

Moving-load impact of classic hinged-hinged slab beam and demarcation to gravity stretching retention effect

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ABSTRACT This paper investigates the displacement and bending moment impact amplification of the classic hinged-hinged beam to moving load and demarcates their applicable range to dead load gravity stretching retention effect. A modified Euler–Bernoulli beam model with an extension to consider stretching retention effect due to beam flexure is developed. A multi-harmonic frequency-multiplication modal forced oscillation theory is adopted to analyze the impact spectrum of the classic beam under single moving force. The applicable range of the computed impact spectrum is demarcated based on the evaluation of the additional response and stretching force increment. This study proposes to compute the peak response occurrence spatial positions besides the impact factors for safety and fatigue evaluation. The results for the computed bending moment response in the low moving speed region tell, the normalized peak response occurrence spatial position curves are of the similar shape and magnitude as the corresponding normalized peak response occurrence instant curves for the classic beam. A case study on the related demarcation analysis present one design of a slender steel beam that the classic Euler beam model is applicable to the moving load impact analysis for the scenarios of low moving speed around its self-weight equilibrium state.

KEYWORDS impact spectrum, Euler–Bernoulli beam, flexure stretching, moving load, harmonic analysis

1 Introduction

The Euler–Bernoulli beam is a commonly used structural mechanics model that is considered to be of good robustness for bridge and engineering structure analysis and design under different load combinations. Moving load is one of the most general type of live load for engineering structures. Considering the short-duration action feature, it is a continuous focus to investigate the impact amplification of beam-like structures under moving loads for engineering purposes. The impact amplification can be evaluated using the impact factor which is a function of the generalized moving speed of the vehicle load. Although the inertial effect and internal stiffness of the vehicle load will induce additional oscillation patterns [1–5] for the studied dynamic system, the moving concentrated force model is generally adopted to be of reasonable precision for engineering simplified analysis.

Similar to the general concept of impact factor [6–9] for moving-load excited bridge deformation response evaluation, the impact factor adopted in this study for the bending moment moving-load impact effect evaluation is defined to be

$$\mu_B(\beta) = \frac{\max[B_D(x, t, \beta)] - \max[B_S(x, x_0)]}{\max[B_S(x, x_0)]}, \quad (1)$$

where B_D is the dynamic bending moment response of the bridge under moving load of a generalized moving speed β ; B_S is the static bending moment response of the bridge under a concentrated force at x_0 ; $\beta = \bar{\omega}/\omega_1$ is the moving load forcing frequency ratio which can be regarded as a generalized moving speed; $\bar{\omega} = 2\pi\bar{s}/L$ is the forcing frequency under moving force; ω_1 is the fundamental frequency of the bridge; $\bar{s}$ is the moving speed of the moving load; L is the span length of the bridge. The operator of $\max(\cdot)$ means to take the maximum value considering the variation of two variables. As the stress resultants, especially the bending moment which is

proportional to the deflection curvature, the contributions of the high-order modes become important. Moreover, the non-smoothness at the load action point and the discontinuity at the fixed end points for the bending moment diagram require more terms of the series expansion approximation [10–12]. The involvement of more modes and more harmonic components make the shock behavior and the impact amplification complex.

Large amounts of related literatures highlight the importance and diversity of the problems. Besides the obvious factor related to the moving-load induced dynamic excitation, the generalized moving speed of the vehicle, many additional factors are demonstrated to be influential on the variation of the moving-load impact factors according to the reported computational and experimental results. These factors include bridge span length and superstructure type [7–10,13], vehicle models considered [5,6,8,14], road surface roughness or track irregularities [7,10], the initial bumping of the vehicle [14], the axle distance and span length ratio [15], etc. The mechanism of the impact effect considering the combinational influence of one or several factors is complex. It seems that it is appropriate to conduct the research work to deal with the related problem case by case.

Concerning of structural geometric nonlinearity, the dead load gravity retention effect is another category of important factor for the behavior analysis of slender beam-like structures. The gravity stiffness is the most popularly accepted concept in this category for structural analysis. Generally, the related concept is adopted for static design and analysis of large-span complex system suspension bridges [16,17]. However, some theoretical investigation on slender beam structures showed that structural geometric nonlinearity will notably modulate the free and forced vibrations even for simple beams of static or postbuckling configurations [18–20]. The related investigation for structures in different loading scenarios is still a challenge.

In this study, moving-load induced impact spectrum of

the supporting single-span hinged-hinged (HH) beam bridge considering structure dead load gravity stretching retention is investigated. The classic axially-loaded Euler–Bernoulli beam model with an extension to consider the nonlinear stretching effect is developed for the analysis. The multi-harmonic frequency-multiplication modal forced oscillation (MFMFO) theory [5] is adopted to analyze the impact spectrum of the linear beam with small deformation. Considering the importance of the stress and stress resultants under moving load for bridge design purpose, both the deformation and bending moment responses are considered. The formation of the impact factor curves for beam modal coordinates, deformation and bending moment responses is discussed. Finite element method (FEM) structure modeling based numerical integration is conducted to verify the impact factor curve computed from modal superposition. The impact factor curves as well as the peak response occurrence spatial positions and temporal instants of single-span HH beam are presented. The applicable range of the computed impact curve is demarcated considering structure gravity stretching retention.

2 Analytical model

The studied structure is a HH isotropic beam with uniform cross section (Fig. 1(a)). The basic assumptions adopted for the analysis are:

- 1) the plane section assumption of a general Euler beam is hold;
- 2) the beam is in small deflection;
- 3) the axial strain of the beam is a constant along the axis;
- 4) the rotation inertial of the thin beam can be neglected;
- 5) dead load gravity effect is more important than the moving load effect;
- 6) no structural system transition during its erection.

For a slender isotropic beam of Young's modulus E ,

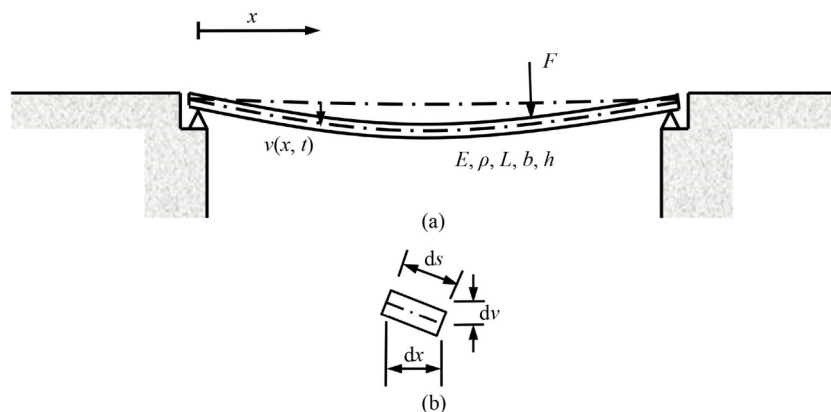


Fig. 1 A HH slab beam carrying single moving force: (a) the layout of the system; (b) one deformed micro-element.

mass density ρ , the area of cross-section A , and the moment of inertia I , it can be modeled as an Euler–Bernoulli beam with an axial force N considering the stretching effect. The governing equation of motion (EOM) for the vertical deformation $v(x, t)$ of the beam under the uniformly distributed dead load q and the axial force N can be expressed as [21]

$$EI \frac{\partial^4 v}{\partial x^4} + \rho A \frac{\partial^2 v}{\partial t^2} - N \frac{\partial^2 v}{\partial x^2} = q, \quad (2a)$$

$$v(0, t) = 0, v''(0, t) = 0, v(L, t) = 0, v''(L, t) = 0. \quad (2b)$$

Considering no structural system transition during the erection and in-operation stages (the assumption 5) in this study, N due to uniform axial stretching of the beam during its bending under gravity load (Fig. 1(b)) can be expressed as

$$N = \frac{EA}{L} \int_0^L \left(\frac{ds}{dx} - 1 \right) dx \doteq \frac{EA}{2L} \int_0^L \left(\frac{dv}{dx} \right)^2 dx, \quad (3)$$

where $ds = \sqrt{(dx)^2 + (dv)^2}$. Considering this effect, the law of linear superposition fails. For the considered loading process, $v(x, t) = \bar{v}(x) + \tilde{v}(x, t)$, where $\bar{v}(x)$ is the static deformation when q is the dead load gravity, and $\tilde{v}(x, t)$ is the dynamic response under moving load when $q = F\delta(x - \bar{s}t)$, where F is the magnitude of the moving force. It is required to take account of the dead load gravity stretching retention effect for moving load excited response analysis.

Based on the assumptions adopted for this study, around the static equilibrium position under structure dead load gravity, $\tilde{v}(x, t)$, excited by single moving force F , is governed by the EOM of the following form

$$EI \tilde{v}'''' + \rho A \ddot{\tilde{v}} - \frac{EA}{2L} \left(\int_0^L \tilde{v}'^2 dx \tilde{v}'' + 2\tilde{v}'' \int_0^L \tilde{v}' \tilde{v}' dx + \tilde{v}'' \int_0^L \tilde{v}'^2 dx + 2\tilde{v}'' \int_0^L \tilde{v}' \tilde{v}' dx + \tilde{v}'' \int_0^L \tilde{v}'^2 dx \right) = F\delta(x - \bar{s}t), \quad (4a)$$

$$\text{B.C.: } \tilde{v}(0, t) = 0, \tilde{v}''(0, t) = 0, \tilde{v}(L, t) = 0, \tilde{v}''(L, t) = 0, \quad (4b)$$

$$\text{I.C.: } \tilde{v}(x, 0) = v_0(x), \dot{\tilde{v}}(x, 0) = u_0(x), \quad (4c)$$

where prime denotes the differentiation with respect to the axis coordinate x , dot denotes the differentiation with respect to time t .

The first two terms in the left-hand-side of Eq. (4a) are the classic Euler–Bernoulli beam dynamics model terms. The second 5 terms are the terms due to the gravity stretching effects, which are functions of structural deformation response. Concerning of the assumption of

small deformation, these terms, which are the third degree of polynomial of beam deformation responses, can be considered to be micro-correction to the classic beam dominated by the first two terms. Introduce a small dimensionless quantity ε , the dynamic response of the system, relative to a pre-set characteristics length l^* , is evaluated up to the scale of ε . Therefore, an asymptotic expansion on beam dynamic deformation response relative to l^* is made to be $\tilde{v}/l^* = \varepsilon \tilde{v}_1 + \varepsilon^2 \tilde{v}_2$. As the static deformation under dead load gravity load is generally more important as assumed, $\bar{v}$ relative to l^* is considered to be evaluated at the scale of $\varepsilon^{1/2}$ and is expressed as $\bar{v}/l^* = \varepsilon^{1/2} \bar{v}_1 + \varepsilon \bar{v}_2$. The linearized EOMs governing the response of $\tilde{v}_1$ and $\tilde{v}_2$ can thus be expressed as

$$\varepsilon: EI \tilde{v}_1'''' + \rho A \ddot{\tilde{v}}_1 = \tilde{F} \delta(x - \bar{s}t), \quad (5a)$$

$$\varepsilon^2: EI \tilde{v}_2'''' + \rho A \ddot{\tilde{v}}_2 = \frac{EA l^{*2}}{2L} \int_0^L \bar{v}_1'^2 dx \tilde{v}_1'' + \frac{EA l^{*2}}{L} \bar{v}_1'' \int_0^L \bar{v}_1' \tilde{v}_1' dx, \quad (5b)$$

where $\tilde{F} = F/\varepsilon l^*$. For the studied HH beam under gravity load, the static deflection, $\bar{v}_1$, which satisfies the small-deflection assumption, can be approximately expressed to be

$$\bar{v}_1(x) = \frac{q}{24EI\varepsilon^{1/2}l^*} (x^4 - 2Lx^3 + L^3x). \quad (6)$$

Herein, q is introduced to take account of uniformly distributed gravity load of beam self-weight and some secondary dead load, such as the bridge deck pavement.

Considering that the value of ε is a small quantity, the dominant component of this system is governed by the classic linear time-invariant Euler–Bernoulli beam dynamics model as shown in Eq. (5a). It is thus convenient and reasonable to evaluate the impact effects to the short-duration passing load based on this dominant component. If the components determined by Eq. (5b) is comparably notable according to the dynamic computation, the additional response due to gravity stretching effect cannot be neglected. Further detailed analysis need to be conducted case by case. In this study, the applicable range of the classic beam model governed by Eq. (5a) and the computed impact spectrum is demarcated for a HH beam.

3 Fundamental mode impact effect

3.1 Occurrence position

For the dominant component governed by the classic Euler beam model of Eq. (5a), the response can be expressed according to the mode summation as

$$\tilde{v}_1(x, t) = \sum_r \phi_r(x) \tilde{v}_{1,r}(t), \quad (7)$$

where $\phi_r(x)$ and $\tilde{v}_{1,r}(t)$ are the mode shape and modal coordinate response of the r th mode according to the classic model. For the studied HH beam, $\phi_r(x) = \sin(r\pi x/L)$.

When $\bar{s} \rightarrow 0$, $\tilde{v}_{1,r}(t) \doteq \tilde{F}/K_{rr}\phi_r(\bar{s}t)$ according to the reciprocity theorem [22], where $K_{rr} = EI \int_0^L \phi_r(x) \phi_r''''(x) dx$ is the r th modal stiffness of the beam. Taking the differentiations of the above expression to the axial coordinate x , structure rotation and curvature responses are obtained. The bending moment response $B_D(x, t)$ under the moving force can be computed using the following formula

$$B_D(x, t) = EI \sum_r \phi_r''(x) \tilde{v}_{1,r}(t), \quad (8)$$

where EI is the flexural rigidity of the uniform beam bridge and $\phi_r''(x)$ is the mode shape curvature of the r th mode. The mode shape $\phi_r(x)$ for a single-span HH beam can be unifiedly expressed as the following Fourier series

$$\phi_r(x) = \sum_{n=0}^{\infty} A_{n,r} \sin(2n\pi\bar{x} + \theta_{n,r}), \quad (9)$$

where $\bar{x} = x/L$,

$$A_{n,r} = \sqrt{a_{n,r}^2 + b_{n,r}^2}, \quad (10a)$$

$$\theta_{n,r} = \arctan(a_{n,r}, b_{n,r}), \quad (10b)$$

where $a_{n,r}$ and $b_{n,r}$ are the Fourier coefficients of the n th harmonic component for the r th mode as expressed in Ref. [5].

According to Eq. (1), the impact factor is evaluated at both the peak response occurrence spatial positions and time instants. For single-span beam, as both structure deformation and bending moment response under moving force is dominated by the fundamental mode, the peak response occurrence spatial positions can be predicted to be around the peak value points of the fundamental mode shape and curvature at the mid-span position of the beam.

3.2 Occurrence instant

As the impact amplification effect on beam deformation and bending moment response can be regarded as to evaluate the envelop of peak value of modal responses with different weights, the evaluation of the modal impact effect, especially for the fundamental mode, is important for the studied single-span HH beam. Generally, for the r th mode, the modal coordinate oscillation responses $v_r(t)$ induced by a moving load can be analyzed in two stages: the forced oscillation stage when the moving load is on the bridge and the free vibration stage after the load moves off the bridge. The maximum impact effect may

occur in the on-beam stage or in the off-beam stage. The impact effect of single moving load on the modal coordinate response can be evaluated according to the maximum value of the dynamic response ratio time history $R_r(t)$ due under a moving load.

In the on-beam stage, the response of the r th modal coordinates of the modal frequency ω_r and modal damping ratio ξ_r under a moving force is derived to be composed of multiple frequency-multiplication modulated harmonics and one single-frequency transient harmonics. The related dynamic response ratio time history can be expressed as

$$R_r(\bar{t}) = \sum_n \rho_{n,r} \sin(2n\pi\bar{t} + \psi_{n,r}) + A_{c,r} \sin\left(\frac{2\pi}{\beta/\beta_r^D} \bar{t} + \psi_{c,r}\right) \exp\left(-\frac{2\pi\xi_r}{\beta/\beta_r} \bar{t}\right), \quad (11)$$

where $R_r = \tilde{v}_{1,r} / \frac{\tilde{F} p_{\phi_r}}{K_{rr}}$, $\bar{t} = t/T$

$$\rho_{n,r} = \frac{A_{n,r}}{p_{\phi_r} \sqrt{(1 - (n\beta/\beta_r)^2)^2 + (2n\xi_r\beta/\beta_r)^2}}, \quad (12a)$$

$$\psi_{n,r} = \theta_{n,r} - \arctan(2n\xi_r\beta/\beta_r, (1 - n^2\beta^2/\beta_r^2)), \quad (12b)$$

where $\beta_r^D = \omega_r^D/\omega_1$, $\beta_r = \omega_r/\omega_1$, $T = L/\bar{s}$, $\omega_r^D = \omega_r \sqrt{1 - \xi_r^2}$, $p_{\phi_r} = \max(\phi_r(x))$, $A_{c,r}$ and $\psi_{c,r}$ are constants determined from the given initial conditions. If the beam is at rest originally, $A_{c,r}$ and $\psi_{c,r}$ can be expressed as

$$A_{c,r} = -\sqrt{\left[\sum_n \rho_{n,r} \sin \psi_{n,r}\right]^2 + \left[\sum_n \rho_{n,r} C_{n,r} \sin(\psi_{n,r} + \vartheta_{n,r})\right]^2}, \quad (13a)$$

$$\psi_{c,r} = \arctan\left(-\sum_n \rho_{n,r} \sin \psi_{n,r}, -\sum_n \rho_{n,r} C_{n,r} \sin(\psi_{n,r} + \vartheta_{n,r})\right), \quad (13b)$$

where $C_{n,r} = \sqrt{n^2(\beta/\beta_r^D)^2 + \xi_r^2} / (1 - \xi_r^2)$, $\vartheta_{n,r} = \arctan(n\beta/\beta_r^D, \xi_r/\sqrt{1 - \xi_r^2})$.

For the static or pseudo-static cases when $\beta \rightarrow 0$, $\rho_{n,r} \rightarrow A_{n,r}/p_{\phi_r}$, $\psi_{n,r} \rightarrow \theta_{n,r}$, $A_{c,r} \rightarrow 0$. The comparison of Eqs. (9) and (11) tells that $v_{1,r}(x_0)$ ($x_0 \in [0, L]$) presents the same shape as the $\phi_r(x)$. For the dynamic cases with different β , the amplitude amplification and phase angle of different harmonic components will distort the response shape and induce dynamic amplification. Figure 2 presents the computed dynamic amplification and phase angle curves for the multi-harmonic forced components of the fundamental mode for four typical single-span HH uniform beams under moving force of the generalized moving speed β . As shown in Fig. 2(a), for the studied

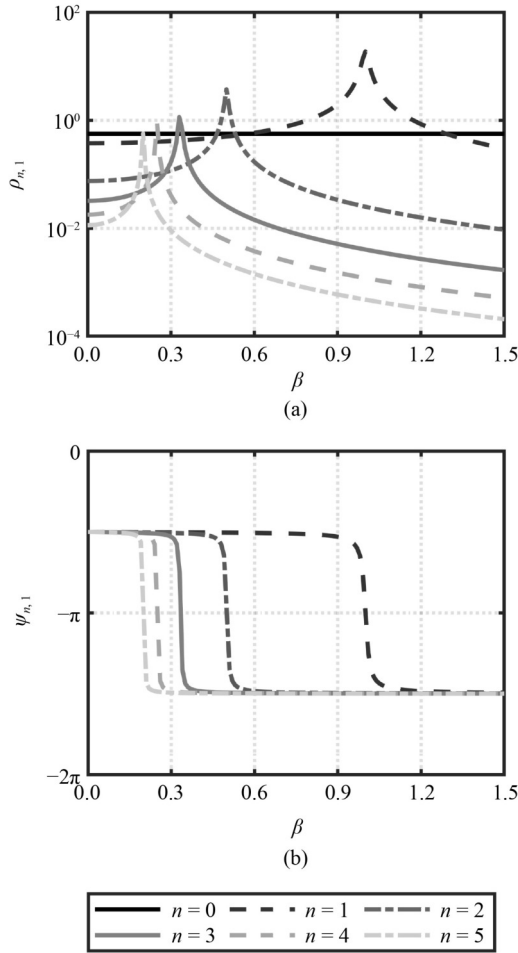


Fig. 2 The computed (a) dynamic amplification curves and (b) phase angle curves for the multi-harmonic forced oscillation components of the fundamental mode of the HH beam under moving force ($\xi_1 = 1\%$).

HH beam, the importance of the high-order harmonic oscillations will decrease with the increase of n . Even considering the dynamic amplified peak response, only the first three components ($n = 0, 1, 2$) are important for the modal response estimation. The phase angle curves in Fig. 2(b) shows multiple in-phase to out-of-phase transition processes of different harmonic components.

After the concentrated force moves off the bridge, the bridge will make a free vibration after the short-duration shock due to the moving load. The oscillation of the r th modal coordinates is a single modulated harmonics. The related dynamic response ratio time history can be expressed as

$$R_r(\bar{t}) = \left\{ R_r(1) \cos \left[\frac{2\pi}{\beta_r^D} (\bar{t} - 1) \right] + \left[\frac{\dot{R}_r(1) + \xi_r \omega_r R_r(1)}{\omega_r^D} \right] \cdot \sin \left[\frac{2\pi}{\beta_r^D} (\bar{t} - 1) \right] \right\} \exp \left[-\frac{2\pi \xi_r}{\beta_r} (\bar{t} - 1) \right], \quad (14)$$

where $R_r(1)$ and $\dot{R}_r(1)$ are the displacement and velocity

response ratios of the r th modal coordinate at the instant when the load moves off the bridge. They can be calculated from Eq. (11).

3.3 Modal impact factor

For the on-beam stage, the peak response occurrence instant can be determined when the following differentiation of R_r in Eq. (11) to $\bar{t}$ equals to zero

$$\frac{dR_r(\bar{t})}{d\bar{t}} = 0. \quad (15)$$

Considering the multi-harmonic structure of R_r , multiple value of $\bar{t}$ will be found from Eq. (15), which corresponds to multiple local peak of R_r . The maximum one among them will be denoted as $R_{r,\max}^I$ to evaluate the impact effect. In the off-beam stage, the peak response equals to the magnitude of the free vibration oscillation and can be conservatively estimated by neglecting the influence of modal damping as

$$R_{r,\max}^{II} \sqrt{R_r^2(1) + \dot{R}_r^2(1)/\omega_r^2}. \quad (16)$$

The moving load induced impact factor of bridge modal coordinate μ_r will be the more important one among $R_{r,\max}^I$ and $R_{r,\max}^{II}$ as

$$\mu_r = \max([R_{r,\max}^I, R_{r,\max}^{II}]). \quad (17)$$

For HH beam, Fig. 3 presents the dynamic response ratio time histories for the fundamental mode under moving force when β varies from 0.16 to 1.0. According to the peak response occurrence feature, all of the responses are packed into three groups as presented. The top figure presents the first group of cases when the peak value of the dynamic response ratio is the first local peak of the time history records. As shown, in this group denoted to be the I-1 group, β varies from 0.4 to 1.0 and the peak value of R_1 increases with the increase of β . The middle figure presents the second group of cases when the peak value of the dynamic response ratio is the second peak of the time history records. As shown, in this group denoted to be the I-2 group, β varies from 0.23 to 0.38. With the increase of β , the peak value of R_1 will increase first and then decrease. The bottom figure presents the third group of cases when the peak value of the dynamic response ratio is the third peak of the time history records. As shown, in this I-3 group, β varies from 0.16 to 0.22. With the increase of β , the peak value of R_1 will also increase first and then decrease. Comparing the impact effects of these three groups, it is clear that the dynamic increments of the I-1 group are most notable. In each group, the peak response occurrence relative time instants monotonically increase. For all the presented

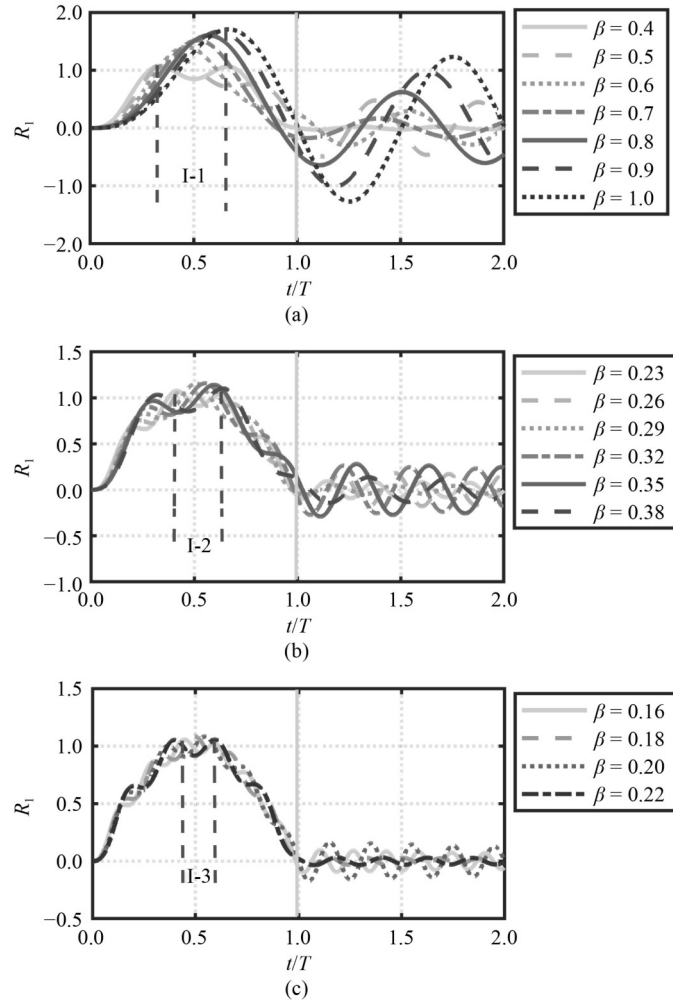


Fig. 3 The dynamic response ratio time histories for the fundamental mode of the HH uniform beam bridge with $\xi_1 = 1\%$ under moving force: (a) when β varies from 0.4 to 1.0; (b) when β varies from 0.23 to 0.38; (c) when β varies from 0.16 to 0.22.

cases when $\beta \leq 1$, the peak responses occur all in the on-beam stage.

Figure 4 summarizes the formed impact factors and the corresponding peak impact effect occurrence instant curves along with the variation of β from 0 to 1.5 for the fundamental modal coordinate of the single-span HH uniform beam bridge with $\xi_1 = 1\%$ under moving force. The transition points between the I-1, I-2, and I-3 groups are indicated. When β is below 0.15, some fluctuations of less importance on the computed impact factor curve and the corresponding peak impact effect occurrence instant scatter plot are observable. They are due to the amplification and transition of the high harmonics.

4 Classic beam impact spectrum

Based on the superposition of structural modal coordinate responses, the deformation and bending moment responses can be computed and evaluated. The related impact factor curves are then computed.

4.1 Deformation impact spectrum

The impact factor of beam dynamic deformation response $v_D(x, t, \beta)$ under a concentrated moving force with a generalized moving speed β is defined as

$$\mu_d(\beta) = \frac{\max[v_D(x, t, \beta)] - \max[v_s(x, x_0)]}{\max[v_s(x, x_0)]}. \quad (18)$$

Figure 5 presents the impact factor curves and the corresponding peak impact effect occurrence positions and instants for structure deformation responses of the single-span HH beam bridge with $\xi_n = 1\%$ under moving force. Four curves, three based on modal superposition considering one mode, two modes and ten modes, and one based on the FEM structure modeling and numerical integration response computation, are presented. The μ_d curves as shown at the top tell that for bridge deformation response, the fundamental mode provide the main skeleton to form the curve. The shape and the transition points of the μ_d curves from multiple modes and from FEM structural modeling based computations match with

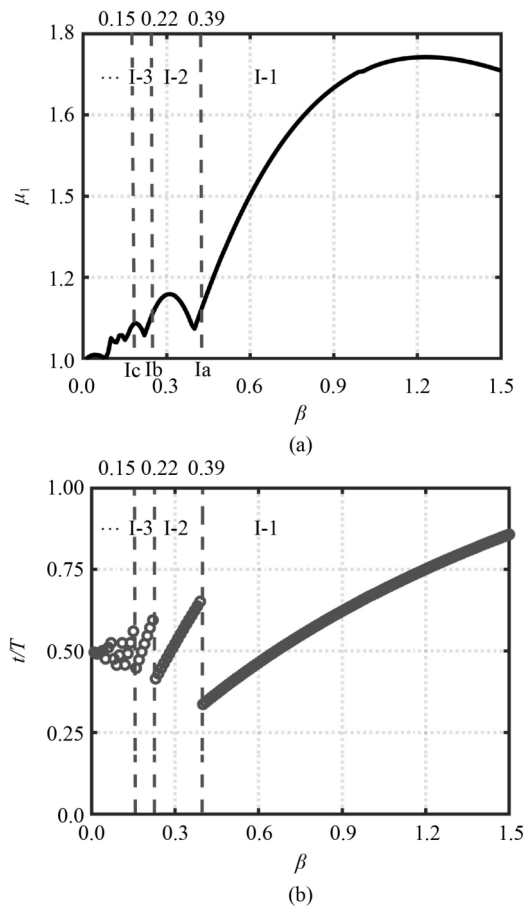


Fig. 4 The computed (a) impact spectrum, and the corresponding (b) normalized peak impact effect occurrence instants varying curve for the fundamental mode of the classic HH uniform beam bridge with $\xi_1 = 1\%$ under moving force.

the μ_d curve from the fundamental mode in a good precision, especially in the low speed region. For the region with high moving speed, it seems that the μ_d curve from the fundamental mode will provide an overestimation of the impact effect. The μ_d curve from the first ten modes provides a more accurate estimation according to the comparison with the μ_d curve from FEM structural modeling based computations.

Figure 5(b) about the peak response occurrence position tells that the x/L computation result from the fundamental mode show the constant term of the x/L curve and the contribution from the second mode provide the main skeleton to the variation of the x/L curve. One interesting phenomenon is the transition points of the x/L curve match exactly with the transition points of the μ_d curve. That means at the transition points for peak response occurrence, not only the time instant (as shown in Fig. 5(c)) but also the spatial position will make a discontinuous change at the same value of β . For this studied case, these peak response occurrence spatial position transitions can be captured if two modes are considered. For the peak response occurrence time instant

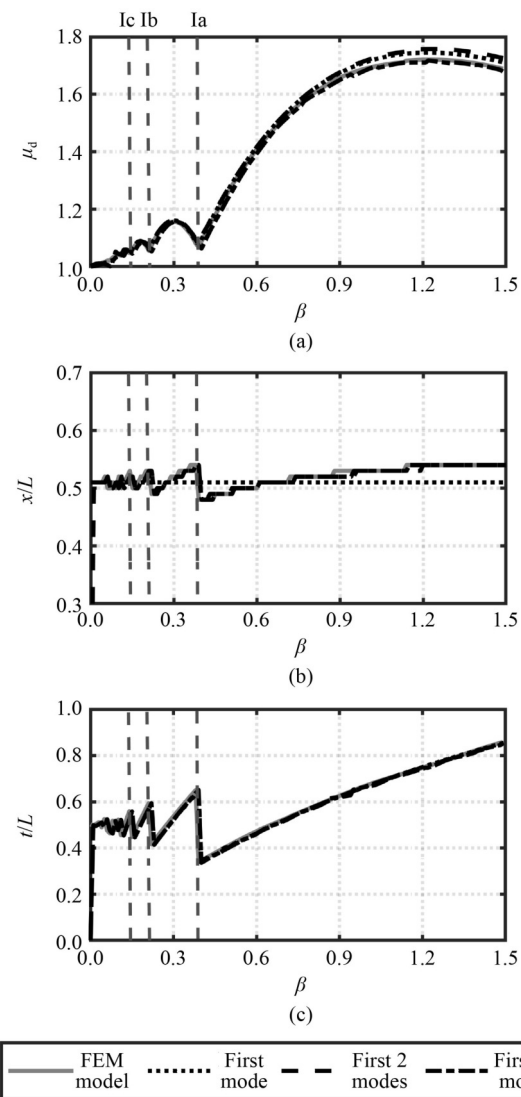


Fig. 5 The computed (a) impact spectrums, the corresponding (b) normalized peak impact effect occurrence positions varying curves, and (c) normalized peak impact effect occurrence instants varying curves for structure deformation responses of the classic HH beam bridge with $\xi_r = 1\%$ under moving force.

curve as shown in Fig. 5(c), it seems that one mode computation of the time instant curve will provide a good estimation of enough precision.

4.2 Bending moment impact spectrum

Compared with the deformation response impact factor, the bending moment response impact factor μ_B is more important for bridge design and is more sensitive to the high-order modes. The formula to compute the bending moment impact factor is Eq. (1). Figure 6 shows the computed impact factors and the corresponding peak impact effect occurrence positions and instants for structure bending moment responses of the single-span HH uniform beam bridge with $\xi_n = 1\%$ under moving

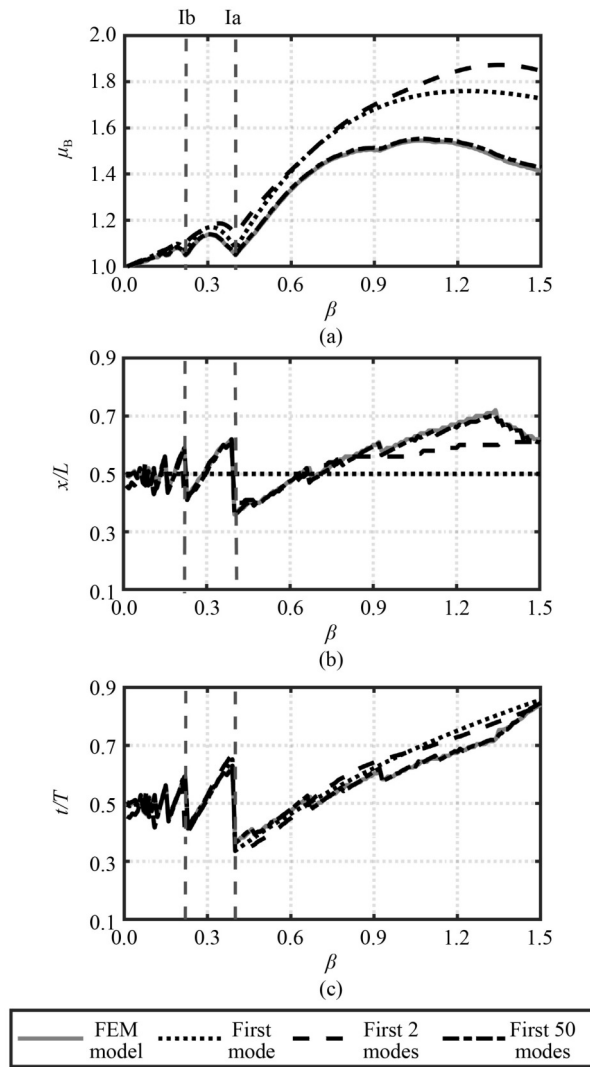


Fig. 6 The computed (a) impact spectrums, the corresponding (b) normalized peak impact occurrence positions varying curves, and (c) normalized peak impact occurrence instants varying curves for the bending moment responses of the classic HH beam with $\xi_r = 1\%$ under moving force.

force. Four curves computed from modal superposition of the first, the first two, the first 50 modes, and from FEM structure modeling based numerical integration are presented in the figures.

Figure 6(a) shows the μ_B curves. As shown, although the computational result from the fundamental mode is generally a reasonable overestimation result with the similar trend. Obvious underestimation exists, especially around the transition points. Some transition points, for example the point Ic cannot be clearly distinguished from the multiple mode summation or FEM based computation results. It seems that the computed μ_B curve from the first 50 modes is a good estimation compared with the FEM structure modeling based computational results. The reason can be illustrated using the percentage ratios of the peak value of the flexibility influence function of the r th

mode and the first r modes to the peak value of the flexibility influence function of the first 100 modes and the percentage ratios of the peak value of the flexibility curvature influence function of the r th mode and the first r modes to the peak value of the flexibility curvature influence function of the first 100 modes as shown in Fig. 7. Herein, the flexibility influence function of the r th mode is defined as $f_r(x, x_0) = \phi_r(x)\phi_r(x_0)/K_{rr}$, the flexibility influence function of the first r modes is defined as $F_r(x, x_0) = \sum \phi_r(x)\phi_r(x_0)/K_{rr}$, the flexibility curvature influence of the r th mode is defined as $f_r''(x, x_0) = \phi_r''(x)\phi_r(x_0)/K_{rr}$, and the flexibility curvature influence of the first r modes is defined as $F_r''(x, x_0) = \sum \phi_r''(x)\phi_r(x_0)/K_{rr}$. As shown, for the modal flexibility influence function, which is related to the modal deformation response, the percentage ratio of its peak value goes up to 98% when considering 2 modes. For the modal flexibility curvature influence function related to the static modal bending moment response, the percentage ratio of its peak value goes to 99.4% after considering 50 modes.

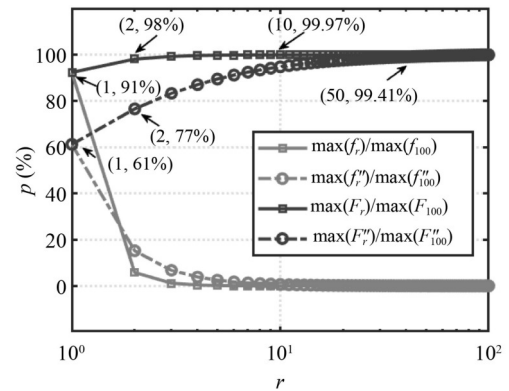


Fig. 7 The percentage ratios of the peak value of the flexibility influence function of the r th mode and the first r modes to the peak value of the flexibility influence function of the first 100 modes and the percentage ratios of the peak value of the flexibility curvature influence function of the r th mode and the first r modes to the peak value of the flexibility curvature influence function of the first 100 modes.

If the peak response occurrence spatial positions and time instants are checked, Figs. 6(b) and 6(c) show the results. It seems that the computed normalized peak response occurrence spatial positions and time instants curves have a startling likeness to one another, especially when $\beta < 1$. The curve shape, transition point position, and the curve magnitudes at the transition points are all the similar. Moreover, the peak response occurrence time instant curves tell that the computational results based on the first modal response can provide a good estimation of the curve shape. The determination of the peak response occurrence position is very important for the bending

moment dynamic load effect envelope estimation for local reinforcing and stiffener placement.

5 Demarcation analysis to gravity stretching retention effect: A case study

Generally speaking, for a supporting beam of a given size, as the increase of the dead load, measured by the load multipliers λ_q in this study, the dead load gravity stretching retention effect will be more and more important. The additional response $\tilde{v}_2(x, t)$ will thus be more and more important and distort the impact spectrum computed using the classic beam model. The applicable range of the classic model can be demarcated by evaluating the relative magnitude of $\tilde{v}_2(x, t)$. For bending moment impact evaluation purpose in this study, the criteria of the demarcation is set to evaluate the following additional response peak factor η_B

$$\eta_B = \max_x \left(\frac{\max_t B_{D,2}(x, t)}{\max_t B_{D,1}(x, t)} \right), \quad (19)$$

where $B_{D,1}(x, t) = -EI\tilde{v}_1''(x, t)$, and $B_{D,2}(x, t) = -EI\tilde{v}_2''(x, t)$. If η_B is smaller than some preset value, the impact spectrum analysis based on the classic beam model is considered to be reliable in this study. Herein,

$\tilde{v}_2(x, t) = \sum \phi_r(x) \tilde{v}_{2,r}(t)$, where the secondary modal components, $\tilde{v}_{2,r}(t)$, are computed according to the following equation derived from Eq. (5b)

$$K_{rr}\tilde{v}_{2,r} + M_{rr}\ddot{\tilde{v}}_{2,r} = \sum_s P_{rs}\tilde{v}_{1,s}, \quad (20)$$

where $M_{rr} = \rho A \int_0^L \phi_r(x) \phi_r(x) dx$, $P_{rs} = \frac{EA I^{*2}}{2L} \int_0^L \phi_r(x) \int_0^L \tilde{v}_1'(x)^2 dx + \frac{EA I^{*2}}{L} \int_0^L \phi_r(x) \tilde{v}_1''(x) \int_0^L \tilde{v}_1'(x) \phi_s'(x) dx dx$.

As shown above, concerning of the flexure-induced stretching retention effect for the studied beam, the existence of self-weight deflection will make cross-mode transfer coefficients P_{rs} nonzero. The additional cross-mode oscillations occur and may distort the computed impact spectrums.

As a demo, a case study on a steel slab beam under a moving force of $F = 2.2$ N is discussed in this paper. The slab beam is of thickness $h = 9$ mm, width $b = 10$ cm, and length $L = 1$ m. The characteristics length is set to be the radius of gyration to the z -axis of the slab $l^* = r$. The dimensionless quantity is set to be $\varepsilon = (r/L)^{0.5}$. Figure 8(a) presents the surface plot of η_B to moving load normalized forcing frequency β and dead load multiplier λ_q for the studied slab beam. When the demarcation criteria is set to be $\eta_B \leq 1$, the contour plot of η_B shown in Fig. 8(b) presents the applicable range of the impact

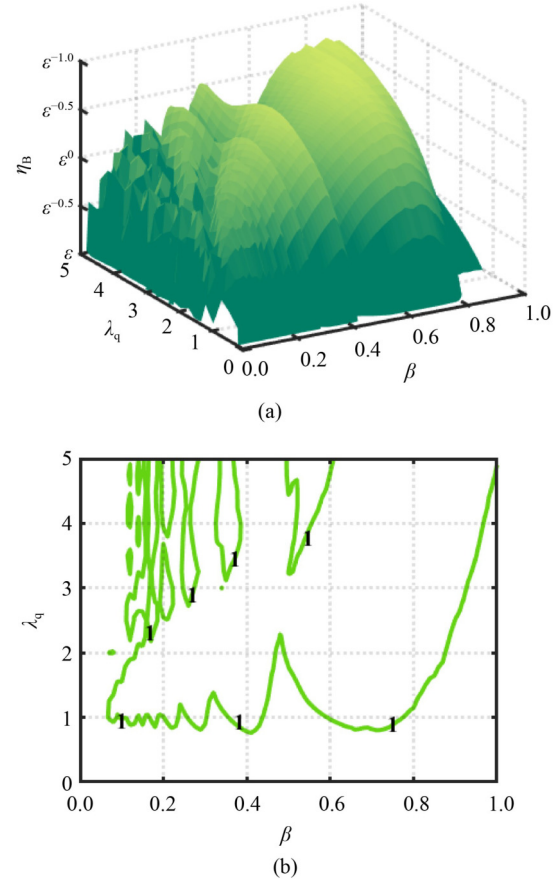


Fig. 8 (a) Surface and (b) contour plots of the additional response peak factor η_B considering gravity stretching retention effect to moving load normalized forcing frequency β and dead load multiplier λ_q for the studied HH slab beam in dead-load initial stretching state.

spectrum computed from the classic beam model concerning of deal load gravity stretching retention effect. As shown, for the studied slab beam, in the low moving speed region where $\beta \leq 0.1$ or in the region where $\lambda_q \leq 1$, additional response concerning of dead load gravity stretching retention effect can be considered to be a small quantity compared with the principal components computed from the classic model.

The determination of the applicable range of the moving load magnitude can be conducted by evaluating the following stretching force increment under moving load compared with the stretching force under dead load from the computed response components

$$\tilde{N}(t) = \sum_r \frac{EA}{L} \int_0^L \tilde{v}_1' \phi_r' dx \tilde{v}_{1,r}(t). \quad (21)$$

Around the edge line of λ_q from the first step analysis, Fig. 9(a) presents the moving load incremental stretching force peak factor η_N to the moving load normalized forcing frequency β and the live load multiplier λ_F for the studied HH steel slab beam when $\lambda_q = 1$. Herein, η_N

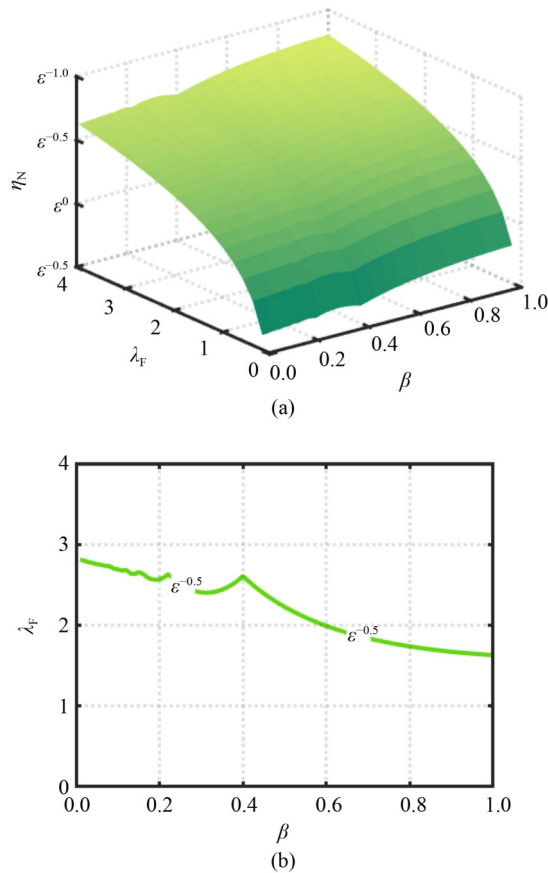


Fig. 9 (a) Surface and (b) contour plots of the moving load incremental stretching force peak factor η_N to moving load normalized forcing frequency β and moving load multiplier λ_F for the studied HH slab beam in dead-load initial stretching state when $\lambda_q = 1$.

is defined to be

$$\eta_N = \frac{\max_t \tilde{N}(t)}{\bar{N}}, \quad (22)$$

$$\bar{N} = \frac{EA}{2L} \int_0^L \bar{v}_1'^2 dx. \quad (23)$$

When the criteria is set to be $\eta_N \leq \varepsilon^{-0.5}$, the contour plot shown in Fig. 9(b) presents the edge curve of λ_F to β . For the studied slab beam, if λ_F is smaller than the bound value, the stretching force increment under moving force can be considered to be a small quantity to disturb the system. As shown, for the studied case when $\lambda_q = 1$, the edge curve of λ_F generally presents a declined trend with slight fluctuations along with the increase of β .

6 Concluding remarks

This study investigates the impact amplification of the classic single-span HH beam bridge under single moving

concentrated force. The impact factors of structure fundamental mode, deformation and bending moment responses are computed based on a closed-form MFMFO solution. Based on the analysis of the computational results, the following concluding remarks can be made.

1) The dynamic amplification of different high-harmonic modal oscillation components in different forcing frequency will produce the multi-bulged shapes of the impact factor curves for beam fundamental modal response and deformation responses dominated by the fundamental mode. For beam bending moment response, the influence of the higher modes on the related impact factor curves in the region with medium to high moving speeds is important.

2) For the positive bending moment response of the studied single-span HH beam bridge under a moving force with a low moving speed, the normalized peak response occurrence spatial position curves are of the similar shape and magnitude as the corresponding normalized peak response occurrence instant curves. The computed peak response occurrence time instant based on the first modal response can provide a good estimation of the peak response occurrence position curve for beam resistance design.

3) The demarcation analysis shows that the impact spectrum computed from the classic beam model is applicable either in the low moving speed region or in the region where there is no heavy secondary nonstructural dead load for the studied HH steel slab beam. The applicable upper bound of the moving load magnitude multiplier λ_F will decrease along with the increase of β for the studied case when $\lambda_q = 1$.

As the transverse loads will induce axial stretching of the HH beam as considered in this study, the beam now is an eccentric tension-flexural component considering its supporting conditions. The stress redistribution of the related structure concerning of gravity stretching retention effects under dynamic loads need to be evaluated to ensure structural sustainable and resilient development in operation.

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