Front. Optoelectron. China 2010, 3(3): 264–269
DOI 10.1007/s12200-010-0095-8

RESEARCH ARTICLE

Use of adaptive RLS, LMS, and NLMS algorithms for nonlinearity modeling in a modified laser interferometer

Saeed OLYAEE (✉), Mohammad Shams Esfand ABADI, Samaneh HAMEDI, Fatemeh FINIZADEH

Nano-Photonics and Optoelectronics Research Laboratory, Faculty of Electrical and Computer Engineering, Shahid Rajaee Teacher Training University (SRTTU), Lavizan 16788, Tehran, Iran

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Abstract Laser heterodyne interferometer is one kind of nano-metrology systems which has been widely used in industry for high-accuracy displacement measurements. The accuracy of the nano-metrology systems based on the laser heterodyne interferometers can be effectively limited by the periodic nonlinearity. In this paper, we present the nonlinearity modeling of the nano-metrology interferometric system using some adaptive filters. The adaptive algorithms consist of the least mean squares (LMS), normalized least mean squares (NLMS), and recursive least squares (RLS). It is shown that the RLS algorithm can obtain optimal modeling parameters of nonlinearity.

Keywords adaptive algorithms, recursive least squares (RLS), least mean squares (LMS), normalized least mean squares (NLMS), modified laser interferometer, nonlinearity modeling

1 Introduction

The nano-scale interferometry requires a displacement metrology system with linearity less than a few nanometers. Measuring interferometers are under development to be used in quantum mechanics, plasma physics, remote sensing, lithography as process of making integrated circuits, studying the velocity of sound in a fluid, locating the position, and properties of objects in space [1,2].

A persistent difficulty in picometer performance with laser interferometric metrology systems is the problem of periodic nonlinearity error which may be caused by small amounts of polarization mixing of the measurement and reference signals [3,4]. It was shown that the principal measurement nonlinearity in a heterodyne interferometer arises from elliptical polarization and nonorthogonality of the two frequencies laser input. Many efforts have been done to model and to compensate the nonlinearity error by inserting more complex optical setup and complex computational analysis [5–16].

Adaptive filters approach can be used to model the nonlinearity error. Heo et al. employed a recursive least squares (RLS) algorithm-based adaptive filter combined with a neural network back-propagation algorithm to compensate the nonlinearity in the laser heterodyne interferometer in 2007 [17]. Then Lee et al. reduced the nonlinearity error and stabilized the measurement data more by using the total least squares (TLS) algorithm [18].

Adaptive signal processing and, particularly, adaptive filtering provide a powerful approach to many signal processing problems [19]. The capacity of the adaptive algorithms to operate when limited a priori information is available makes them ideally matched to many practical applications [20]. The adaptive filters have become standard solutions in many fields in the last decades [21]. The adaptive filters with finite-impulse response (FIR) are now widely used in many applications such as noise cancellation, system identification and more [22]. The demands made of adaptive filters by the deployment of new technologies call for ever-increasing performance, modeling of longer impulse responses, and lower computational complexity.

Important performance measures for adaptive filters characterize the initial convergence rate, the residual error after convergence, the ability to track time-varying systems, and the computational complexity. The most widely used adaptive filter is the least mean squares (LMS) algorithm [20]. The LMS algorithm originally developed by Widrow [19,23] uses a gradient descent method to update the weights in a transversal adaptive filter structure. In contrast to the nonadaptive schemes, this approach takes into account the nonstationarity inherent in practical signals. In addition, it does not make any prior assumptions

Received January 20, 2010; accepted March 20, 2010

E-mail: s_olyaee@srttu.edu

regarding the statistics of the data and the degradations. The celebrated LMS algorithm and the normalized version (NLMS) [21] are the most common in practice because of their straightforward implementation and relatively low complexity compared to the better performing but substantially more complex least squares algorithms. The recursive least squares (RLS) algorithm as one of the adaptive filter algorithms has been first presented by Plackett. This method (RLS) is broadly used for signal processing in electronic engineering and shows a good performance and a fast adaptation within a stationary environment [21].

We have recently presented the nonlinearity modeling of the nano-metrology system only by using the RLS adaptive algorithm, and we have compared the results with nonlinearity modeling using the neural networks [24]. The results have shown that the RLS algorithm has faster conversion speed and lower steady-state mean square error (MSE) in the nonlinearity modeling, compared with the neural network approach. In the present paper, we describe our approach to modeling the periodic nonlinearity with emphasis on signal processing and the development of adaptive filter techniques to detect the error. We use the classical algorithms including LMS, NLMS, and RLS. Therefore, the main contribution of this paper is the application of different algorithms for nonlinearity error modeling in a modified nano-metrology system (Fig. 1). These techniques are generally valid for the nonlinearity caused by possible optical sources, such as ellipticity and nonorthogonality of incident laser polarization states. Through some experiments, it is verified that the proposed algorithms can model nonlinear factors.

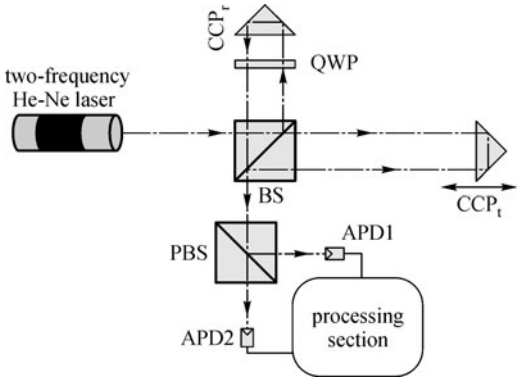


Fig. 1 Optical setup of nanometrology system based on modified laser heterodyne interferometer

The rest of this paper is organized as follows: the next section presents the optical structure of the nano-metrology system and its nonlinearity based on the modified laser heterodyne interferometer. Then, Sect. 3 reviews the LMS, NLMS, and RLS algorithms. The results are presented in Sect. 4, and finally, the conclusions are drawn from this investigation in Sect. 5.

2 Analysis of nonlinearity error

In the laser heterodyne interferometers with two different frequencies, non-ideal polarized beams can induce the nonlinearity error. In other words, nonlinearity resulting from elliptical polarization and nonorthogonality has the significant rule in producing error [25,26]. The optical source is a stabilized two-mode He-Ne laser at a 632.8-nm wavelength. The electric fields of the two beams emerged from laser are given by [25]

$$E_{\omega_L} = \begin{bmatrix} 1 \\ \alpha \end{bmatrix} \exp(i\omega_L t),$$
$$E_{\omega_H} = \begin{bmatrix} \beta \\ 1 \end{bmatrix} \exp(i\omega_H t), \tag{1}$$

where ω_L and ω_H are the optical frequencies of two modes; $\alpha = |\alpha|e^{i\phi_\alpha}$ and $\beta = |\beta|e^{i\phi_\beta}$ are the elliptical polarization parameters of the laser source. If the laser beam satisfies the orthogonal condition, it can be described as [25]

$$\begin{bmatrix} 1 \\ \alpha \end{bmatrix} [\beta \quad 1] = 0. \tag{2}$$

The optical setup of the nanometrology system based on the improved laser heterodyne interferometer is represented in Fig. 1. The electrical field of laser output is proceeding directly through the non-polarizing beam splitter (BS), and then the beams are led to the target and reference corner cube prism (CCP). The beam in the reference path passes through the quarter-wave plane (QWP) to change its polarization direction. Reflecting off the reference and target corner cube prisms, and reflecting off the beam splitter to the detectors, separate the interfered signal into x and y directions. As shown in Fig. 1, the first avalanche photodiode (APD) measures beams along the x polarization direction, and the second APD measures the rest along the y polarization direction. The beat signals produced by two avalanche photodiodes are respectively given by [27,28]

$$I_{mx} \propto \cos[(\omega_L - \omega_H)t - \Delta\phi + \psi_1], \tag{3}$$

$$I_{my} \propto \cos[(\omega_L - \omega_H)t + \Delta\phi + \psi_2], \tag{4}$$

where ψ_1 and ψ_2 are the phase shift nonlinearities; $\Delta\phi$ is the phase change resulting from optical path difference (OPD) between the reference and target paths error-free. Two photocurrents are led to signal processing section analyzing nonlinearity error and measuring displacement of target corner cube prism (CCP). The phase shift resulting from different optical paths which is concluded from electrical and processing part of the measurement system is given by

$$\Delta\Phi = 2\Delta\phi + (\psi_1 - \psi_2), \tag{5}$$

where the first term refers to the increased resolution compared to the conventional two-mode interferometers, and the second term ($\Psi_{nl} = \psi_1 - \psi_2$) is the periodic nonlinearity error. The displacement measurement of the target is then given by

$$\Delta z = \frac{\Delta\Phi\lambda_2}{4n\pi}, \tag{6}$$

where n is the refractive index of medium; λ_2 is the second wavelength of the laser source. Two beams, which are regarded as orthogonal to each other in a heterodyne interferometer, are not perfectly separated in two paths. Consequently, the polarization mixing occurs. This leads to a nonlinearity which limits the accuracy of the displacement measurement in the heterodyne interferometer. Therefore, it is important that we know the output phase difference precisely and distinguish it from nonlinearity.

3 Nonlinearity modeling by adaptive algorithms

3.1 LMS algorithm

The LMS algorithm is now briefly reviewed and then introduced the NLMS and RLS algorithms. The main benefit reported to motivate the introduction of NLMS and RLS schemes is that they are more robust to model nonlinearity in the modified laser heterodyne interferometer.

The LMS is one of the most popular adaptive algorithms due to its simplicity and robustness. While the simplicity of an LMS algorithm is generally judged by the small number of additions and multiplications required, the robustness of an LMS algorithm is judged in two ways: first, by how fast it converges, and second, by how small the mean square error (MSE) becomes.

Figure 2 shows the block diagram of the nonlinearity modeling system using the adaptive filter setup in which the error signal $e(n)$ and the expected signal $d(n)$ are defined in the negative feedback. We will attempt to model the desired signal $d(n)$ (here it is the expected nonlinearity error) by using the FIR filters, as

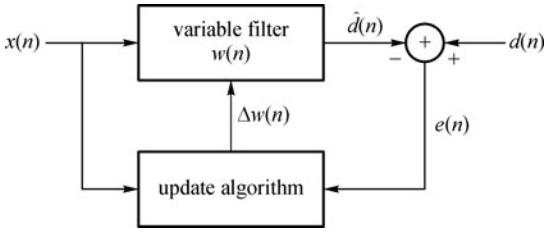


Fig. 2 Block diagram of nonlinearity modeling system using adaptive filter setup

$$\hat{d}(n) = y(n) = w(n)^{\mathrm{T}} x(n), \tag{7}$$

where $x(n)$ is the input signal vector (an $L\times 1$ matrix in which L is the filter order); $\hat{d}(n)$ or $y(n)$ is the estimated-desired signal. An adaptive filter $w(n)$ is used to model by adaptively minimizing the square of the error signal.

The error implicitly depends on the filter coefficients through the estimate $\hat{d}(n)$,

$$e(n) = d(n) - \hat{d}(n). \tag{8}$$

The LMS tap update equation is then given by

$$w(n) = w(n-1) + \mu x(n) e(n), \tag{9}$$

where μ is the step size. Two factors determining the robustness of an LMS algorithm, convergence rate and MSE, generally depend on its step size. As widely known, an LMS algorithm with a large step size, in general, converges fast, but the MSE becomes large. On the other hand, an LMS algorithm with a small step size tends to converge slowly, but the MSE becomes small [21,22].

3.2 NLMS algorithm

The NLMS algorithm is like the LMS algorithm, but defining $\|x(n)\|_2^2$ as the squared norm of $x(n)$, the NLMS tap update equation is then given by

$$w(n) = w(n-1) + \mu \frac{x(n)e(n)}{\|x(n)\|_2^2 + \delta}, \tag{10}$$

where δ is the regularization parameter and has a very small value.

3.3 RLS algorithm

An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated. The resulting rate of convergence is therefore typically an order-of-magnitude faster than other algorithm. This improvement in performance, however, is achieved at the expense of a large increase in computational complexity. The recursive least squares algorithm is used in adaptive filters to find the filter coefficients that relate to recursively producing the least squares (minimum of the sum of the absolute squared) of the error signal (difference between the desired and the actual signals). This is in contrast to other algorithms that aim to reduce the mean square error. Our goal is to estimate the parameters of the filter (w), and at each time n we refer to the new least squares estimate by $w(n)$. As time evolves, we would like to avoid completely redoing the least squares algorithm to find the new estimate for $w(n)$, in terms of $w(n-1)$. The idea behind RLS filters is to minimize a cost function by appropriately selecting the filter coefficients $w(n)$, updating the filter as new data

arrive. The filter vector update equation for the RLS algorithm is represented by

$$w(n) = w(n-1) + k(n)e(n), \tag{11}$$

where

$$k(n) = \frac{\eta^{-1}p(n-1)x(n)}{1+\eta^{-1}x^{\mathrm{T}}(n)p(n-1)x(n)}, \tag{12}$$

and

$$p(n) = \eta^{-1}p(n-1) - \eta^{-1}k(n)x^{\mathrm{T}}(n)p(n-1), \tag{13}$$

where η is the forgetting factor and is a positive constant close to but less than 1. The smaller the value of η, the smaller the contribution of previous samples. This makes the filter more sensitive to recent samples, which means more fluctuations in the filter coefficients.

4 Experimental results

In this section, LMS, NLMS, and RLS algorithms have been used on some deviation parameters to compare their performance of nonlinearity. The output photocurrents of detectors ($x(n) = [I_{mx}(n), I_{my}(n), I_{mx}(n-1)]^{\mathrm{T}}$) are considered as input of algorithms; the order of all filters is 3 ($L = 3$), and the nonlinearity error ($d(n) = \Psi_{nl}(n)$) is considered as the desired output of the systems. Figures 3 and 4 show the estimated nonlinearity errors from the ellipticity and non-orthogonally polarized beams, with adaptive algorithms. In Fig. 3, the real deviations are assumed to be $|\alpha| = 0.02$, $|\beta| = 0.04$ and $\phi_\alpha = \phi_\beta = 0.5$ rad. The expected and modeled periodic nonlinearities resulting from ellipticity and non-orthogonality deviations as $|\alpha| = 0.02$, $|\beta| = 0.01$ and $\phi_\alpha = 0.5$ rad, $\phi_\beta = 0.6$ rad are depicted in Fig. 4. For both cases, the nonlinearity is modeled by using LMS, NLMS, and RLS algorithms.

The order of all adaptive filters is 3 ($L = 3$). For all input data, the step size for the LMS and NLMS algorithms is 0.5 ($\mu = 0.5$), and the forgetting factor for the RLS algorithm is 0.9 ($\eta = 0.9$). In the right panel of Figs. 3 and 4, the differences between the expected output and modeled nonlinearity error are plotted. According to the figures, the nonlinearity is better modeled with the RLS algorithm than the LMS algorithm.

To better compare results, here, we calculate the mean square error for all samples as

$$\mathrm{MSE} = 10\log\left(\frac{1}{n}\sum_{i=1}^{n}e(i)^2\right), \tag{14}$$

where n is the number of total samples, and $e(i)$ is the error value (difference between the expected and output values) for the ith sample. The time consumed for running the program and the mean square error for adaptive algorithms

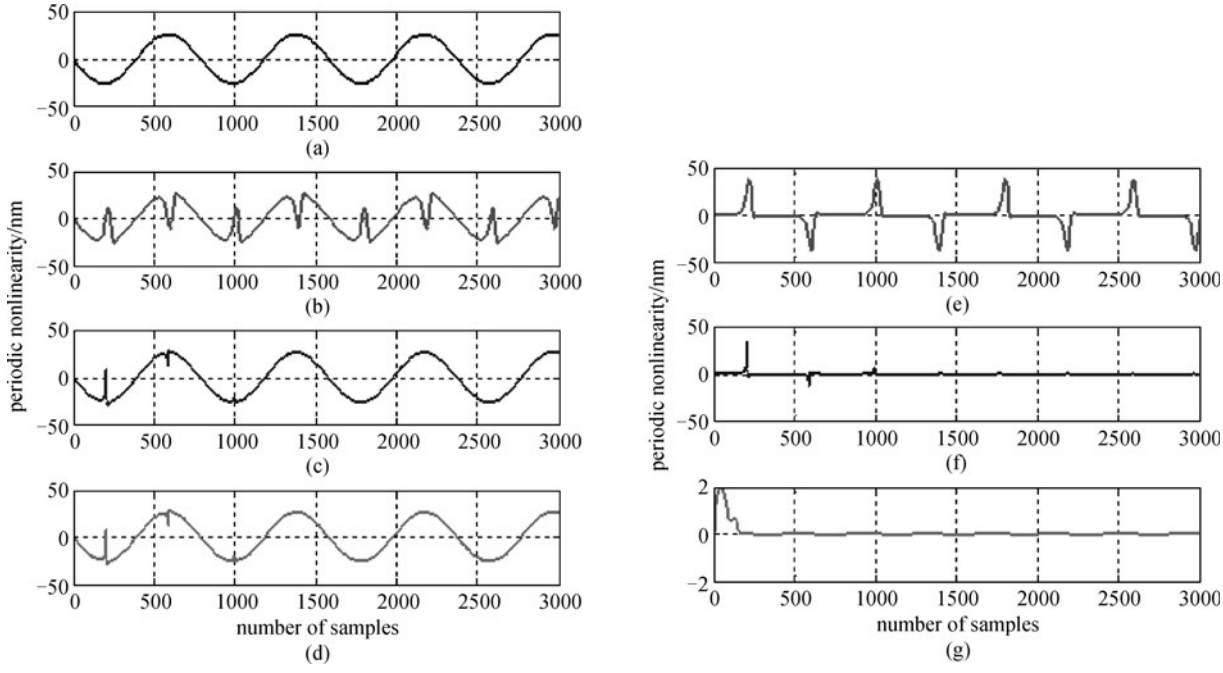


Fig. 3 Nonlinearity modeling in the nano-metrology system based on modified laser heterodyne interferometer in terms of the number of samples by adaptive algorithms for $|\alpha| = 0.02$, $|\beta| = 0.04$ and $\phi_\alpha = 0.5$ rad, $\phi_\beta = 0.5$ rad. (a) Desired output of the algorithm; (b) nonlinearity modeled by using LMS; (c) nonlinearity modeled by using NLMS; (d) nonlinearity modeled by using RLS; (e) difference between expected error and modeled nonlinearities of LMS; (f) difference between expected error and modeled nonlinearities of NLMS; (g) difference between expected error and modeled nonlinearities of RLS

are summarized in Table 1. The properties of the computer that is used for modeling are core2 CPU, 3 GHz RAM. According to the time consumed in the results, it is concluded that the adaptive NLMS method is faster than other algorithms, and the adaptive RLS method has minimum mean square error.

5 Conclusions

In this paper, the nonlinearities resulting from non-ideal polarized modes in a nano-metrology system based on the improved laser heterodyne interferometer have been modeled by adaptive algorithms (LMS, NLMS, and

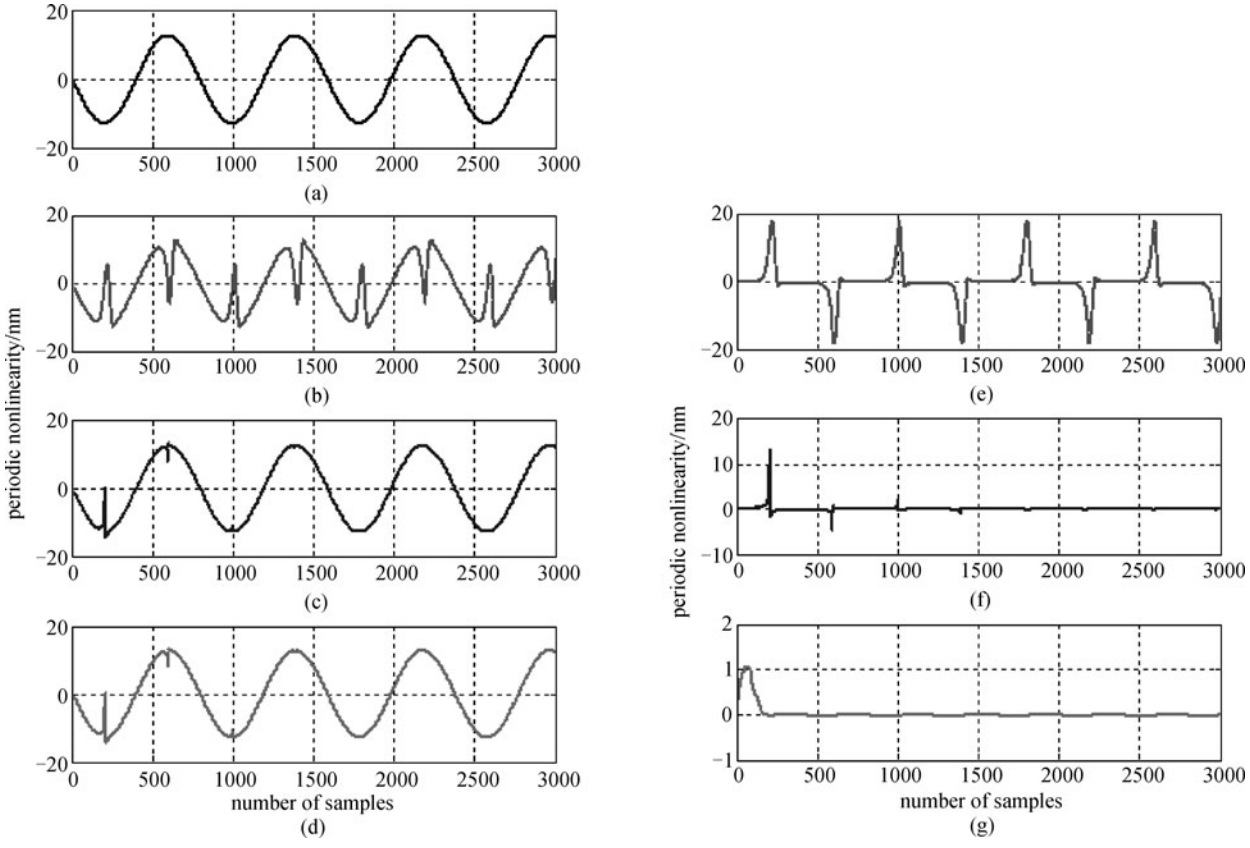


Fig. 4 Nonlinearity modeling in nano-metrology system based on modified laser heterodyne interferometer in terms of the number of samples by adaptive algorithms for $|\alpha| = 0.02$, $|\beta| = 0.01$ and $\phi_\alpha = 0.5$ rad, $\phi_\beta = 0.6$ rad. (a) Desired output of the algorithm; (b) nonlinearity modeled by using LMS; (c) nonlinearity modeled by using NLMS; (d) nonlinearity modeled by using RLS; (e) difference between expected error and modeled nonlinearities of LMS; (f) difference between expected error and modeled nonlinearities of NLMS; (g) difference between expected error and modeled nonlinearities of RLS

Table 1 Properties of adaptive algorithms to model nonlinearity for some deviation parameters

	condition				LMS ($\mu = 0.5$)		NLMS ($\mu = 0.5$)		RLS ($\eta = 0.9$)	
	$\|\alpha\|$	$\|\beta\|$	ϕ_α	ϕ_β	mean square error/dB	consumption time/s	mean square error/dB	consumption time/s	mean square error/dB	consumption time/s
1	0.002	0.002	0.50	0.45	−51.3220	0.201572	−71.0967	0.037671	−79.4539	0.084065
2	0.010	0.010	0.50	0.45	−37.3433	0.057803	−57.7430	0.038221	−65.5588	0.080104
3	0.020	0.010	0.50	0.45	−33.8595	0.059866	−54.0151	0.038076	−64.0895	0.113923
4	0.010	0.020	0.50	0.50	−33.9343	0.062952	−54.1662	0.036431	−64.1155	0.077829
5	0.040	0.000	0.50	0.50	−31.4309	0.062743	−50.1885	0.040322	−64.4580	0.080174
6	0.020	0.040	0.50	0.50	−27.9147	0.058910	−46.7239	0.038487	−59.5126	0.080459
7	0.020	0.010	0.50	0.60	−34.1084	0.057273	−54.4531	0.037180	−63.8682	0.092611
	mean				−35.7019	0.080160	−55.4838	0.03805	−65.8652	0.0870236

RLS). The results from these algorithms were compared. It was shown that the very small ripple of the LMS algorithm result has not appeared by using NLMS and RLS algorithms. It was also represented that the best results for nonlinearity modeling can be achieved by the adaptive RLS algorithm in a short time of about 87 ms.

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