the transmission of competency with the analytical rigor applied to formal network flow optimization, ensuring that the curriculum architecture actively facilitates, rather than passively constrains, the intended pedagogical outcomes. In practical university governance, however,

implementing such topological adjustments is subject to concrete institutional constraints, including accreditation-mandated credit-hour ceilings, departmental faculty staffing capacities, and multi-year curriculum-revision cycles. Introducing parallel elective conduits or integrative project courses cannot be achieved by simply adding network edges in isolation; it requires reallocating bounded credit quotas and balancing teaching workloads across faculty teams. Therefore, the FDN model should be utilized strictly as a quantitative *decision-support tool* that informs, rather than replaces, expert pedagogical judgment and deliberative curriculum committee review.

5.4 Methodological Boundaries and Limitations

Several methodological boundaries must be noted to prevent over-interpreting the topological audit results. First, regarding the applicable boundaries of the fluid-network analogy, constructs such as steady-state flow load, competency bandwidth (w_{ij}), and topological impedance (d_{ij}) are interpretive representations of computable graph-theoretic quantities—specifically, cosine similarity weights, reciprocal edge distances, and shortest-path centralities. Student competency acquisition is an active educational process rather than passive physical fluid transport. Because the FDN model operates on static syllabus documents, it does not capture classroom-level and individual variables—such as instructional delivery quality, assessment design, student motivation, prior academic preparation, and peer interaction—that substantially govern real-world learning outcomes.

Second, the model outputs identify a priori structural vulnerabilities in curriculum plans rather than empirically verified learning bottlenecks. While a high C_{B} value indicates that a course sits at the intersection of multiple prerequisite chains, whether this structural concentration translates into actual course failures or progression delays depends on how the course is taught and supported in practice. Validating the empirical link between network centrality metrics and student academic performance requires longitudinal educational records (e.g., course grade distributions, failure/retake rates, and cohort progression trajectories), which were not available in the current document-based dataset.

Third, the empirical evaluation is based on purposive sampling of three undergraduate tracks within a single mechanical engineering discipline at one university. While this controlled scope isolates structural variations under a unified credit system, it limits direct external validity across institutions with different credit structures, modularization policies, or non-engineering epistemological traditions. Furthermore, in tracks lacking official prerequisite flowcharts (Case C), the inferred topology depends on both the granularity of expert syllabus ratings (High/Medium/Low) and the analyst-selected threshold δ. Cross-institutional or cross-disciplinary applications therefore require local parameter recalibration and sensitivity verification.

6 Conclusion

This study establishes the FDN model not merely as a visualization technique, but as a rigorous quantitative framework for auditing the structural integrity of engineering curricula. By mapping the competency flows within three distinct tracks of the *Mechanical Design, Manufacturing, and Automation* discipline, we demonstrated that curriculum architecture shapes the structural pathways of competency transmission. The application of metrics, particularly Betweenness Centrality (C_{B}), Weighted Fuzzy PageRank (PR), and topological connectivity, allows educators to move beyond static course lists to

reveal the structural dynamics of competency transmission, thereby identifying potential prerequisite barriers and structural load concentrations that traditional compliance audits frequently overlook.

The comparative analysis identified three fundamental topological archetypes, each representing a distinct resolution to the curriculum design trade-offs. The *Deep Chain* model (Case A), defined by its highly concentrated critical path, excels at enforcing standardization and depth but suffers from structural vulnerability due to singular topological bottlenecks, necessitating highly targeted faculty interventions. Conversely, the *Implicit Grid* model (Case C) offers substantial flexibility and resilience through its diffuse connectivity but risks an *integration deficit* due to the absence of a cohesive structural spine, requiring administrative enforcement of integrative capstone modules. The *Research Spine* model (Case B) emerges as a structural synthesis, utilizing a dual-stream architecture to balance the rigorous demands of theoretical mechanics with the cumulative nature of innovation training, albeit at a substantially higher instructional resource demand.

Ultimately, the significance of this work lies in its methodological shift. By framing curriculum design as a topological engineering problem, we provide administrators with a decision-support tool to screen for potential structural vulnerabilities—whether they be critical load concentrations or integration deficits—during the curriculum-planning stage. As engineering education faces increasing complexity and structural inflation, the FDN model offers a pathway to transform curriculum reform from a heuristic practice into a rigorous, data-driven methodology, ensuring that educational structures are optimized for the efficient, resilient, and coherent transmission of competency. While the current framework establishes a robust quantitative baseline for a priori curriculum auditing, future research will prioritize the integration of longitudinal, multi-cohort student performance data—including course pass/fail rates, grade distributions, and semester-by-semester progression records—to empirically test the relationship between these topological network metrics and actual academic trajectories across diverse institutional and disciplinary contexts.

Appendix A. Course ID Mapping Tables

This appendix provides the complete mapping between the numerical Course IDs used in the topological visualizations and the actual course names for each of the three analyzed curriculum tracks.

Appendix A.1. Case A: Advanced Electronic Manufacturing

Table A.1: Complete Course ID mapping for Case A (The Deep Chain). The primary topological bottleneck (C_B peak) is italicized.

ID	*Course Name*	*ID*	*Course Name*	*ID*	*Course Name*
01	Physical Education (I)	18	University Physics A (II)	35	Intelligent Manufacturing Equipment I
02	C++ Programming	19	Circuit Theory	36	Professional Social Practice
03	Morality & Law	20	Complex Functions	37	Physical Education (III)
04	Calculus B (I)	21	Engineering Materials	38	Electronic Materials
05	Linear Algebra B	22	Engineering Chemistry	39	Mechatronics II
06	Situation & Policy	23	Engineering Training I	40	Micro-Electronic Manufacturing Process
07	Chinese	24	Theoretical Mechanics II	41	Advanced Electronic Manufacturing I (A)
08	English I	25	Marxism Principles	42	Advanced Electronic Manufacturing I (B)
09	University Physics A (I)	26	Physics Experiment (II)	43	Intelligent Manufacturing Equipment II
10	Probability & Statistics B	27	Xi Jinping Thought	44	Production Internship
11	Mechanical Engineering Introduction	28	Material Mechanics	45	Mechatronics III
12	Mechanical Design I	29	Engineering Mechanics Experiment	46	Advanced Electronic Manufacturing II (A)
13	Calculus B (II)	30	Mechatronics I	47	Advanced Electronic Manufacturing II (B)
14	Physics Experiment (I)	31	Design Innovation	48	Advanced Electronic Manufacturing III (A)
15	History	*32*	*Mechanical Design Theory II*	49	Advanced Electronic Manufacturing III (B)
16	English II	33	Mao Zedong Thought	50	Professional Engineering Training
17	Physical Education (II)	34	AI Introduction	51	Graduation Thesis

Appendix A.2. Case B: Intelligent Manufacturing Qiming Class

Table A.2: Complete Course ID mapping for Case B (The Research Spine). The foundational research hub is italicized.

ID	*Course Name*	*ID*	*Course Name*	*ID*	*Course Name*
01	Physical Education (I)	20	Circuit Theory	39	Mechatronics II
02	Advanced Mathematics A (I)	21	Complex Functions	40	Research Training II
03	Scientific Thinking & Method	22	Engineering Materials	41	Fluid Mechanics I
04	Software Technology Base	*23*	*Research Practice*	42	Thermal Engineering Base
05	Morality & Law	24	Theoretical Mechanics II	43	Intelligent Manufacturing Equipment II
06	Linear Algebra B	25	Marxism Principles	44	Sensor Technology
07	Chinese	26	AI Introduction	45	Mechanical Design III
08	English I	27	Physics Experiment (II)	46	Computational Methods II
09	University Physics A (I)	28	Xi Jinping Thought	47	Flexible Electronic Manufacturing
10	Probability & Statistics B	29	Material Mechanics	48	Production System Modeling
11	Advanced Mathematics A (II)	30	Engineering Mechanics Experiment	49	Research Training III
12	Engineering Training I	31	Mechatronics I	50	Mechatronics III
13	Mechanical Engineering Introduction	32	Mechanical Intelligent Systems I	51	Robotics
14	Mechanical Design I	33	Mechanical Design II	52	Mechanical System Dynamics
15	Physics Experiment (I)	34	Research Training I	53	Computer Numerical Control Technology
16	History	35	Mao Zedong Thought	54	Hydraulic & Pneumatic
17	Generative Design	36	Intelligent Manufacturing Equipment I	55	Graduation Thesis
18	Physical Education (II)	37	Decision & Design		
19	University Physics A (II)	38	Physical Education (III)		

Appendix A.3. Case C: Standard Intelligent Manufacturing

Table A.3: Complete Course ID mapping for Case C (The Implicit Grid). The inferred local functional hubs are italicized.

ID	*Course Name*	*ID*	*Course Name*	*ID*	*Course Name*
01	Physical Education (I)	20	Complex Functions	39	Design Innovation II
02	C++ Programming	21	Engineering Materials	40	Fluid Mechanics I
03	Morality & Law	22	Engineering Training I	41	Thermal Engineering Base
04	Calculus B (I)	23	Theoretical Mechanics II	42	Intelligent Equipment II
05	Linear Algebra B	24	Marxism Principles	43	Computational Methods II
06	Situation & Policy	25	AI Introduction	44	Mechanical Intelligent Systems I
07	Chinese	26	Physics Experiment (II)	45	Production Internship
08	English I	27	Xi Jinping Thought	46	Modeling Training
09	University Physics A (I)	28	Material Mechanics	47	Mechatronics III
10	Probability & Statistics B	29	Engineering Mechanics Experiment	48	Robotics
11	Mechanical Engineering Introduction	*30*	*Mechatronics I*	49	Mechanical Design III
12	Mechanical Design I	31	Design Innovation	50	Mechanical System Dynamics
13	Calculus B (II)	*32*	*Mechanical Design II*	51	Computer Numerical Control Technology
14	Physics Experiment (I)	33	Mao Zedong Thought	52	Hydraulic & Pneumatic
15	History	34	Intelligent Manufacturing Introduction	53	Intelligent Equipment III
16	English II	*35*	*Intelligent Equipment I*	54	Professional Engineering Training
17	Physical Education (II)	36	Social Practice	55	Graduation Thesis
18	University Physics A (II)	37	Physical Education (III)		
19	Circuit Theory	*38*	*Mechatronics II*		

Abbreviations

AI	Artificial Intelligence
FDN	Fluid Dynamics Network
GMIR	Graded Mean Integration Representation
H	High support level
ID	Course identifier
L	Low support level
M	Medium support level
PR	PageRank

Variables

$\tilde{A}$	Generic triangular fuzzy number.
$\tilde{A}_{\mathrm{H}}$	Triangular fuzzy number for the High support label.
$\tilde{A}_{\mathrm{L}}$	Triangular fuzzy number for the Low support label.
$\tilde{A}_{\mathrm{M}}$	Triangular fuzzy number for the Medium support label.
α	PageRank damping factor.
c_i	Competency vector of course i.
$C_{\mathrm{B}}(v_i)$	Normalized Betweenness Centrality of node v_i.
d	Dimension of the competency vector (11 in this study).
δ	Semantic similarity threshold used in Case C.
E	Directed edge set of the curriculum network.
E_{explicit}	Explicitly documented prerequisite edge set.
E_{implicit}	Inferred prerequisite edge set.
$\mathbb{F}$	Fuzzy space used to represent linguistic evaluations.
G	Directed, weighted curriculum network, $G = (V, E, W)$.
k	Graduation-attribute index.
l, m, u	Lower, modal, and upper parameters of a triangular fuzzy number.
μ_{H} , μ_{M} , μ_{L}	Defuzzified GMIR scalar weights for High, Medium, and Low support labels.
N	Total number of courses in the network.
$\mathrm{N}_{\text{in}}(v_i)$	Set of advanced courses that depend on course v_i.
$\mathrm{N}_{\text{out}}(v_j)$	Set of foundational prerequisites demanded by course v_j.
n	Number of vertices or matrix dimension.
$PR(v_i)$	Reverse Weighted Fuzzy PageRank score of course v_i.
σ_{st}	Number of shortest-impedance paths from source v_s to target v_t.
$T(v_i)$	Semester index of course v_i.
V	Vertex set of the curriculum network.
v_i	Node representing course i.
W	Adjacency weight matrix of directed competency-transfer capacities.
w_{ij}	Weight of the directed edge from v_i to v_j.
x	Defuzzified scalar associated with a linguistic evaluation.

d_{ij}	Topological impedance from v_i to v_j; $d_{ij} = 1/w_{ij}$ for $w_{ij} > 0$.
$\mathcal{S}(c_i, c_j)$	Cosine similarity between competency vectors c_i and c_j.
$\sigma_{st}(v_i)$	Number of shortest-impedance paths from v_s to v_t that pass through v_i.
$\mathbb{I}(\cdot)$	Topological indicator; 1 for an edge in E and 0 otherwise.
W^{T}	Transpose of W, used to compute reverse-demand PageRank.
F_{ij}	Competency-flow notation used in Section 4.
$S_1, \ldots, S_8$	Semester-layer labels for Semesters 1 to 8 in Figures 4–6.
e_{ij}	Directed prerequisite edge from v_i to v_j in Figures 1 and 2.
θ, θ_{ij}	Angle between competency vectors; θ in Figure 2 and θ_{ij} in Figure 1.
$\mathbb{R}^d$	d-dimensional real vector space containing the competency vectors.
v_s, v_t	Source and target course nodes for shortest-impedance paths.
E_{chart}	Prerequisite edge set obtained from explicit charts in Figure 1.

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