

Non-invasive prosthesis pose visualization in joint replacement surgery based on multi-sensor fusion

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Abstract

With the aging population, demand for artificial joint replacement is growing. In total hip arthroplasty (THA), implanting the acetabular cup into the safe zone (inclination: $40^\circ \pm 10^\circ$, anteversion: $15^\circ \pm 10^\circ$) is critical for success, but existing technologies suffer from low accuracy or radiation invasiveness. To address this, a non-invasive prosthesis pose visualization method based on multi-sensor fusion is proposed, aiming for real-time, radiation-free and high-precision monitoring. A complete hardware-software system is constructed, consisting of an inertial measurement unit (IMU), a microprocessor, a bluetooth transceiver and a computer. An extended Kalman filter (EKF) is employed for IMU data fusion, and a non-invasive registration method is proposed to establish the coordinate transformation, enabling calculation and visualization of the cup's inclination and anteversion. Experiments showed that the root mean square error (RMSE) of self-pose estimation was within 0.423° , the RMSE of the inclination and anteversion were 0.278° and 0.296° respectively, and the average error vector magnitude was 0.373° , all within the safe zone tolerance, rendering the proposed method promising as an effective surgical tool.

Keywords total hip arthroplasty (THA), pose visualization, inertial measurement unit (IMU), data fusion, non-invasive registration

1 Introduction

With the aging population in China and even globally, the rate of osteoarthritis and joint injuries is increasing yearly, and artificial joint replacement has become the primary method for joint function recovery and pain relief^[1]. According to the American Joint Replacement Registry, approximately 572 000 hip replacements are performed in the US annually^[1-2]. In China, the number of hip and knee arthroplasty surgeries had reached about 1.75 million cases till

2022, and the number is still growing rapidly^[3].

Taking THA as an example, the process of artificial joint replacement can be summarized into three steps as shown in Fig. 1. First, resect the diseased joint tissues (including the femoral head and acetabulum); second, implant the joint prostheses and adjust them to appropriate positions; finally, assemble the joint, confirm that the alignment and range of motion are good, and then close the incision. In the second step, implanting the prosthesis into the “safe zone” (i.e., the acetabular cup prosthesis is within the range of inclination angle $40^\circ \pm 10^\circ$ and anteversion angle $15^\circ \pm 10^\circ$) is the key to the success of the surgery^[4]. Otherwise, it may lead to complications such as dislocation and prosthesis impingement, and even a

secondary revision surgery^[5]. Due to the differences in medical standards across different regions in China, the incidence of complications after THA ranges from 2% to 10%, of which approximately 17% are dislocations^[6].

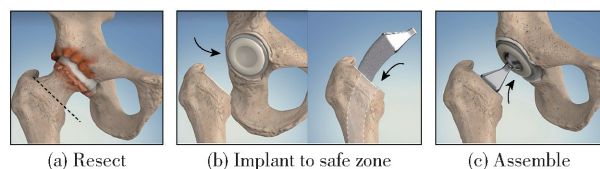


Fig. 1 Three-step process of THA

Current prosthesis placement technologies include traditional tools and computer-aided navigation systems. For THA, mechanical alignment guides and protractors are common traditional tools with low cost^[7-9], but only provide passive visual references and require anatomical landmark identification, so they highly depend on surgeon's experience and only suit preoperative planning or simple cases^[10]. On the other hand, computer-aided navigation systems are currently the most accurate technology, with a displacement error within 1 mm and an angular error within 1°^[11]. However, they have not been widely adopted due to their cost, complexity, time demands and radiation exposure^[12-13]. Moreover, they require the implantation of markers in the patient's bones as anatomical landmarks, which may cause additional damage to the bones and increase the risk of pin tract complications^[12].

With the development of inertial measurement units (IMUs), IMU-based pose measurement devices and smart prostheses have emerged as a key research trend. In 2017, a Tsinghua University team developed a smart trial capable of measuring its own pose relative to the acetabular cup in real time by integrating an image sensor and an IMU into a femoral head prosthesis^[14]. In 2021, the team proposed an IMU-based system for anterior pelvic plane measurement, which enables real-time measurement of the femoral head prosthesis pose when combined with the IMU module fixed on the femur^[15]. In 2025, a team from Beijing Jishuitan Hospital proposed an IMU-based smart hip trial system, and verified its ability to measure intraoperative cup inclination and

anteversion through in vitro experiments^[13]. However, these technologies remain in the experimental stage, and all require integrating sensors and circuits into traditional joint prostheses, which means that additional adaptations are needed for prostheses of different sizes and materials.

As the existing technologies have notable limitations in clinical application, this study focuses on hip replacement and proposes a prosthesis pose visualization method, aiming to realize real-time, radiation-free, portable, and high-precision intraoperative prosthesis pose monitoring, thereby improving the success rate of surgery.

2 Problem and system overview

2.1 Definition of the visualization problem

Since the “safe zone” is defined by the inclination and anteversion angles of the acetabular cup, the key to achieve the pose visualization of the joint prostheses is real-time estimation of the two angles. There are several definitions of inclination and anteversion angles, including operative, radiographic and anatomical definitions^[16]. In this paper, the operative inclination θ_i and anteversion θ_a are the selected definition, which is illustrated in Fig. 2.

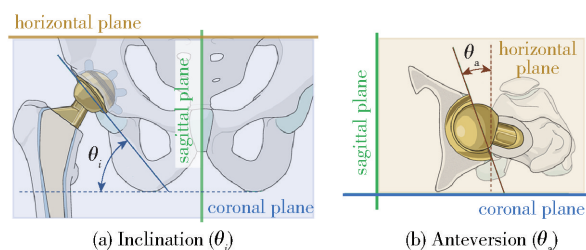


Fig. 2 Operative definition of cup inclination and anteversion

For mathematical description, a body coordinate system (denoted as $\mathbf{B}$) is necessary to be established using the coronal, sagittal, and horizontal planes of the human body, as shown in Fig. 3. Its positive x -axis is orthogonal to the sagittal plane and points to the body's right, the positive y -axis is orthogonal to the coronal plane and points forward, and the positive z -axis is the vertical upward direction along the spinal axis.

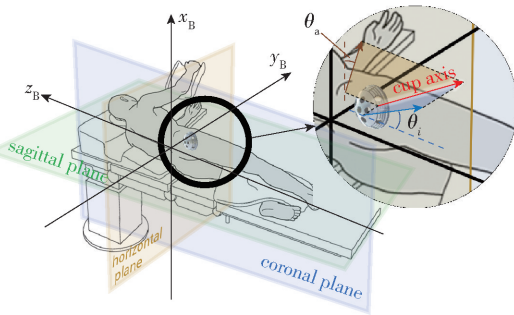


Fig. 3 Definition of the body coordinate system

The red arrow in Fig. 3 is the axis of the acetabular cup (i. e. , the normal vector), which can be used to indicate the orientation of the cup. Assume that $\mathbf{n}$ denotes the cup axis vector and $\mathbf{n}^B = (n_x^B, n_y^B, n_z^B)$, where the superscript B indicates the coordinates defined in system B . According to geometric relationships, the inclination θ_i equals the angle between the z_B -axis and the projection $\mathbf{n}_{xoz}^B$ on the coronal plane, while the anteversion θ_a equals the angle between the x_B -axis and the projection $\mathbf{n}_{xoy}^B$ on the horizontal plane. So the vector $\mathbf{n}$ is projected onto the two planes as the blue and brown arrows shown in Fig. 3. The projection results are shown as

$$\left. \begin{aligned} \mathbf{n}_{xoz}^B &= (n_x^B, 0, n_z^B) \\ \mathbf{n}_{xoy}^B &= (n_x^B, n_y^B, 0) \end{aligned} \right\} \quad (1)$$

and therefore the values of θ_i and θ_a can be obtained through

$$\left. \begin{aligned} \theta_i &= \cos^{-1} \left(\frac{\mathbf{n}_{xoz}^B \cdot \mathbf{e}_{Bz}^B}{|\mathbf{n}_{xoz}^B|} \right) = \cos^{-1} \left(\frac{-n_z^B}{\sqrt{n_x^{B^2} + n_z^{B^2}}} \right) \\ \theta_a &= \cos^{-1} \left(\frac{\mathbf{n}_{xoy}^B \cdot \mathbf{e}_{Bx}^B}{|\mathbf{n}_{xoy}^B|} \right) = \cos^{-1} \left(\frac{n_x^B}{\sqrt{n_x^{B^2} + n_y^{B^2}}} \right) \end{aligned} \right\} \quad (2)$$

where $\mathbf{e}_{Bx}^B$ and $\mathbf{e}_{Bz}^B$ represent the unit vectors in the positive x -axis and negative z -axis directions of B respectively.

Overall, the visualization problem is first transformed into the problem of solving the inclination and anteversion of the acetabular cup, and finally defined as the problem of obtaining the coordinates of the vector $\mathbf{n}$ in B . The pose visualization method proposed in this paper addresses this issue through three steps. First, design an IMU-based device to achieve real-time pose estimation of its own sensor coordinate system I relative to the world coordinate

system W ; second, propose a non-invasive registration method to establish the transformation relationship between the patient's body coordinate system B and the world coordinate system W (the relationship is assumed to remain unchanged during surgery); finally, mount the device on the cup impactor in a specific orientation to acquire the real-time coordinates in W of the cup axis vector $\mathbf{n}$, and then transform these values to those in B using the relationship obtained from the registration phase.

2.2 System architecture

Pose estimation is commonly used in automatic control fields such as navigation, robotics, aerospace vehicles, and unmanned driving, but it cannot be directly applied to this pose visualization system as the application scenario differs significantly from that in the human body. Taking the tolerance of the safe zone and the accuracy of computer-aided navigation into consideration, the accuracy that the system intends to achieve is specified as: the angular error should be within $\pm 1^\circ$ and be maintained within $\pm 5^\circ$ within 20 min.

The system consists of two parts; hardware and software, as shown in Fig. 4. The hardware part is responsible for collecting motion data and fusing it through an extended Kalman filter (EKF) to achieve self-pose estimation. All the hardware modules are finally packaged into the small blue device shown in Fig. 4. The software part is located on the PC side, where the processor executes the registration algorithm and uses the result as a parameter for the subsequent real-time estimation of the prosthesis pose. Among many sensors capable of pose estimation, the IMU (model MPU9250) is selected as the sensor solution, which includes a 3-axis gyroscope, a 3-axis accelerometer, and a 3-axis magnetometer. At the same time, the microprocessor (STM32F103) of the control module is responsible for controlling the sensor and Bluetooth communication module, realizing functions such as real-time collection, processing, storage, and transmission of prosthesis pose-related data. The software function block diagram of the control module is shown in Fig. 5, including the

initialization of each module, IMU data collection, pose estimation, data fusion, pose visualization, data

storage, communication, etc.

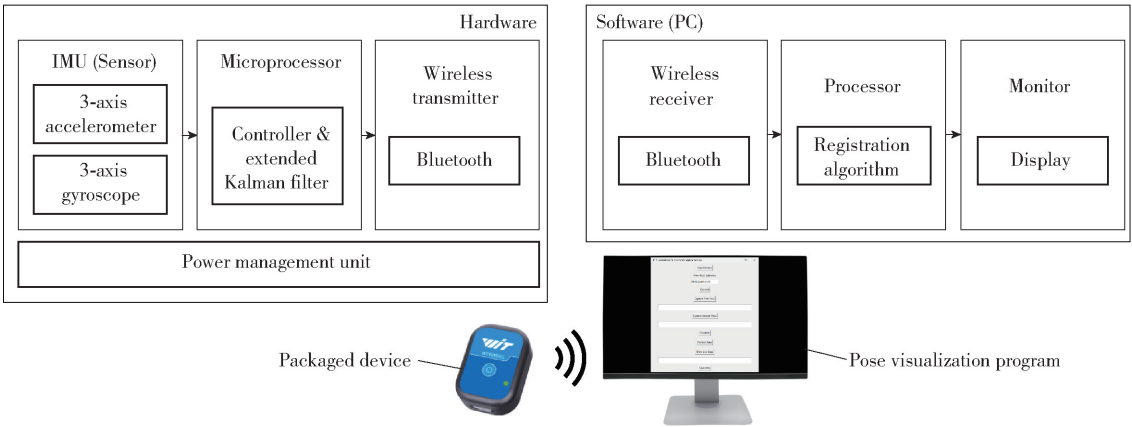


Fig. 4 Architecture of the IMU-based prosthesis pose visualization device

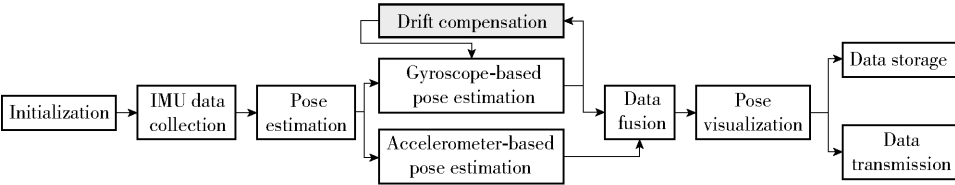


Fig. 5 Software function block diagram of the control module

3 Method of self-pose estimation

3.1 Mathematical representation of pose

The self-pose estimation is to obtain the transformation relationship between the sensor coordinate system and the world coordinate system. For this purpose, it is necessary to define the world coordinate system W and the IMU sensor coordinate system I , as shown in Fig. 6. The system W adopts the east-north-up (ENU) coordinate system, while the system I adopts the body-fixed coordinate system of the sensor itself. The definitions of W , I , and B are summarized in Table 1.

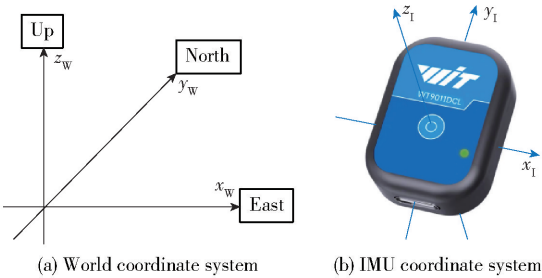


Fig. 6 Definition of the coordinate system W and I

Table 1 A summary of the three coordinate systems definitions

Name & notation	Definition
World coordinate system W	x_W : pointing east
	y_W : pointing north
	z_W : pointing up, orthogonal to the local Earth surface
IMU coordinate system I	x_I : pointing right (from front-facing direction)
	y_I : pointing up (from front-facing direction)
	z_I : pointing forward (from front-facing direction)
Body coordinate system B	x_B : pointing to the body's right
	y_B : pointing forward
	z_B : pointing up along the spinal axis

There are several mathematical tools for describing the pose of a rigid body, including rotation matrices, Euler angles, quaternions, etc., and these tools can be converted into each other^[17]. Among them, the Euler angles^[18] used in this paper are defined by intrinsic rotations in the rotation axis order of z - y - x , also known as the heading angle, consisting of yaw, pitch and roll, denoted as (α, β, γ) . The angles decompose the rotation motion into three sequential parts. First, rotate around its own z -axis by the angle

γ (yaw), denoted as $\mathbf{R}_z(\gamma)$; second, rotate around the updated y -axis by the angle β (pitch), denoted as $\mathbf{R}_y(\beta)$; finally, rotate around the rotated x -axis by the angle α (roll), denoted as $\mathbf{R}_x(\alpha)$. The combined rotation $\mathbf{R}_{zyx}$ obtained by these three rotations is shown as

$$\mathbf{R}_{zyx} = \mathbf{R}_z(\gamma) \mathbf{R}_y(\beta) \mathbf{R}_x(\alpha) = \begin{bmatrix} c_\beta c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma & c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma \\ c_\beta s_\gamma & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma \\ -s_\beta & s_\alpha c_\beta & c_\alpha c_\beta \end{bmatrix} \quad (3)$$

where c_α and s_α are abbreviations of $\cos \alpha$ and $\sin \alpha$ respectively.

Through Eq. (3), each set of Euler angles output from the IMU-based device can be converted into a rotation matrix $\mathbf{R}_I^W$, which not only represents the rotation motion from the world coordinate system W to the IMU coordinate system I , but also represents the pose of I relative to W . Assume that $\mathbf{p}$ is a fixed vector and $\mathbf{p}^W$ represents its coordinates in W , $\mathbf{R}_1^W$ and $\mathbf{R}_2^W$ are the rotation matrices (in intrinsic rotation form) of the device in two different poses, and $\mathbf{p}^1$ and $\mathbf{p}^2$ are the coordinates of $\mathbf{p}$ in I for the two poses respectively. The transformation relationship between these coordinates are as follows:

$$\mathbf{p}^W = \mathbf{R}_1^W \mathbf{p}^1 = \mathbf{R}_1^W \mathbf{R}_2^1 \mathbf{p}^2 = \mathbf{R}_2^W \mathbf{p}^2 \quad (4)$$

In Eq. (4), the matrix $\mathbf{R}_2^1$ can be used to represent the relative pose of the device before and after the rotation. It is calculated by

$$\mathbf{R}_2^1 = \mathbf{R}_1^{W^{-1}} \mathbf{R}_2^W = \mathbf{R}_1^{WT} \mathbf{R}_2^W \quad (5)$$

where $\mathbf{R}_1^{WT}$ represents the transpose matrix of $\mathbf{R}_1^W$, and $\mathbf{R}_1^{WT} = \mathbf{R}_1^{W^{-1}}$ as it is a rotation matrix.

3.2 Principle of IMU-based self-pose estimation

The 3-axis accelerometer and 3-axis gyroscope in the IMU are the main sensors used for self-pose estimation. The accelerometer works when the device is stationary or moves at a very slow acceleration. Therefore, the reading of the accelerometer at this time is considered to be the component of gravitational acceleration $\mathbf{g}$ in each axis of the IMU. If $\mathbf{g}^I$ and $\mathbf{g}^W$ represent the coordinates of $\mathbf{g}$ in I and W respectively, their relationships with the Euler angles (α, β, γ) of the device can be summarized as follows:

$$\mathbf{g}^I = [g_x \ g_y \ g_z]^T = \mathbf{R}_W^I \mathbf{g}^W = \mathbf{R}_{zyx}^T \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} = \begin{bmatrix} g \sin \beta \\ -g \sin \alpha \cos \beta \\ -g \cos \alpha \cos \beta \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \tan^{-1} \left(\frac{g_y}{g_z} \right) \\ \tan^{-1} \left(-\frac{g_x}{\sqrt{g_y^2 + g_z^2}} \right) \end{bmatrix} \quad (7)$$

Among Eq. (6) and Eq. (7), g is the magnitude of gravitational acceleration, $[g_x, g_y, g_z]$ is the reading of the accelerometer, and $\mathbf{R}_W^I$ is the transpose of $\mathbf{R}_I^W$, which can be calculated by Eq. (3) and the Euler angles (α, β, γ) . The range of the angle α is set within $[-90^\circ, 90^\circ]$ because the prosthesis will not have a roll angle exceeding 90° during normal movement. However, the yaw angle γ cannot be obtained only by the accelerometer, and if the device is affected by additional acceleration, this method will fail.

The principle of the gyroscope can be simply described as obtaining the angle variation by integrating the angular velocity and using it to update the device's pose. Assume that $[\omega_x \ \omega_y \ \omega_z]$ is the three-axis angular velocity measured by the gyroscope and (α, β, γ) are the Euler angles of the current device's pose. Since the device is not an inertial system, there is a relationship between the reading of the gyroscope and its actual pose increment $(\dot{\alpha}, \dot{\beta}, \dot{\gamma})$, which refers to the variation of the pose angle within the sampling interval of the gyroscope.

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

By rearranging Eq. (8), a nonlinear differential equation is obtained.

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \frac{\cos \gamma}{\cos \beta} & \frac{\sin \gamma}{\cos \beta} & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ -\cos \gamma & -\sin \gamma & 1 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (9)$$

Theoretically, solving this equation yields the device's pose increment, which can update its pose, but the solution process is overly complex. In practice, converting Eq. (9) from Euler angles to the quaternion form can simplify the calculation, and more details are omitted here.

It is not difficult to find the drawback of the gyroscope, which will continuously accumulate small measured values over time when the device is stationary, leading to large errors, known as drift errors.

3.3 Data fusion algorithm

To utilize the complementarity of the accelerometer and gyroscope, the quaternion-based EKF^[19] is introduced to fuse the IMU data, and its algorithm flowchart is shown in Fig. 7.

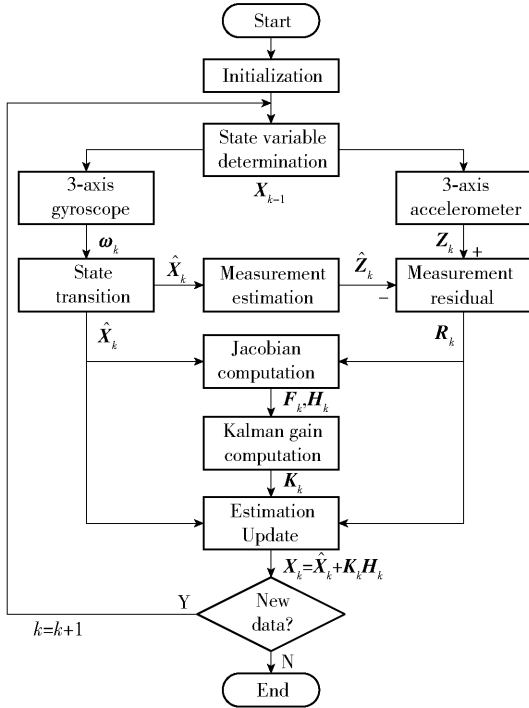


Fig. 7 Flowchart of the EKF algorithm

In Kalman filtering, the first thing is to determine the state prediction equation and observation equation. The parameter to be measured is the pose quaternion $\mathbf{q}_k$ of the IMU coordinate system $\mathbf{I}$ relative to $\mathbf{W}$. Considering the drift error ${}^g\mathbf{b}_k$ of the gyroscope, the state variable of the system can be set as $\mathbf{X}_k = [\mathbf{q}_k, {}^g\mathbf{b}_k]$, and the state prediction equation can be obtained as

$$\mathbf{X}_{k+1} = \begin{bmatrix} \mathbf{q}_{k+1} \\ {}^g\mathbf{b}_{k+1} \end{bmatrix} = \mathbf{f}(\mathbf{X}_k, \boldsymbol{\omega}_{k+1}) + \mathbf{w}_{k+1} = \begin{bmatrix} \mathbf{I} + \frac{T_s}{2} \boldsymbol{\Omega}(\hat{\boldsymbol{\omega}}_{k+1}) & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{q}_k \\ {}^g\mathbf{b}_k \end{bmatrix} + \mathbf{w}_{k+1} \quad (10)$$

where $\mathbf{f}$ is the state prediction function, $\mathbf{w}_{k+1}$ is the Gaussian noise component, $\boldsymbol{\omega}_{k+1}$ is the three-axis angular velocity data measured by the gyroscope, and $\hat{\boldsymbol{\omega}}_{k+1} = \boldsymbol{\omega}_{k+1} - {}^g\mathbf{b}_{k+1}$ is the gyroscope data after drift correction.

According to Eq. (6) and Eq. (7), each reading of the accelerometer corresponds to a certain pose result, so the observation equation can be written as $\mathbf{Z}_{k+1} = \mathbf{R}_w^T [\mathbf{q}_{k+1}] \mathbf{g} + \mathbf{v}_{k+1}$ (11) where $\mathbf{v}_{k+1}$ is the Gaussian noise component, $\mathbf{g}$ is the gravitational acceleration in $\mathbf{W}$, and $\mathbf{R}_w^T [\mathbf{q}_{k+1}]$ is the rotation matrix from $\mathbf{W}$ to $\mathbf{I}$ represented by quaternions.

Based on Eq. (10) and Eq. (11), iteration can be started and the steps are as follows.

- 1) Extract the optimized result $\mathbf{X}_k$ from the previous step.
- 2) Linearize the system. Since the state prediction equation is nonlinear, Kalman filter cannot be directly used. Linearize the state variable $\mathbf{X}_k$ by first calculating its Jacobian matrix as

$$\mathbf{F}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{X}_k} \quad (12)$$

- 3) Use the linearized system to execute the prediction process of Kalman filter through

$$\hat{\mathbf{X}}_{k+1} = \begin{bmatrix} \mathbf{I} + \frac{T_s}{2} \boldsymbol{\Omega}(\hat{\boldsymbol{\omega}}_{k+1}) & 0 \\ 0 & \mathbf{I} \end{bmatrix} \mathbf{X}_k \quad (13)$$

$$\mathbf{P}_{k+1|k} = \mathbf{F}_k \mathbf{P}_k \mathbf{F}_k^T + \mathbf{Q}_k \quad (14)$$

Among them, $\mathbf{Q}_k$ is the covariance matrix of the noise component $\mathbf{w}_{k+1}$, and $\mathbf{P}_k$ is the covariance matrix of the optimized result $\mathbf{X}_k$ from the previous step. Based on these, the covariance matrix $\mathbf{P}_{k+1|k}$ of the predicted value $\hat{\mathbf{X}}_{k+1}$ can be calculated.

- 4) Similarly, find the Jacobian matrix of the observation system $\hat{\mathbf{Z}}_{k+1} = \mathbf{h}(\hat{\mathbf{X}}_{k+1})$ for linearization:

$$\mathbf{H}_{k+1} = \frac{\partial \mathbf{h}_{k+1}}{\partial \hat{\mathbf{X}}_{k+1}} \quad (15)$$

- 5) Solve the Kalman gain and complete one cycle of Kalman filtering, as shown in Eqs. (16) – (18),

$$\mathbf{X}_{k+1} = \hat{\mathbf{X}}_{k+1} + \mathbf{K}_{k+1} \mathbf{R}_{k+1} \quad (16)$$

$$\mathbf{R}_{k+1} = \mathbf{Z}_{k+1} - \mathbf{h}(\hat{\mathbf{X}}_{k+1}) \quad (17)$$

$$\mathbf{P}_{k+1} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{H}_{k+1}] \mathbf{P}_{k+1|k} \quad (18)$$

where $\mathbf{K}_{k+1}$ is the Kalman gain coefficient of the $(k+1)$ th iteration.

After several iterations, the optimized result variable $\mathbf{X}_k$ output by Kalman filtering will tend to converge, and the optimized pose quaternion $\mathbf{q}_k$ is obtained. To realize the Euler angle output, just convert the pose quaternion $\mathbf{q} = [q_0, q_1, q_2, q_3]^T$ to the Euler angles (α, β, γ) ,

$$\left. \begin{aligned} \alpha &= \text{Atan}2(2(q_0 q_1 + q_2 q_3), 1 - 2(q_1^2 + q_2^2)) \\ \beta &= \arcsin(2(q_0 q_2 - q_3 q_1)) \\ \gamma &= \text{Atan}2(2(q_0 q_3 + q_1 q_2), 1 - 2(q_2^2 + q_3^2)) \end{aligned} \right\} \quad (19)$$

The real-time performance of the algorithm is limited by the data acquisition of the IMU, the signal processing of the microprocessor, and the wireless transmission of Bluetooth. The sampling rates of the gyroscope and accelerometer are both set to 200 Hz, so as to balance the data acquisition delay and the processing performance of the microprocessor. Meanwhile, a high-precision timestamp is added to each collected IMU data frame to ensure data synchronization and alignment. The delay introduced by the microprocessor in executing the EKF fusion algorithm is approximately 20 ms, while the wireless transmission delay of the Bluetooth 5.0 technology adopted is about 60 ms. In summary, the total delay of the Euler angle output is within 100 ms. Considering that the change rate of the patient's joint pose during surgery is relatively slow, this delay can meet the real-time requirements of the system.

4 Method of registration and pose visualization

Using the IMU sensor and pose estimation algorithm, the device has already realized the measurement of its own pose in $\mathbf{W}$. Simply by mounting the device in a specific pose onto the acetabular cup impactor, the measurement of the cup's pose in $\mathbf{W}$ can be completed. However, the device cannot yet directly obtain the cup's pose relative to the body coordinate system $\mathbf{B}$, and thus cannot assist surgeons in implanting the cup into the safe zone. Therefore, a registration phase is

needed to establish the transformation relationship between $\mathbf{B}$ and $\mathbf{W}$, and then the cup's pose relative to $\mathbf{W}$ can be converted to that relative to $\mathbf{B}$, achieving the visualization of the cup's implantation pose.

4.1 Principle of registration

Assuming that the patient and the operating table are relatively stationary, the pose of the operating table is used to represent the patient's pose in the proposed method. Patients are often positioned in either the supine position or the lateral position during surgeries. For ease of discussion, this paper uses the left lateral position as an example to explain the principle.

In the left lateral position as shown in Fig. 3, the long axis of the operating table is considered parallel to the z_B axis. Mount the device on the operating table in any pose, then tilt the operating table around its long axis (i. e., z_B axis) by a small angle (about 10°), and record the Euler angles $(\alpha_1, \beta_1, \gamma_1)$ and $(\alpha_2, \beta_2, \gamma_2)$ before and after the rotation respectively. According to Eq. (3) and Eq. (5) mentioned earlier, the relative pose transformation matrix before and after rotation is shown as

$$\mathbf{R}_2^n = \mathbf{R}_x^T(\alpha_1) \mathbf{R}_y^T(\beta_1) \mathbf{R}_z^T(\gamma_1) \mathbf{R}_z(\gamma_2) \mathbf{R}_y(\beta_2) \mathbf{R}_x(\alpha_2) \quad (20)$$

Convert the rotation matrix $\mathbf{R}_2^n$ into the axis-angle form, that is, parameterize this rotation as a counterclockwise rotation by the angle φ around the rotation axis (unit vector) $\mathbf{u} = (u_x, u_y, u_z)^T$ in space. Then the angle φ and rotation axis $\mathbf{u}$ can be obtained through

$$\left. \begin{aligned} \cos \varphi &= \frac{1}{2} [\text{tr}(\mathbf{R}_2^n) - 1] \\ \sin \varphi &= \sqrt{1 - \cos^2 \varphi} \end{aligned} \right\} \quad (21)$$

$$\left. \begin{aligned} u_x &= \frac{1}{2 \sin \varphi} [\mathbf{R}_2^n(3, 2) - \mathbf{R}_2^n(2, 3)] \\ u_y &= \frac{1}{2 \sin \varphi} [\mathbf{R}_2^n(1, 3) - \mathbf{R}_2^n(3, 1)] \\ u_z &= \frac{1}{2 \sin \varphi} [\mathbf{R}_2^n(2, 1) - \mathbf{R}_2^n(1, 2)] \end{aligned} \right\} \quad (22)$$

In the above equations, $\text{tr}(\mathbf{R}_2^n)$ represents the trace of the matrix $\mathbf{R}_2^n$, and $\mathbf{R}_2^n(i, j)$ represents its element in the i th row and the j th column. As $\mathbf{R}_2^n$ is an intrinsic rotation matrix, the coordinates of $\mathbf{u}$ (i. e., $\mathbf{e}_{B_z}$) in

Eq. (22) are actually from $\mathbf{I}$ before the rotation, and should be converted to $\mathbf{W}$ through the transformation matrix $\mathbf{R}_{\mathbf{I}}^{\mathbf{W}}$

$$\mathbf{e}_{B_z}^{\mathbf{W}} = \mathbf{u}^{\mathbf{W}} = \mathbf{R}_{\mathbf{I}}^{\mathbf{W}} \mathbf{e}_{B_z}^{\mathbf{I}} = \mathbf{R}_{\mathbf{I}}^{\mathbf{W}} \mathbf{u} \quad (23)$$

Consider that the operating table is placed horizontally, the x_B axis is parallel to the z_W axis, so the coordinates of the unit vector $\mathbf{e}_{B_x}^{\mathbf{W}}$ on the x_B axis in $\mathbf{W}$ can be written as:

$$\mathbf{e}_{B_x}^{\mathbf{W}} = \mathbf{e}_{W_z}^{\mathbf{W}} = (0, 0, 1)^T \quad (24)$$

Thus, the direction of the y_B axis can be derived according to the x_B axis and z_B axis

$$\mathbf{e}_{B_y}^{\mathbf{W}} = \mathbf{e}_{B_z}^{\mathbf{W}} \times \mathbf{e}_{B_x}^{\mathbf{W}} \quad (25)$$

Now the transformation matrix $\mathbf{R}_B^{\mathbf{W}}$ of the body coordinate system $\mathbf{B}$ relative to the world coordinate system $\mathbf{W}$ can be finally obtained according to the definition of the rotation matrix.

$$\mathbf{R}_B^{\mathbf{W}} = \mathbf{R}_B^{\mathbf{W}} [\mathbf{e}_{B_x}^{\mathbf{B}} \quad \mathbf{e}_{B_y}^{\mathbf{B}} \quad \mathbf{e}_{B_z}^{\mathbf{B}}] = [\mathbf{e}_{B_x}^{\mathbf{W}} \quad \mathbf{e}_{B_y}^{\mathbf{W}} \quad \mathbf{e}_{B_z}^{\mathbf{W}}] \quad (26)$$

In Eq. (26), $\mathbf{e}_{B_x}^{\mathbf{B}}$, $\mathbf{e}_{B_y}^{\mathbf{B}}$ and $\mathbf{e}_{B_z}^{\mathbf{B}}$ are the unit orthogonal basis vectors of $\mathbf{B}$, so the matrix $[\mathbf{e}_{B_x}^{\mathbf{B}} \quad \mathbf{e}_{B_y}^{\mathbf{B}} \quad \mathbf{e}_{B_z}^{\mathbf{B}}]$ should be an identity matrix. With the transformation relationship $\mathbf{R}_B^{\mathbf{W}}$ between $\mathbf{B}$ and $\mathbf{W}$ established, the registration phase is completed.

In special cases (such as pelvic deformity, scoliosis, or intraoperative positional instability in patients), non-invasive registration cannot be completed by substituting the operating table pose for the patient's pose. In such cases, it is necessary to switch to the invasive method: first, insert bone screws into appropriate positions in the patient's pelvis, fix the device onto the bone screws, then tilt the patient's position, and complete the registration using the same principle as mentioned above.

4.2 Pose measurement and visualization of the cup

After the registration phase, the device is mounted on the acetabular cup impactor with the x - o - y plane of the IMU coordinate system $\mathbf{I}$ orthogonal to the acetabular cup axis so that the cup axis can be represented by the z -axis of $\mathbf{I}$, i. e., $\mathbf{n} = \mathbf{e}_{I_z}$. Obviously, the coordinates of $\mathbf{n}$ in $\mathbf{I}$ are $\mathbf{n}^{\mathbf{I}} = \mathbf{e}_{I_z}^{\mathbf{I}} = (0, 0, 1)^T$.

Now the device can continuously output Euler angles $(\alpha_c, \beta_c, \gamma_c)$ to represent the real-time pose of $\mathbf{I}$ relative

to $\mathbf{W}$, and the coordinates of $\mathbf{n}$ in $\mathbf{W}$ can be further obtained.

$$\mathbf{R}_{I_c}^{\mathbf{W}} = \mathbf{R}_z(\gamma_c) \mathbf{R}_y(\beta_c) \mathbf{R}_x(\alpha_c) \quad (27)$$

$$\mathbf{n}^{\mathbf{W}} = \mathbf{R}_{I_c}^{\mathbf{W}} \mathbf{n}^{\mathbf{I}} = \mathbf{R}_{I_c}^{\mathbf{W}} \mathbf{e}_{I_z}^{\mathbf{I}} \quad (28)$$

Combining the transformation relationship $\mathbf{R}_B^{\mathbf{W}}$ between $\mathbf{B}$ and $\mathbf{W}$ in Eq. (26) established during registration and the coordinates of $\mathbf{n}^{\mathbf{W}}$ in Eq. (28), the coordinates of $\mathbf{n}$ in $\mathbf{B}$ can be obtained

$$\mathbf{n}^{\mathbf{B}} = \mathbf{R}_B^{\mathbf{B}} \mathbf{n}^{\mathbf{W}} = \mathbf{R}_B^{\mathbf{W}^{-1}} \mathbf{n}^{\mathbf{W}} = \mathbf{R}_B^{\mathbf{W}^T} \mathbf{R}_{I_c}^{\mathbf{W}} \mathbf{n}^{\mathbf{I}} \quad (29)$$

So far, the coordinates of $\mathbf{n}^{\mathbf{B}} = (n_x^{\mathbf{B}}, n_y^{\mathbf{B}}, n_z^{\mathbf{B}})^T$ have been solved. As mentioned earlier in the problem definition, the cup's inclination θ_i and anteversion θ_a can finally be calculated by Eq. (2) and the real-time pose estimation of the acetabular cup is completed.

To assist surgeons in completing the surgery, a reasonable way to feed back the measured pose information to the surgeons is necessary. In the proposed method, a PC is used as a tool to visualize the cup's pose in real time through numerical display and 3D animation. The interface of the program written in Python for registration and numerical display of cup's pose is shown in Fig. 8, which includes functions such as registration, real-time pose display, and process pose data storage.

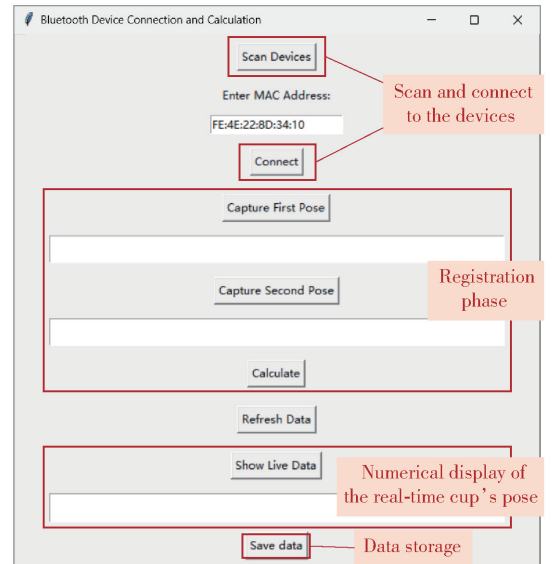


Fig. 8 Program interface for real-time numerical display of pose

A real-time 3D animation demonstration system is also designed for pose visualization as shown in Fig. 9.

It is developed through the open-source software VOFA-Plus, which supports the independent addition of 3D controls, parameter setting, and dynamic display. Cooperating with the sensor fixed on the bone model, it can realize the real-time reproduction of the bone model pose.

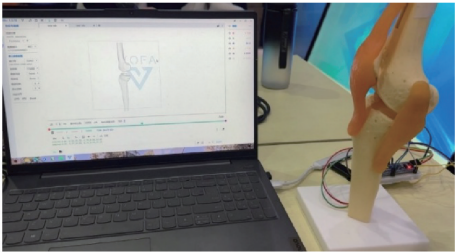


Fig. 9 3D animation demonstration system used with bone models for real-time pose visualization of the joint

5 Experimental results and analysis

To verify the effectiveness of the prosthesis pose visualization method proposed in this paper, a 3-axis tilt table (as shown in Fig. 10) is used. This precision instrument consists of three layers: the bottom two layers are two orthogonally stacked single-axis tilt tables, and the third layer is a circular turntable that can rotate 360° around its vertical axis.

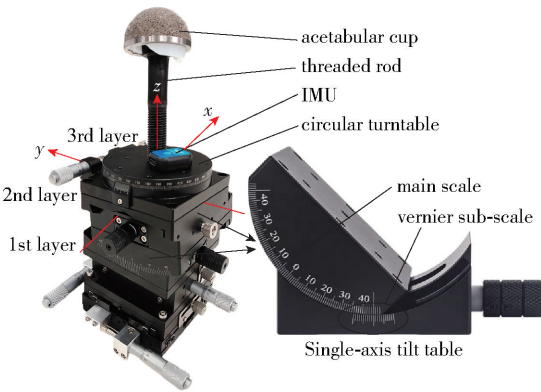


Fig. 10 The 3-axis tilt table used in experiments

Each layer is equipped with a main scale and a vernier scale to improve the accuracy of angle reading, with a resolution of 0.1°. There is a threaded rod in the center of the tilt table, one end connected to the acetabular cup prosthesis, and the other end screwed vertically into the center of the circular turntable,

which is used to simulate the acetabular cup axis during surgery.

This instrument can simulate both extrinsic rotation and intrinsic rotation in three dimensions. According to its three-layer stacked structure, if rotating in the order of z - y - x axes, it corresponds to extrinsic rotation; if rotating in the order of x - y - z axes, it corresponds to intrinsic rotation. A series of experiments were conducted with this instrument.

5.1 Experiment 1: accuracy of basic self-pose estimation

This experiment aims to test the performance of the IMU-based device in estimating its own rotation angle. The device was mounted on the 3-axis tilt table in any initial pose, and then the tilt table was controlled to rotate around the three axes respectively. More than 10 tests were conducted about each axis. During this process, record the rotation angle value of the tilt table (true value, denoted as y_i) and the angle value output by the device (estimated value, denoted as $\hat{y}_i$) to compare the error between them. The test results are shown in Table 2, where the root mean square error (RMSE) and mean absolute percentage error (MAPE) are calculated respectively.

$$e_{\text{RMSE}} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2} \tag{30}$$

$$e_{\text{MAPE}} = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \right| \tag{31}$$

The results show that the estimated angle $\hat{y}_i$ is in good agreement with the true angle y_i regardless of rotation around any axis, and the absolute error is maintained within $\pm 1^\circ$.

Table 2 Angle estimation accuracy of rotations in the three axes

Rotation axis	RMSE/(°)	MAPE/%
z -axis	0.400	2.8
y -axis	0.409	3.3
x -axis	0.423	3.6

5.2 Experiment 2: estimation of rotation axis

This experiment aims to test the accuracy of

obtaining the coordinates of the z_B -axis of $\mathbf{B}$ by rotating the operating table, which is the key step during the registration phase. Mount the device on the turntable of the 3-axis tilt table, and then rotate the tilt table around its y -axis to simulate the tilt of the operating table around its long axis.

The experiments were repeatedly conducted with the tilt table being placed at four different positions on the test tabletop. The differences among them are as follows. At position 1, the tilt table always rotated by a fixed 10° about its y -axis, but the device was mounted to the tilt table in varying poses each time to test whether the pose of device affects the rotation axis vector acquisition; at positions 2–4, the device always maintains a fixed pose, while the rotation angle of the tilt table around its y -axis varies from 3° to 15° , aiming to study the influence of different rotation angles. The rotation axis vector coordinates obtained under different positions, poses, and rotation angles are plotted in a scatter diagram, as shown in Fig. 11.

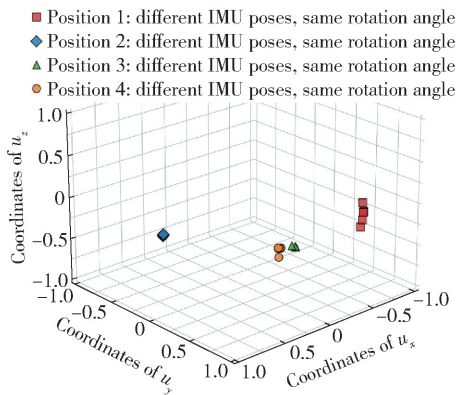


Fig. 11 Coordinates of rotation axis unit vector $\mathbf{u}^w$ in $\{\mathbf{W}\}$ acquired during registration phase

Since the tilt table is placed on a horizontal test tabletop, its y -axis is almost horizontal. Therefore, the u_z^w of the rotation axis vector $\mathbf{u}^w$ should be close to zero, and only u_x^w and u_y^w change with the position. Among all 20 test results, only 4 cases showed slight outliers, i. e., $|u_z^w| > 0.1$, and the remaining 16 results had $|u_z^w|$ within 0.04. These slight outliers may be caused by measurement jitter or noise. In addition, when the rotation angle is extremely small, $|u_z^w|$ tends to increase, which indicates that the

rotation angle of the operating table around the axis during registration should not be too small.

5.3 Experiment 3: measurement accuracy of the cup's inclination θ_i and anteversion θ_a

This experiment aims to verify the measurement accuracy of the acetabular cup's θ_i and θ_a . The 3-axis tilt table is used to simulate the body coordinate system $\mathbf{B}$, and the threaded rod screwed into the center is used as the acetabular cup axis, representing the implantation angle of the acetabular cup. From the geometric relationship, it can be known that the inclination θ_i and anteversion θ_a are equivalent to the rotation angles of the tilt table around its x -axis and y -axis respectively. After registration, the device was mounted face up on the top circular turntable to ensure that its z_I -axis is parallel to the acetabular cup axis. Rotate the tilt table around its x -axis and y -axis to make the acetabular cup axis in a series of preset positions (changing from $(25^\circ, 5^\circ)$ to $(40^\circ, 20^\circ)$ at 5° intervals). At the same time, the PC receives the Euler angle data in real time and calculates the θ_i and θ_a . Plot the estimated values (abbreviated as Est. Value) and the true values on the tilt table in the form of a 2D vector (θ_i, θ_a) on a 2D coordinate diagram to obtain the error vector, as shown in Fig. 12.

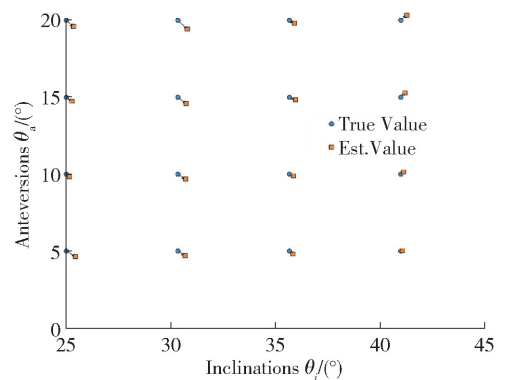


Fig. 12 Comparison of estimated (θ_i, θ_a) values and true values, and corresponding 2D error vectors

The measurement accuracy is evaluated by the RMSE of the two angles and the modulus of the error vector, as shown in Table 3. The results show that the method proposed in this paper has high measurement accuracy. The RMSE of θ_i and θ_a are both lower than

0.3° , and the modulus of the error vector at all test positions is generally lower than 0.714° . This result is much smaller than the $\pm 10^{\circ}$ tolerance range specified by the “safe zone”.

Table 3 Measurement accuracy of cup inclination and anteversion

RMSE of		$ ((\theta_i, \theta_a))_{\text{Est.}} - ((\theta_i, \theta_a))_{\text{True}} $		
$\theta_i/(^{\circ})$	$\theta_a/(^{\circ})$	Min/ $(^{\circ})$	Mean/ $(^{\circ})$	Max/ $(^{\circ})$
0.278	0.296	0.073	0.373	0.714

5.4 Experiment 4: influence of IMU drift errors

Due to the drift error in the IMU, which accumulates over time, the method proposed in this paper may become less effective during long time use. Therefore, this experiment was designed to test the impact of IMU drift. Similar to the 3rd experiment, the θ_i and θ_a were measured at a series of preset acetabular cup angles with the device pose unchanged. The difference in this experiment was that the procedure was conducted again after 10 and 20 min. The influence of drift was analyzed by comparing the error vectors of the estimated values at different time as shown in Fig. 13.

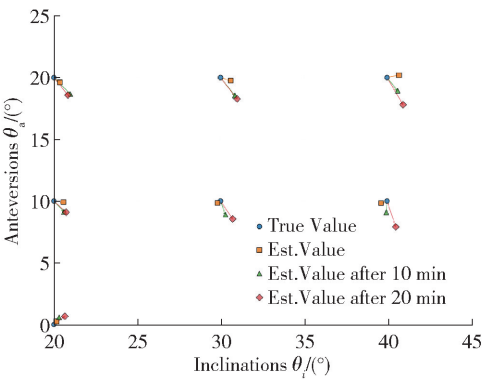


Fig. 13 Comparison of error vectors at different time

The RMSE of measurements and the mean magnitude of error vectors obtained at different time points are listed in Table 4. As evident from the results, IMU drift has a marked negative influence on measurement accuracy: all error results increase progressively as test duration increases. Nevertheless, after a 20-min test period, although the average error grows to roughly 3.5 times its initial value, all absolute errors are confined within $\pm 3^{\circ}$, which can well satisfy the accuracy criteria for acetabular cup orientation measurement. Considering that the acetabular cup implantation procedure takes approximately 30 min, it is recommended that surgeons perform secondary registration midway through the operation to recalibrate IMU angular measurements.

Table 4 Measurement accuracy of θ_i and θ_a in different time

Test time/ min	θ_i RMSE/ $(^{\circ})$	θ_a RMSE/ $(^{\circ})$	Average error vector magnitude/ $(^{\circ})$
0	0.469	0.234	0.494
10	0.618	1.085	1.199
20	0.791	1.586	1.704

5.5 Comparison and discussion

The comparison between this work and other existing technologies is presented in Table 5. This work shows advantages in multiple aspects, including high prosthesis compatibility, non-invasiveness, and no radiation exposure. In particular, its angular measurement error is significantly lower than that of other technologies. However, it must be noted that this excellent angular measurement accuracy is associated with the experimental environment. This work was tested on a precise 3-axis tilt table, which eliminated errors caused by patient position shifts and human soft tissue compression.

Table 5 Performance comparison among different prostheses pose estimation technologies for THA

Method	Sensor Type	Angular error	Prosthesis adaptability	Invasiveness	Radiation exposure	Experimental environment
This work	IMU	RMSE: $\theta_i = 0.278^{\circ}$, $\theta_a = 0.296^{\circ}$	high	non-invasive	no	3-axis tilt table
IMU-Based real-time measuring system ^[15]	IMU	RMSE: $\theta_i = 3.26^{\circ}$, $\theta_a = 3.05^{\circ}$	low	non-invasive	no	customized measuring device
IMU-Based smart trial system ^[13]	IMU	MAE: $\theta_i = 2.3^{\circ}$, $\theta_a = 2.2^{\circ}$	low	non-invasive	no	in vitro tests on sawbones
Computer-assisted systems ^[11]	camera	MAE: $\theta_i = 1.8^{\circ}$, $\theta_a = 2.0^{\circ}$	high	invasive	yes	real cases (25 patients)

Therefore, the previously mentioned experiments primarily verified the theoretical correctness and feasibility of the proposed method under an ideal condition. Meanwhile, it also indicates that this work needs to be further validated through in vitro tests and clinical trials in the future to examine its effectiveness in complex real surgical scenarios.

6 Conclusions and future work

Aiming at the key problem of prosthesis implantation into the safe zone and the limitations of traditional technologies, this study focuses on THA and proposes a prosthesis pose visualization method based on multi-sensor fusion. A complete hardware-software system was constructed. The hardware mainly consists of the MPU9250 IMU and the STM32F103 microprocessor to realize the collection and processing of motion data. The EKF algorithm is applied to fuse accelerometer and gyroscope data and realize basic self-pose estimation. A non-invasive registration method is proposed to establish the transformation relationship between the patient's body coordinate system and the world coordinate system, enabling further calculation of the acetabular cup's real-time inclination and anteversion. Numerical display and 3D animation demonstration programs are designed to finally realize the visual feedback of the prosthesis pose to surgeons.

Four experiments using a 3-axis tilt table verified the proposed method. The IMU's reliability in its self-pose estimation was validated with the RMSE ranging from 0.400° to 0.423° . Experiments at four tilt table positions showed that the coordinates of the rotation axis vector could be accurately acquired, with slight variations caused by different IMU poses and rotation angles. Another test demonstrated high accuracy in measuring cup orientations (RMSE: 0.278° for inclination, 0.296° for anteversion; average error vector: 0.373°), well within clinical tolerance. The 20-min robustness tests showed that IMU drift caused errors to increase by approximately 3.5 times. To address this, it is recommended to perform secondary

registration every 15 minute to reduce drift errors.

The proposed method does not require the implantation of bone markers, which means it is non-invasive and radiation-free. It shows important clinical value and social significance as it not only provides a new navigation scheme for joint replacement surgery, but also offers a reference for the application of multi-sensor fusion in orthopedic surgery. However, this device is currently a demonstration system and this method still lacks clinical trials. In the future, various clinical trials need to be conducted to study the soft tissue compression compensation for obese patients and adaptive algorithms for multiple surgical approaches, and further verify its effectiveness in patients with different weights, body types, and surgical approaches. After productization and medical device certification, the proposed method is expected to become a standardized clinical navigation tool.

Acknowledgements

This work was supported by the Fujian Provincial Natural Science Foundation (2021J01335).

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(Editor: Luo Lang)