

Spin waves in the two-dimensional Ammann–Beenker quasicrystals

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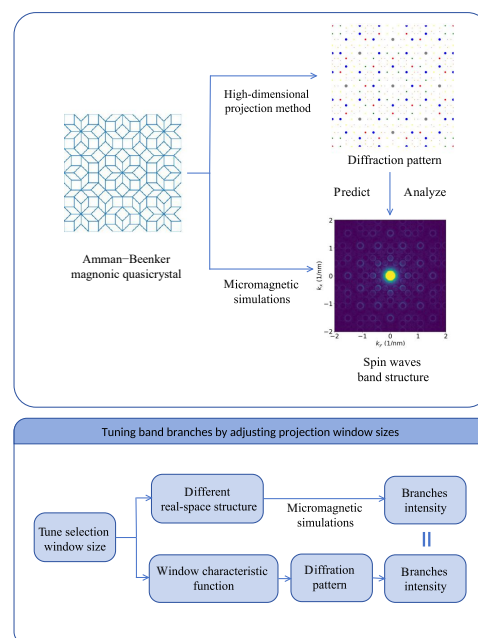
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ABSTRACT

We investigate spin wave (SW) spectra in two-dimensional (2D) magnonic quasicrystals (MQCs) based on Ammann–Beenker (AB) tiling using micromagnetic simulations. The AB MQC structure is realized as an antidot lattice (ADL) etched into a thin Permalloy (Py) strip, generated via projection from a four-dimensional (4D) hypercube onto a 2D plane within a selection window in perpendicular space. Our simulations reveal SW dispersion with distinct circular ring patterns in frequency slices. The centers of these rings coincide with high-intensity diffraction points predicted by the higher-dimensional projection method, and their intensity correlates with corresponding diffraction peaks, confirming origins from Bragg reflection. The spectra exhibit numerous band gaps and fractal characteristics inherent to quasiperiodicity. Crucially, band gap tuning can be achieved by scaling the perpendicular-space selection window, which modifies diffraction intensities and thereby adjusts the SW band structure. This introduces a new design degree of freedom for reconfigurable magnonic devices. Our results demonstrate the effectiveness of the higher-dimensional projection method for analyzing MQC SW dispersion and highlight the potential of quasiperiodic modulation in engineering SW spectra for low-cost, reconfigurable spintronic applications.

Keywords spin wave, magnonic quasicrystals, micromagnetic simulation



1 Introduction

Quasicrystals (QCs) are composed of several identical repeating units with quasi-periodicity, self-similarity, a special diffraction pattern and long-range order [1]. QC does not have symmetry of space translation and allows rotational symmetries prohibited in crystal such as 5-fold, 8-fold, and 10-fold rotational symmetry. In 1984, Shechtman *et al.* [2] discovered QCs for the first time in the rapid cooling solidification process of Al-Mn alloy with a special ratio. Since then, hundreds of QCs with various compositions and symmetries have been discovered. Due to their special structure, QCs have many unique properties, such as high hardness, low surface

energy and high oxidation resistance [3].

Magnonics, which leverages spin waves (SWs) for information processing with lower energy consumption, higher integration density, and reconfigurability than conventional electronics, shows transformative potential in RF communication systems, neuromorphic computing architectures, and quantum-enhanced signal processing [4]. Magnonic crystals (MCs), artificial structures designed to manipulate SWs (magnons) through periodic modulation of magnetic parameters (e.g., magnetization, anisotropy, or magnetic fields), are classified by dimensionality into 1D (e.g., stripe arrays, multilayers), 2D (e.g., nanodisc arrays, antidot lattices(ADLs)), and 3D (e.g., multilayers, colloidal crystals) [5]. Materials



designed in this way are also called metamaterials. In ferromagnetic materials, nanoscale holes are formed at the lattice sites of the crystal lattice to create an ADL. The properties of ADL are closely related to the direction and magnitude of the magnetic field, the lattice constant, the shape of the hole [6], and the lattice type [7–9].

In addition to periodic modulation that can be applied in MCs, quasicrystalline modulation can also be used. Magnetic materials with quasicrystalline modulation are called MQCs. Similar concepts have also been applied to photonic QCs [10–12] and phononic QCs [13–17].

In recent years, studies on different MQCs have found that they have complex local SW spectra, characterized by a large number of band gaps and fractal characteristics [18]. For example, MQC based on the Fibonacci sequence [19] exhibits a magnetic band gap structure in the micromagnetic simulation. A SW propagation behavior with nonreciprocity [20] different from the conventional one is observed in MQCs based on the Fibonacci sequence [21]. A 2D MQC based on Penrose tiling was reported, where multiband transport was observed by broadband and phase-resolved spectroscopy. This study employed electron-beam lithography (EBL) combined with ion-beam etching (IBE) to fabricate periodic/quasiperiodic nanotrough arrays with diameters of 300 nm, periodicities of 900 nm, and depths of 23 nm [22]. Chiral MQCs based on Ir/Fe/Pt(Ir) and Au/Fe/Pt(Au) multilayers were recently reported by Wei *et al.* [23], where fractal band structures, tunable flat bands, and nonreciprocal spin-wave propagation were observed via Brillouin light scattering.

In this work, we investigate the SW propagation and band structure engineering in 2D MQCs based on AB tiling, utilizing micromagnetic simulations. While SWs in ferromagnetic films typically exhibit quadratic dispersion, we observe a complex succession of band gaps and fractal-like energy spectra induced by quasiperiodic modulation. Crucially, we demonstrate that the band gaps originate from Bragg reflections at high-intensity diffraction points predicted by the high-dimensional projection method, and their widths can be dynamically tuned by scaling the perpendicular-space selection window. The dispersion relations display circular ring patterns in frequency slices, whose intensities correlate with the diffraction peak magnitudes. By adjusting the size of the selection window in perpendicular space, we obtained two new types of AB QC structures. Their diffraction spot positions in reciprocal space remain unchanged, but they exhibit distinct and predictable diffraction intensity distributions, thereby enabling the modulation of the energy bands. Narrowing the selection window makes the widths of different band gaps more comparable, whereas widening the selection window significantly enhances the first major band gap.

We present our results as follows. In Section 2, the micromagnetic simulation methodology based on the

Landau–Lifshitz–Gilbert (LLG) equation is elaborated in detail. In Section 3, we report the computational results: we start with the calculated band structure of the AB MQC and the mathematically derived diffraction pattern obtained via the high-dimensional projection method, then proceed with a discussion of the band gap formation mechanism, supported by real-space dynamic magnetization profiles that confirm Bragg reflection origins. We subsequently introduce the method for calculating the reciprocal space image via projection from a 4D hypercube, taking the AB QC as an example, and discuss the role of the window characteristic function in determining diffraction intensities. A systematic finite-size convergence analysis is then presented to validate the robustness of the observed band structure against simulation domain effects. Finally, we demonstrate the methods and results of band structure modulation achieved by adjusting the size of the projection window. In Section 4, we summarize our conclusions and discuss the universal applicability of this approach to other quasiperiodic systems.

2 Model

Micromagnetic simulations of SW spectra in QC were performed using MuMax3 [24] based on the LLG equation (1)

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{eff} + \alpha \mathbf{m}(\mathbf{r}, t) \times \frac{\partial \mathbf{m}(\mathbf{r}, t)}{\partial t}, \quad (1)$$

where $\mathbf{m}$ is the magnetization, α is damping parameter, $\mathbf{H}_{eff}$ is the effective field. The effective field is obtained by the following equation:

$$\mathbf{H}_{eff} = -\frac{1}{\mu_0 M_s} \frac{\delta E}{\delta \mathbf{m}}, \quad (2)$$

where μ_0 is the vacuum permeability and M_s is the saturation magnetization, E is the total energy. In the Heisenberg model, for a ferromagnetic system, assuming that $\mathbf{S}_i$ and $\mathbf{S}_j$ are spins on two lattice sites, The exchange energy E is given by the following formula:

$$E = -J \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (3)$$

where J is the Heisenberg exchange interaction. The Heisenberg interaction can be extended to the LLG equation based on the continuous model and expressed as an effective field. The effective field of the exchange interaction in the continuous model is

$$\mathbf{H} = \frac{2A}{M_s} \nabla^2 \mathbf{m}, \quad (4)$$

where $A = 2Js^2z/(\mu_0 a)$, s is the amplitude of spin, and z denotes the number of lattice points in a cell. a is the

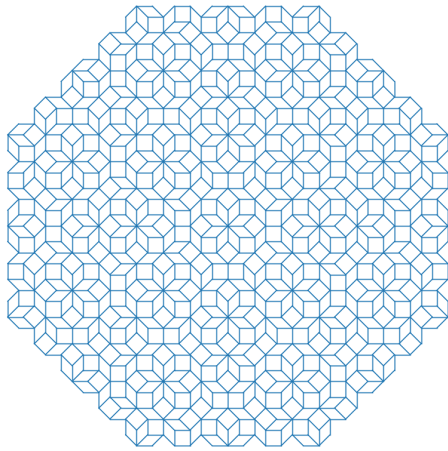


Fig. 1 Ammann-Beenker tiling.

lattice constant.

Representative examples of 2D QCs include Penrose QCs with fivefold rotational symmetry and AB QCs with eightfold rotational symmetry. Here, we choose the AB QC as the research object and create nanotroughs at the lattice points of the 2D AB quasilattice, forming an ADL. The structure of the AB QC is shown in Fig. 1. In the calculation, the distance between two adjacent points connected by line segments, which is the QC constant l , is set to 12 nm.

The 12 nm antidot spacing approaches the physical limit of current nanofabrication, making experimental realization challenging in the short term. While EBL can achieve sub-10 nm resolution, faithfully transferring such dense patterns into Py thin films remains fundamentally difficult. For larger-scale structures (hundreds of nanometers to micrometers), however, the predicted SW spectra are readily accessible via Brillouin light scattering, time-resolved magneto-optical Kerr effect, or all-electrical spin-wave spectroscopy — techniques already successful in Penrose-tiling MQCs [22].

More importantly, our window-modulation band engineering strategy is not scale-limited: it applies equally to structures with characteristic sizes ranging from nanometers to micrometers, which are well within the capabilities of current electron-beam lithography and focused-ion-beam milling. This scalability significantly enhances the experimental feasibility and technological relevance of our approach.

The spins are initialized along the z -axis under the action of an external field B_z of 2.086 T along the z -axis. We selected Py material as the ferromagnetic material. The parameters of the Py material used in the simulation were $M_s = 830 \text{ kA} \cdot \text{m}^{-1}$, $\gamma = 2.337 \times 10^5 \text{ m} \cdot (\text{A} \cdot \text{s})^{-1}$, $\alpha = 0.03$, $A = 1.3 \times 10^{-11} \text{ J} \cdot \text{m}^{-1}$ [25], and the simulation time step Δt was 1 ps. A sinc pulse field was applied along the x -axis direction in the center of the sample to excite the SW.

The micromagnetic simulations are calculated in Py strips with dimensions of $512 \text{ nm}(\text{length}) \times 512 \text{ nm}(\text{width}) \times 1 \text{ nm}(\text{thickness})$. The simulation cell sizes are $1 \text{ nm} \times 1 \text{ nm} \times 1 \text{ nm}$. Since the calculated QC is 2D, a thickness of one lattice unit is adopted in the z -direction to reduce the amount of computation. In order to make the dispersion diagram obtained by simulation calculation clearer, damping is selected as 0.03.

A sinc pulse field with a width of 50 ps is applied to the center of the sample, and the pulse center is located at $t = 50 \text{ ps}$ to avoid generating redundant signals at the edges of high frequencies. The pulse signal is given by

$$h_z(t, x, y) = h_z^0 \text{sinc}[2\pi f_c(t - t_0)] \text{sinc}\left[k_c\left(x - \frac{x_{\max}}{2}\right)\right] \text{sinc}\left[k_c\left(y - \frac{y_{\max}}{2}\right)\right],$$

where $h_z^0 = 0.504 \text{ T}$, $f_c = \frac{1}{2\Delta t}$, and k_c depends on the size of the reciprocal space to be simulated. The evolution of magnetic moments $\mathbf{M}(\mathbf{r}, t)$ over time are obtained by sampling points on a grid with uniform intervals. Performing a three-dimensional fast Fourier transform (3D FFT) on $\mathbf{M}(\mathbf{r}, t)$ yields the amplitude at specific frequencies and wavevectors, which corresponds to the dispersion curve.

3 Results

3.1 SWs in AB MQC

We use linear SW approximation to solve the SW spectra. Suppose that at position $\mathbf{r}$ and time t , the magnetization is $\mathbf{M}(\mathbf{r}, t)$. Ignoring the higher-order terms, the magnetic moment can be expanded as $\mathbf{M}(\mathbf{r}, t) = M_z(\mathbf{r})\hat{\mathbf{e}}_z + \mathbf{m}(\mathbf{r}, t)$. M_z is time independent, with $M_z \approx M_s$ and $\mathbf{m}(\mathbf{r}, t) \ll M_z(\mathbf{r})$, which is identical to the linear approximation for SWs.

If it is a periodic ADL, then the perpendicular bisector of the line connecting the nearest diffraction point and $(k_x, k_y) = (0, 0)$ is the boundary of the first Brillouin zone, where an energy gap can be generated. Moreover, the energy bands of the second, third and higher-order Brillouin zones can be folded into the first Brillouin zone for analysis [26]. At the position where the energy gap appears (the boundary of the Brillouin zone), edge modes with a large amplitude distribution appear at the edge of the ADL [27, 28].

It is noteworthy that although energy bands can be folded into the first Brillouin zone for analysis, unlike in lattice models where wavevectors $\mathbf{k}'$ exceeding the first Brillouin zone are governed by Bloch's theorem ($\mathbf{k}' + \mathbf{G} \equiv \mathbf{k}$). In ADL, wavevectors beyond the first Brillouin zone correspond to higher-order modes with shorter wavelengths (smaller than the ADL period) [29], since the unit cell size is smaller than that of the ADL, resulting in a corresponding relationship in real space.

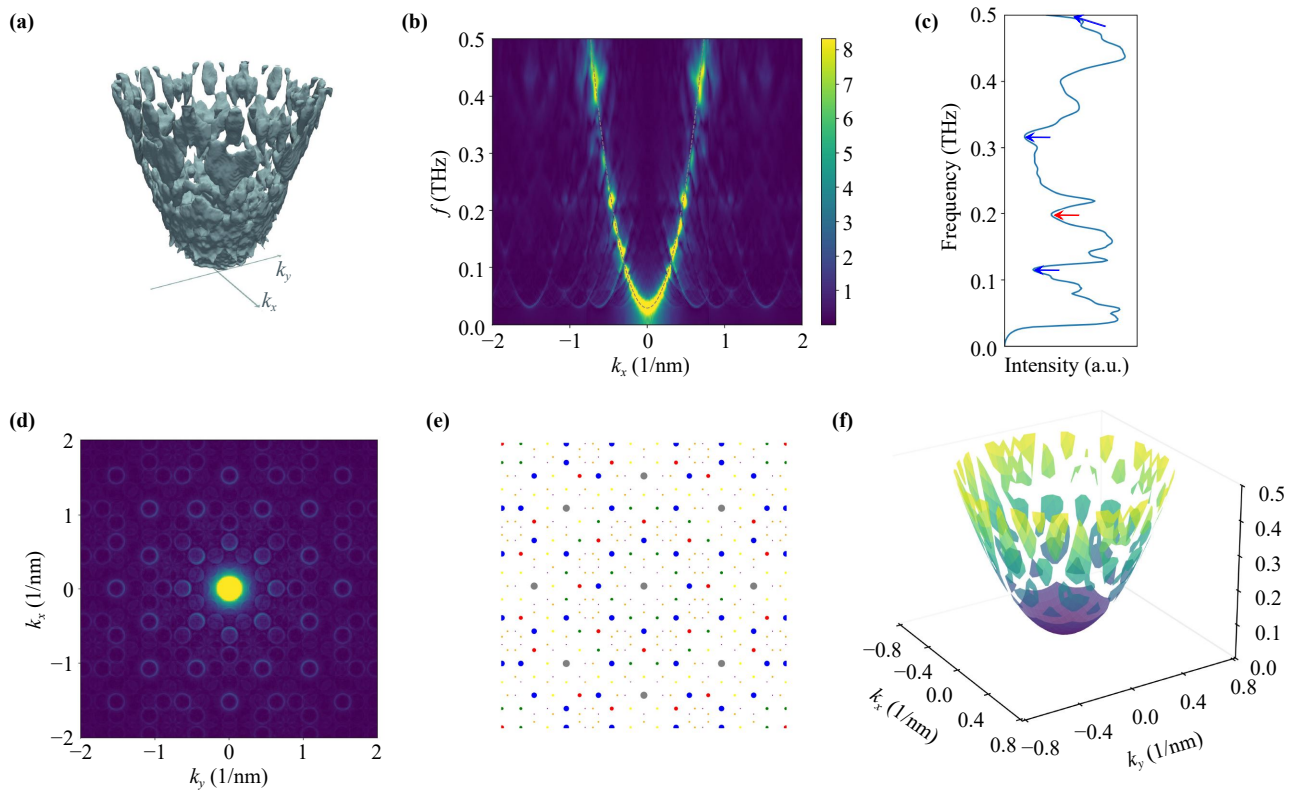


Fig. 2 The calculation results of AB tiling. (a) The SW spectrum of AB MQC obtained by micromagnetic simulation. (b) The slice at $k_y = 0$ position, the $k_x - f$ relationship image. (c) Intensity distribution maps at different frequencies, with small colored arrows in the figures marking the minima. (d) A slice at the frequency of $f = 0.04$ THz shows equal-sized circular rings with varying intensities, and the intensity distribution is consistent with the results of the mathematical diffraction image. (e) The mathematically simulated diffraction pattern of AB QC from the high-dimensional projection method illustrates the reciprocal space geometry of AB MQC. Only high intensity diffraction peaks are plotted, color-coded gray, blue, red, green, yellow, orange, purple in order of decreasing intensity. (f) The SW dispersion results of AB tiling theoretically predicted by the diffraction pattern and Dispersion Formula (5) show that it is up to 0.5 THz. The wave vector is defined as $2\pi/\lambda$, with the unit (rad/nm), abbreviated as (1/nm).

Therefore, excitation spectra of ADL is usually analyzed in the extended Brillouin zone [30, 31]. Furthermore, a quasi-periodic structure does not allow for the definition of a Brillouin zone. Instead, a pseudo-Brillouin zone can be defined using methods analogous to those employed for periodic structures. Therefore, our results are all presented in the extended Brillouin zone.

For an out-of-plane magnetized thin film, the SW dispersion relation can be well approximated as:

$$f_{\text{theory}} = \frac{\gamma}{2\pi} \left(\frac{2A}{M_s} k^2 + B_z - \mu_0 M_s \right), \quad (5)$$

where the term $\frac{2A}{M_s} k^2$ accounts for the exchange interaction term, B_z is the external field, and $-\mu_0 M_s$ represents the static demagnetizing field. The dynamic dipolar term $\mu_0 M_s k_{\parallel}^2 / k^2$ is safely neglected here because, at the nanoscale wavevectors relevant to our system, the exchange contribution overwhelmingly dominates over the long-range dipolar interaction. It can be derived from the formula that the excitation frequency is

$f = 0.029$ THz at $\mathbf{k} = 0$.

The calculated SW spectra of the QC ADL are shown in Fig. 2(a). The SW spectrum exhibits quadratic dispersion, with numerous energy gaps appearing. For further analysis of the spectrum, we plot its slice at $k_y = 0$ as shown in Fig. 2(b) and its frequency slice at $f = 0.04$ THz as shown in Fig. 2(d). In Fig. 2(b), the main branch exhibits a quadratic form, with numerous secondary branches of the same quadratic dispersion emerging. Energy gaps appear at the intersections of the main branch with the other branches. As denoted by the dashed line, the main branch matches the spin-wave dispersion curve [Eq. (5)] derived from the LLG equation for Py thin-film strips.

It can be seen that the emergence of new branches formed by the QC ADL is the reason for the numerous energy gaps and fractal structures in the spectrum.

To confirm the branch positions, a frequency slice at $f = 0.04$ THz (near the minimum of the dispersion curve) is plotted, as shown in Fig. 2(d). It can be observed that the spectrum exhibits eightfold rotational symmetry.

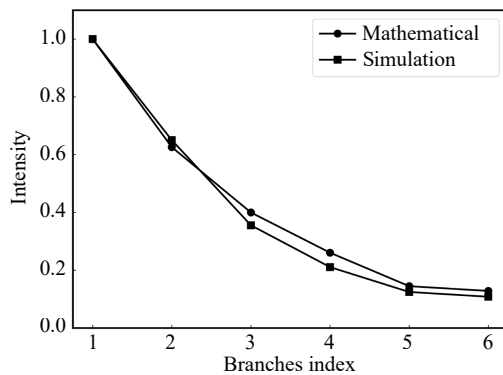


Fig. 3 Normalized intensity of mathematical diffraction spots and micromagnetic SW branches. The definition of branches index are presented in Table 1.

These branches, induced by the QC structure, manifest as rings of equal size in the frequency cross-section, as shown in Fig. 2(d). The position of the ring center corresponds to the location of the lowest excitation energy point for each branch. The pattern formed by the collection of ring centers also exhibits characteristics of the AB tiling, featuring both square and rhombic configurations, which is directly related to the reciprocal space structure of the AB tiling. We calculated the mathematical diffraction pattern of the AB tiling using the high-dimensional projection method, as shown in Fig. 2(e). Details of the high-dimensional projection method are provided in the appendix. From the figure, the reciprocal space structure of the AB tiling can be observed, where the positions of the diffraction spots correspond to those of the ring centers in Fig. 2(d). We classified the intensities of the diffraction spots, marking them with different sizes and colors, with the intensity decreasing in the order of gray, blue, red, green, yellow, orange and purple. It is noteworthy that the intensity of the diffraction spots corresponds one-to-one with the intensity of the branches, and the stronger the diffraction spot, the higher the intensity of the corresponding branch as shown in Fig. 3. The diffraction spot intensities shows excellent agreement with the intensities of the corresponding spin-wave branches.

The new branches generated at the positions of the diffraction spots overlap with the main branches, creating energy gaps. In Fig. 2(c), we show the SW intensity as a function of frequency. Several key energy gaps are marked in the figure. The energy gap induced by the nearest-neighbor blue points $[(k_0, 0)]$ is the first distinct energy gap (located at the pseudo-Brillouin zone boundary), analogous to the first gap of a periodic lattice (at the Brillouin zone boundary), at a frequency of $f = 0.116$ THz.

There exist branches with distinct intensities in the spectrum. Due to the presence of damping, the smaller energy gaps become less distinct or are not observable.

Table 1 Diffraction spots ranked by intensity. The coordinate of one characteristic spot is given for each grade. $k_0 = \frac{\sqrt{2}+1}{2} \frac{2\pi}{l} = 0.632 \text{ nm}^{-1}$. Details for calculating k_0 are provided in the appendix.

Branches index	Marker color	Coordinates
1	Blue	$(k_0, 0)$
2	Red	$(\sqrt{2}k_0, 0)$
3	Green	$(\sqrt{2}k_0, k_0)$
4	Yellow	$(2k_0, 0)$
5	Orange	$(\sqrt{2}k_0, k_0)$
6	Purple	$(\sqrt{2}k_0, (\sqrt{2}-1)k_0)$

As shown in Fig. 2(c), distinct intensity minima exist at the energy gaps corresponding to the stronger branches when $\alpha = 0.03$. There are obvious intensity minima in the energy gaps corresponding to the branches with higher intensity. The energy gaps associated with diffraction points marked in blue in Fig. 2(e) are located at $f = 0.116, 0.317$ THz. Those corresponding to the diffraction points marked in red are at $f = 0.200$ THz. Additionally, several energy gaps exist above 0.5 THz, which lie beyond the range covered by the sampling frequency $f = \frac{1}{2\Delta t}$.

To identify the physical origin of the observed band gaps, we analyze the real-space distribution of the dynamic magnetization at selected characteristic gap frequencies. Figures 4(a)–(c) present the spatial profiles of the oscillating magnetization component $m_x(x, y)$ at the first ($f = 0.116$ THz), second ($f = 0.200$ THz), and third ($f = 0.317$ THz) major band gaps, respectively. Remarkably, all three modes exhibit well-defined, long-range standing-wave patterns that extend throughout the entire simulated domain. The magnetization amplitude is not randomly localized but instead forms periodic-like nodal and anti-nodal lines with an overall octagonal contour, consistent with the eightfold rotational symmetry of the AB QC.

Taking the first band gap as an example [Fig. 4(a)], the nodal lines (dark regions) coincide precisely with the ‘Bragg diffraction planes’, while the anti-nodal lines (bright regions) are located at the centers of the square and rhombic plaquettes. This spatial configuration provides direct evidence of coherent Bragg scattering: spin waves propagating in different directions interfere constructively and destructively to form a stable standing-wave pattern. To quantitatively characterize this standing wave, we extract the line profile of $|m_x|$ along the $y = 0$ cut, as shown in Fig. 4(d). The profile exhibits a clear periodic oscillation, from which we determine a wavelength of $\lambda = 19.8$ nm. This value is consistent with the Bragg condition $\lambda = \pi/k_0$.

A quantitative match between the real-space nodal distribution and the reciprocal-space diffraction pattern confirms that the observed modes are indeed extended

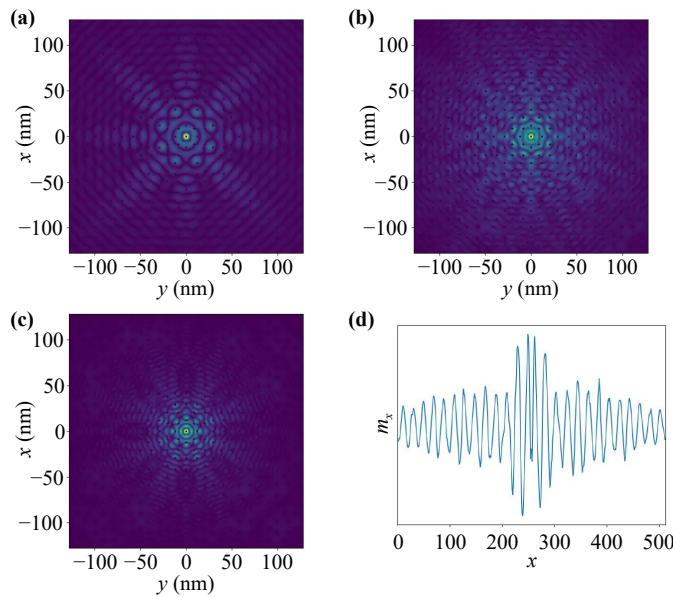


Fig. 4 Real-space distribution of the dynamic magnetization amplitude $|m_x(x, y)|$ at the first (a), second (b), and third (c) major band gaps of the AB MQC. Nodal lines (dark regions) coincide with ‘Bragg diffraction planes’, while antinodal lines (bright regions) occupy the plaquette centers, indicating standing-wave formation. The patterns exhibit an overall octagonal contour consistent with the eightfold symmetry of the AB tiling. (d) Line profile of $|m_x|$ along the $y = 0$ cut extracted from (a), showing a clear periodic oscillation with a spatial period of $\lambda = 19.8$ nm, consistent with the Bragg condition.

standing waves arising from Bragg reflection at the quasiperiodic reciprocal lattice vectors, rather than localized states induced by disorder or structural defects. Combined with the frequency-space analysis presented in the preceding sections, these real-space observations provide compelling evidence that the fractal magnonic band structure of the AB MQC originates from constructive interference of spin waves scattered by the dense set of quasiperiodic reciprocal lattice points, and that the secondary branches in the spectrum are caused by Bragg reflection.

3.2 Diffraction image and window characteristic function

The high-dimensional projection method is an important tool for us to obtain the mathematical diffraction patterns of QC. By choosing a regular octagonal window in perpendicular space and projecting the lattice points inside this window into physical space, the structural pattern of the AB QC can be constructed. The perpendicular space and physical space are illustrated in Fig. 5.

At $k_y = 0$, the several dispersion branches with the highest intensities are marked by colored dashed lines, as shown in Fig. 6(a). The shape of the branches is

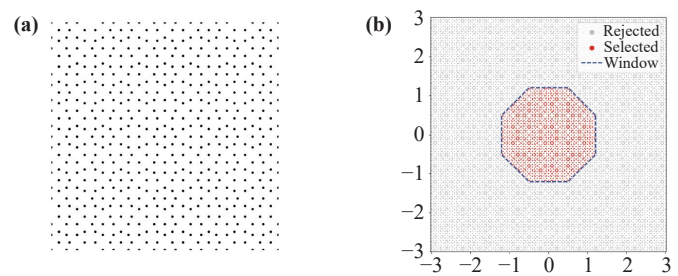


Fig. 5 The projections of AB tiling in physical space and perpendicular space. (a) The partial image of ab tiling in the actual space is obtained by projecting a 4D hypercube onto the actual plane. (b) The projection of a 4D hypercube lattice into perpendicular space. The projection in the perpendicular space is dense; for ease of drawing, a $20 \times 20 \times 20 \times 20$ 4D hypercube lattice is used here. The blue dashed line represents the selection window, the red-highlighted points are those within the selection window, and the transparent red points are those rejected.

consistent with that of the main branch. Compared to periodic lattices, the presence of dispersion branches with distinct intensity magnitudes is a unique characteristic of QC lattices. As revealed by the preceding analysis, the positions and intensities of these branches are directly determined by the positions and intensities of the diffraction points. The positions of the diffraction points are governed by the inherent structure of the AB QC itself, while the intensities of the branches are determined by the selection window, as shown in Eq. (A3). Within the framework of the average unit cell (AUC) method, the position of a branch is interpreted as a modulation relative to the average lattice with a specific quasi-periodic length, and the intensity of the branch is determined by the probability of occurrence associated with that modulation length [32]. Details of the average unit cell method are presented in the appendix.

Many articles point out that the energy bands of QCs exhibit fractal characteristics, with smaller energy gaps observable at smaller scales. This is actually a feature of quasi-periodic modulation: near diffraction points with higher intensities, weaker satellite spots caused by quasi-periodicity appear. Figure 6(c) shows the frequency-intensity distribution map at $\alpha = 0.01$. Compared with $\alpha = 0.03$, more and smaller energy gaps appear, which also reflects the fractal characteristics of the energy bands in QC.

As indicated in Eq. (A3), the positions L^* of diffraction points are distributed quasi-periodically, whereas their intensities are determined by the window characteristic function. In Fig. 6(d), more diffraction spots with weaker intensities in QCs are marked with black dots.

The intensity of diffraction points depends on the characteristic function of the selection window, which is located in the perpendicular space. Therefore, calculating the intensity of a diffraction point requires determining

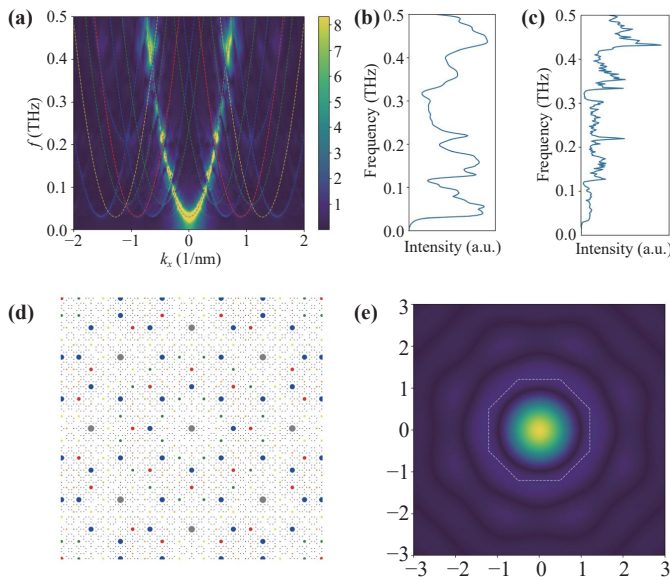


Fig. 6 Satellite points of AB QCs. (a) The k_x - f energy spectrum cross-section at $k_y = 0$, where the shade of color represents the intensity. Additionally, the branches emerging from diffraction points of different intensities are marked by dashed lines in different colors. Only the branches with relatively high intensities, colored blue, red, and green, are shown. Energy gaps appear at the intersections of the white dashed lines and the colored dashed lines. Intensity distribution maps at (b) $\alpha = 0.03$ and (c) $\alpha = 0.01$. (d) The mathematical diffraction image of AB tiling shows more satellite spots marked by black dots. (e) The window characteristic function. The Fourier transform of twice the selection window. The dashed frame in the figure marks the border of a regular octagon with a side length of 1.

its position in the perpendicular space to obtain the value of the corresponding characteristic function. The characteristic function of the selection window in the perpendicular space is shown in Fig. 6(e). In the perpendicular space, points in different regions can be divided by coordination numbers [33], and points in different regions correspond to different types of points in real space [34]. The characteristic function of the selection window is given by the Fourier transform of the selection window. As illustrated in Fig. 6(e), the intensity of the window characteristic function decays radially from the center outward. The dashed boundary forms a regular octagon with unit side length, and the coordinates of diffraction spots in perpendicular space are provided in detail in Fig. A1.

The intensity distribution of diffraction points is related to the characteristic function of the selection window, and the intensity of diffraction points directly affects the energy spectrum bandgap. Therefore, the energy spectrum can be regulated by changing the selection window. According to the properties of the Fourier transform, expanding (reducing) the selection window will cause the dashed frame of the characteristic function

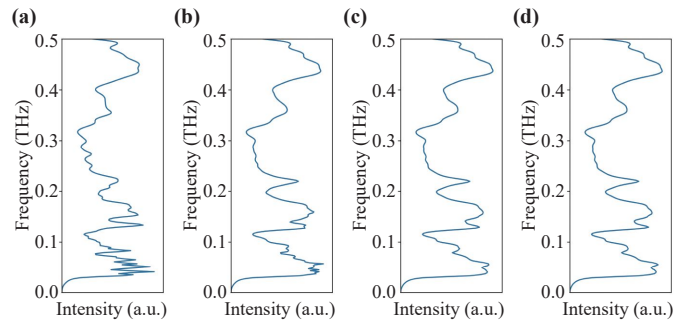


Fig. 7 Intensity-versus-frequency profiles of spin waves in AB MQCs with different system sizes: (a) $128 \text{ nm} \times 128 \text{ nm}$, (b) $256 \text{ nm} \times 256 \text{ nm}$, (c) $512 \text{ nm} \times 512 \text{ nm}$, and (d) $1024 \text{ nm} \times 1024 \text{ nm}$. The vertical dashed lines mark the positions of the primary band gaps. The curves for the 1024 nm and 512 nm samples are nearly coincident across the entire frequency range; the 128 nm and 256 nm samples exhibit large fluctuations in the low-frequency regime due to finite-size boundary effects.

shown in Fig. 6(e) to reduce (expand) proportionally. The dashed frame serves as a reference, within which there are 5 groups of diffraction points with different intensities. Therefore, the intensity of diffraction points can be regulated by adjusting the size of the selection window, so as to realize the adjustment of the energy band structure.

3.3 Finite-size convergence analysis

To examine the influence of the finite simulation domain on the observed band structure, we systematically performed micromagnetic simulations on AB QC antidot arrays with four different system sizes: $128 \text{ nm} \times 128 \text{ nm}$, $256 \text{ nm} \times 256 \text{ nm}$, $512 \text{ nm} \times 512 \text{ nm}$, and $1024 \text{ nm} \times 1024 \text{ nm}$.

Figures 7(a)–(d) present the intensity-versus-frequency profiles of spin waves for the four sample sizes, respectively. It is evident that the curves for the 1024 nm and 512 nm samples are nearly identical across the entire frequency range, with all major band-gap positions and relative depths in excellent agreement. This indicates that the primary band-gap structure is already well converged at sizes of 512 nm and above. In contrast, the 128 nm and 256 nm samples exhibit significant non-physical fluctuations in the low-frequency regime ($f \lesssim 0.15 \text{ THz}$). These deviations arise from the insufficient number of quasiperiodic unit cells in the smaller samples, where long-wavelength spin-wave modes are subject to severe boundary interference and cannot accurately reflect the intrinsic scattering properties of the QC. As the size increases to 512 nm , the boundary effects on the primary gaps are effectively eliminated.

To further quantify the size dependence of branch intensities, Fig. 8 displays the relative intensities of the first six major branches for the four sample sizes,

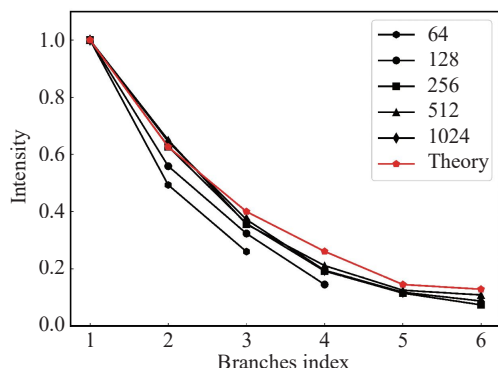


Fig. 8 Normalized intensities of the first six major branches for different system sizes, normalized by the intensity of the first branch. The branch intensities converge from the 512 nm size upward.

normalized by the intensity of the first branch. The results show that the branch intensities converge from 512 nm upward. Taking the second branch as an example, its normalized intensities in the 128, 256, and 512 nm samples are 0.56, 0.63, and 0.65, respectively, gradually approaching a stable value as the size increases. The third through sixth branches exhibit similar convergence behavior. In contrast, the intensities of all branches in the smallest (128 nm) sample deviate significantly from the converged values. Notably, the stronger branches (e.g., the first and second branches) maintain relatively high intensities across all sizes, indicating that the physical origin of the primary band gaps is robust against size variations. In fact, the emergence of the major branches requires only a self-similar structure, as illustrated in Fig. 9, i.e., the minimum size for the major branches to appear is $2 \times 12 \times (1 + \sqrt{2})$ nm.

In summary, the 512 nm $\times$ 512 nm simulation domain strikes a favorable balance between computational efficiency and physical accuracy. It is sufficiently large to capture the primary band-gap structure of the AB MQC. We adopt this size as the basis for our study, and the validation with the 1024 nm sample further ensures that all core conclusions are robust against finite-size effects.

3.4 Tuning band gaps by adjusting projection window sizes

We calculated the regulation of energy gaps achieved by adjusting the selection window size. Here, the calculated results with 1.5 times and 2/3 times its original window size are presented, as shown in Fig. 10, which exhibits significant differences. Firstly, the adjustment of the selection window directly affects the point distribution of the corresponding AB QC in real space. When the selection window is expanded (reduced), the number of lattice points increases (decreases) [see Figs. 10(a–c)].

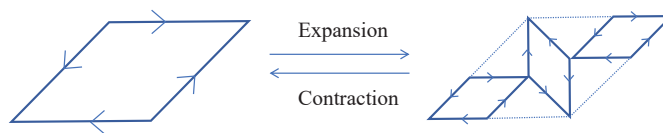


Fig. 9 Schematic illustration of the minimum self-similar structural unit of the AB tiling. The characteristic size is $2 \times 12 \times (1 + \sqrt{2})$ nm, which represents the minimum length scale required for the emergence of the major branches.

The relationship between the reciprocal space and the real space can be derived from Eq. (A1). In reciprocal space, different regions correspond to real space points that have different connectivity numbers [33].

Subsequently, changes in the selection window alter the density of real-space points, which directly affects the spacing between lattice sites. However, due to the unique properties of QCs, such changes do not affect the positions of diffraction spots, but they do influence the intensities of different diffraction spots. The mathematical diffraction patterns are displayed in Figs. 10(d)–(f). Figures 10(g)–(i) shows the energy spectrum diagram of the reciprocal space frequency section near the position of the lowest excitation frequency. After adjusting the size of the selection diagram, the spectrum diagram of the frequency sections are also consistent with their respective diffraction patterns.

In the pristine AB QC, the diffraction points marked in blue and red have relatively large intensities, but there is a significant intensity difference between them. The intensity difference between them can be modulated by adjusting the selection window. When the selection window is reduced to 2/3 of its original size, the intensities of the blue and red marked diffraction points, which had a large intensity difference in the pristine AB QC, become similar, as shown in Fig. 10(e). Their corresponding branches form an image composed of eight connected regular octagons in the frequency section, as shown in Fig. 10(h). At 1.5 times the selection window, the intensity difference between them becomes larger. As shown in Fig. 10(i), several branches with the highest intensity form the canonical structure of the AB QC, which is composed of 45° rhombuses and squares.

It can be seen from this that adjusting the perpendicular-space selection window is an effective way to modify the energy band structure. Compared with the frequency distribution diagram of the pristine AB QC [Fig. 10(m)], when the selection window is 2/3, the widths of the minima of each frequency intensity are closer, as shown in Fig. 10(n). Through this method, more tiny energy gaps can be revealed. As shown in Fig. 10(p), compared with the pristine AB QC, more energy minima appear when $\alpha = 0.03$.

It can thus be concluded that tuning the selection window in perpendicular space is an effective approach to modify the band structure. At $\alpha = 0$ (Fig. 10(o)),

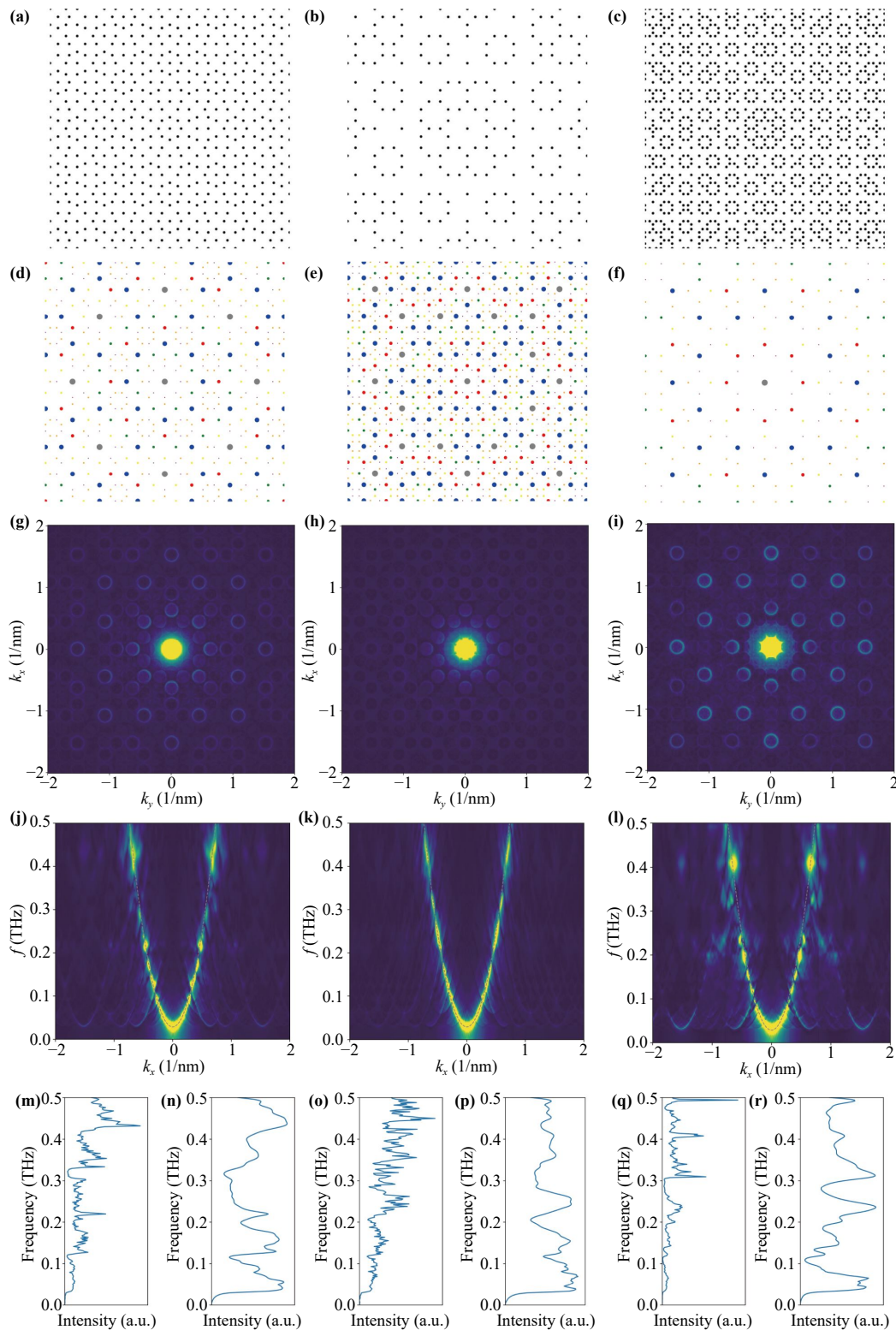


Fig. 10 Tuning the energy gaps of AB QCs by modifying the selection window for perpendicular-space projection. The real-space image corresponding to (a) origin, (b) 2/3-times, and (c) 1.5-times sized selection window. The mathematical diffraction image corresponding to (d) original, (e) 2/3-times, and (f) 1.5-times sized selection window, the diffraction intensity decreases in the order of gray, blue, red, green and purple. The frequency-slice image at $f = 0.04$ THz corresponding to (g) original, (h) 2/3-times, and (i) 1.5-times sized selection window. The k_x - f slice at $k_y = 0$ corresponding to (j) original, (k) 2/3-times, and (l) 1.5-times sized selection window, where the shade of color represents the intensity. In (m, o, q) $\alpha = 0$ and in (n, p, r) $\alpha = 0.03$, SWs intensity at different frequency corresponding to (m, n) origin, (o, p) 2/3-times, and (q, r) 1.5-times sized selection window.

compared with the frequency distribution of the pristine AB QC [Fig. 10(m)], the widths of the minimum values of each frequency intensity are much closer when the selection window is set to 2/3. The pristine AB QC [Fig. 10(o)] shows spectral broadening near 0.116THz and 0.317THz, while the broadening is reduced for the case with the 2/3 selection window [Fig. 10(o)]. More subtle band gaps can be resolved via this strategy. As illustrated in Fig. 10(p), additional energy minima appear compared with the pristine AB QC when $\alpha = 0.03$.

Contrary to the result of reducing the selection window, expanding the selection window to 1.5 times its original size enhances the intensity difference between the red, blue, and gray diffraction spots and other diffraction spots. As shown in Fig. 10(i), compared with the pristine AB QC, only the branches corresponding to a few strongest diffraction spots are displayed, while other branches are suppressed and become indistinguishable due to their relatively lower intensity. From the frequency intensity distribution diagram [Fig. 10(q)], it can be observed that the positions of the energy gaps corresponding to several enhanced diffraction spots have widened.

Furthermore, as illustrated in Figs. 10(q) and (r), several energy gaps exhibit greater stability in contrast to both the pristine QC and the case with a selection window of 2/3. Even with increased damping ($\alpha = 0.03$), these gaps still exhibit a significant intensity difference from other frequencies, which is of great significance in the context of filters, as shown in Fig. 10(r).

To systematically investigate the effect of projection window scaling on the intensities of spin-wave branches, we calculated the branch intensities at multiple intermediate scaling factors ranging from 2/3 to 1.5. Figure 11 presents the normalized intensities of the second and third main branches as a function of the window scaling factor.

As observed in the figure, the branch intensities exhibit an overall continuous and monotonic trend with varying window scaling factors: enlarging the selection window increases the intensity difference between the second and third branches, while reducing the window decreases this difference; at a scaling factor of 1.5, the intensities of the second and third branches tend to converge. This continuous variation demonstrates that the response of branch intensities to window size is smooth and predictable, further validating the reliability of window scaling as a means of band engineering.

It is worth noting that near scaling factors of approximately 1.3 and 1.4, the raw data points exhibit certain simulation artifacts. Specifically, continuous connecting lines appear between diffraction points, giving rise to spurious readings at positions where the theoretical intensity should be zero. Upon examination, this artifact is attributed to the excitation of non-physical high-order

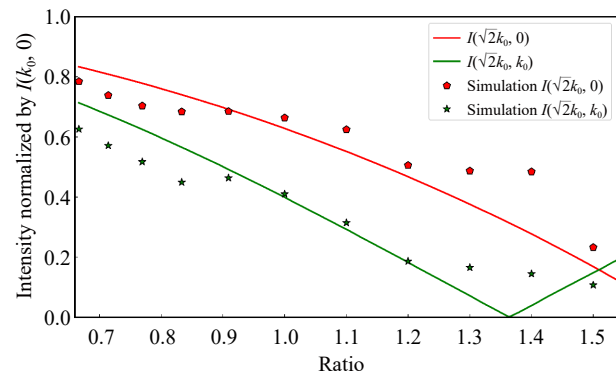


Fig. 11 Normalized intensities of the second and third major branches versus the projection window scaling factor varying from 2/3 to 1.5. All intensities are normalized by $I(k_0, 0)$. Enlarging (reducing) the selection window increases (decreases) the intensity difference between the first and second branches.

numerical noise when the local pulsed source is placed at the serrated grid points of the antidot lattice boundary. As corroborated in Fig. 12, after shifting the excitation source position by 5 nm to the right to avoid the serrated boundary, the spurious readings are completely eliminated, and all data points are found to be in basic agreement with theoretical predictions. The corrected results clearly demonstrate that window scaling provides a continuous and predictable control over branch intensities.

The above findings indicate that the tuning behavior of window scaling can be precisely described by the window characteristic function. Based on this understanding, we can selectively choose scaling factors to achieve targeted manipulation of specific branch intensities, thereby providing theoretical guidance for band structure engineering.

4 Conclusion

This study demonstrates through micromagnetic simulations that in 2D AB MQCs, the SW spectra exhibit uniform-sized circular ring patterns under frequency slices, with the centers of the rings precisely aligned to the high-intensity diffraction points predicted by the higher-dimensional projection method. The intensity correlation between the dispersion branches and diffraction peaks confirms their Bragg reflection origin, thereby verifying the effectiveness of this projection method in the band structure analysis of MQCs. Additionally, the SW spectra further reveal inherent fractal characteristics and numerous band gaps induced by quasiperiodicity. Crucially, band gap engineering can be achieved by adjusting the scale of the perpendicular-space selection window: reducing the window size to 2/3 of its original

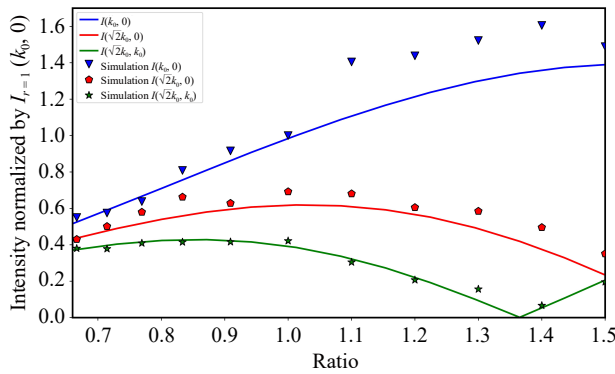


Fig. 12 Intensities of branches of the first, second, and third intensity ranks plotted against the window scaling factor after excitation position optimization. All intensities are normalized to $I_{r=1}(k_0, 0)$, which corresponds to the scaling factor equal to unity. Spurious signals are eliminated after optimization, and experimental data agree well with theoretical results.

dimension enhances the density of mini-gaps by equalizing diffraction intensities, while expanding it to 1.5 times the original size enables the formation of wider band gaps — these band gaps also possess strong damping immunity. This method of adjusting the selection window scale provides a new paradigm for MQC design that goes beyond geometric parameter tuning. Notably, the approach of regulating the band structure by modifying the selection window is realized by altering the reciprocal space structure of the QC and exhibits universality: it is applicable not only to MQCs of different materials and scales but also to photonic QCs and phononic QCs.

Declarations The authors declare that they have no competing interests and there are no conflicts.

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Appendix A: High-dimensional projection method

We briefly introduce the high-dimensional projection method [35] used to calculate the mathematical diffraction spots in Fig. 2(e). AB tiling can be generated by projecting points from a 4D hypercube onto a 2-dimensional plane within a selected window of the perpendicular space, as

described by the following equation:

$$L = \left\{ XP \left| x_{\perp} \in W \right. \right\}, \quad (\text{A1})$$

$$P = \begin{pmatrix} 1 & \cos(\pi/4) & 0 & \cos(3\pi/4) \\ 0 & \sin(\pi/4) & 1 & \sin(3\pi/4) \end{pmatrix}^{\top}, \quad (\text{A2})$$

where P is the projection matrix, $X = (x_1, x_2, x_3, x_4)$ represents the coordinates of the 4D hypercube lattice, $x_{\perp} = XP_{\perp}$ is the coordinate of the perpendicular space projection of the hypercube lattice, and W denotes a regular octagonal selection window in the perpendicular space. As shown in Fig. 5, points of the 4D hypercube lattice within the selection window are projected into physical space to obtain the AB tiling. AB tiling can be represented as a Dirac comb δ_x composed of points as shown in Eq. (A1) and Fig. 1. Combining the high-dimensional projection method and the mathematical calculation of diffraction images, the diffraction measure K of QCs can be expressed by the following formula:

$$K = \sum_{k \in L^*} |A(k)|^2 \delta_k. \quad (\text{A3})$$

Here, L^* is the projection of the high-dimensional dual lattice, and the amplitude is a characteristic function proportional to the Fourier transform of the selection window. In AB tiling, $L^* = \frac{1}{2}L$.

The corresponding main diffraction point is defined as half of the base point, which is shown in Table A1:

$$\mathbf{a}_0 = \frac{1}{2}(1 + 2c, 0) = \left(\frac{1}{2} + c, 0 \right) \approx (1.207, 0). \quad (\text{A4})$$

Therefore, for the AB QC with QC constant l , the principal diffraction spots are given as follows:

$$\mathbf{k}_0 = \frac{2\pi}{l} \mathbf{a}_0 \approx \left(1.207 \frac{2\pi}{l}, 0 \right). \quad (\text{A5})$$

Figure A1 illustrates the positions of diffraction spots corresponding to the first three secondary branches in the perpendicular space.

Table A1 Group 1: Base point $(1 + 2c, 0) = (2.414\ 214, 0)$, where $c = \frac{\sqrt{2}}{2} \approx 0.707\ 106\ 781$.

Real space	Vertical space	4D lattice
$(1 + 2c, 0)$	$(1 - 2c, 0)$	$(1, 1, 0, -1)$
$(1 + c, 1 + c)$	$(1 - c, 1 - c)$	$(1, 0, 1, -1)$
$(0, 1 + 2c)$	$(0, 1 - 2c)$	$(1, -1, 0, -1)$
$(-1 - c, 1 + c)$	$(-1 + c, 1 - c)$	$(1, -1, -1, 0)$
$(-1 - 2c, 0)$	$(-1 + 2c, 0)$	$(1, -1, -1, 1)$
$(-1 - c, -1 - c)$	$(-1 + c, -1 + c)$	$(1, 0, -1, 1)$
$(0, -1 - 2c)$	$(0, -1 + 2c)$	$(1, 1, 0, 1)$
$(1 + c, -1 - c)$	$(1 - c, -1 + c)$	$(1, 1, 1, 0)$

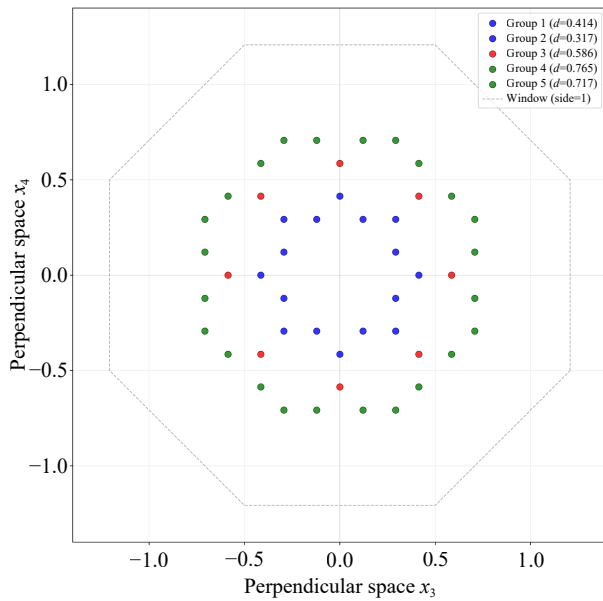


Fig. A1 Positions of diffraction spots for the first three secondary branches in perpendicular space.

Appendix B: Average unit cell method

The AUC method [36, 37] can draw the same conclusions as the high-dimensional projection method, and their calculation formulas are completely equivalent. The average unit cell method does not require high-dimensional methods and directly calculates in the actual physical space. The QC is considered an average crystal plus a quasi-periodic modulation, and the wave vector $\mathbf{k}$ is obtained through the statistically average length of the lattice. The wave vector is modulated by the quasiperiodic component ($\mathbf{k}' = n\mathbf{k} + m\frac{\mathbf{k}}{\tau}$, where n, m is an integer, τ is a coefficient related to the QC structure.) to determine the positions of the diffraction points, and the intensity is determined by the probability of different modulation lengths. AUC can intuitively reveal the significance of the positions and intensities of diffraction points: positions reflect the satellite diffraction points caused by the average lattice and QC modulation, while intensities reflect the probability of different modulation lengths.

We take the one-dimensional silver sequence (1D AB QC) as an example in the following discussion.

Starting from a single letter S and iteratively applying the substitution rules

$$S \rightarrow SL, \quad L \rightarrow SLS,$$

one obtains a non-periodic but long-range ordered one-dimensional sequence, e.g.,

$$S \rightarrow SL \rightarrow SLSLS \rightarrow SLSLSLS \dots$$

Counting the occurrences of the two letters, the substitution matrix is

$$M = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

(with the order S, L), whose largest eigenvalue

$$\lambda = 1 + \sqrt{2}$$

is the inflation factor of the sequence. The corresponding stable frequencies satisfy

$$f_S : f_L = \sqrt{2} : 1,$$

which after normalization gives

$$f_S = 2 - \sqrt{2}, \quad f_L = \sqrt{2} - 1.$$

Setting the short interval length $S = 1$ and the long interval length $L = \sqrt{2}$, the mean atomic spacing is

$$\begin{aligned} a &= f_S \cdot 1 + f_L \cdot \sqrt{2} = (2 - \sqrt{2}) + (\sqrt{2} - 1)\sqrt{2} \\ &= 4 - 2\sqrt{2} \approx 1.1716. \end{aligned}$$

Using the average unit cell method, the sequence is projected onto two reference lattices, giving diffraction peaks at

$$k_{n,m} = nk_0 + mq, \quad n, m \in \mathbb{Z},$$

where the fundamental wavevector is

$$k_0 = \frac{2\pi}{a} = \frac{2\pi}{4 - 2\sqrt{2}},$$

and the satellite wavevector is

$$q = \frac{k_0}{1 + \sqrt{2}}.$$

If the actual short interval length is taken as $S = 12$ nm and the long interval as $L = 12\sqrt{2}$ nm, then all lengths are scaled by 12 and all wavevectors are divided by 12, yielding

$$a = l(4 - 2\sqrt{2}) \approx 14.06 \text{ nm},$$

and

$$k_{0,0} = \frac{\pi}{l(2 - \sqrt{2})} \approx 0.447 \text{ nm}^{-1}.$$

Principal diffraction spots of the 2D AB QC:

$$k_0 = k_{1,1} = \left(\frac{\sqrt{2} + 1}{2} \right) \frac{2\pi}{l} \approx 0.632 \text{ nm}^{-1}.$$

This sequence belongs to the silver ratio QC, whose diffraction is purely point-like.

References

1. D. Levine and P. J. Steinhardt, Quasicrystals: A new



- class of ordered structures, *Phys. Rev. Lett.* 53(26), 2477 (1984)
2. D. Shechtman, I. Blech, D. Gratias, and J. W. Cahn, Metallic phase with long-range orientational order and no translational symmetry, *Phys. Rev. Lett.* 53(20), 1951 (1984)
3. T. Yadav and N. Mukhopadhyay, Quasicrystal: A low-frictional novel material, *Curr. Opin. Chem. Eng.* 19, 163 (2018)
4. A. Barman, G. Gubbiotti, S. Ladak, A. O. Adeyeye, M. Krawczyk, et al., The 2021 magnonics roadmap, *J. Phys.: Condens. Matter* 33(41), 413001 (2021)
5. M. Krawczyk and D. Grundler, Review and prospects of magnonic crystals and devices with reprogrammable band structure, *J. Phys.: Condens. Matter* 26(12), 123202 (2014)
6. A. De, K. Dutta, S. Mondal, S. Barman, Y. Otani, and A. Barman, Magnonic crystals with complex geometry, *Phys. Rev. B* 103(6), 064402 (2021)
7. S. Tacchi, B. Botters, M. Madami, J. W. Kłos, M. L. Sokolovskyy, M. Krawczyk, G. Gubbiotti, G. Carlotti, A. O. Adeyeye, S. Neusser, and D. Grundler, Mode conversion from quantized to propagating spin waves in a rhombic antidot lattice supporting spin wave nanochannels, *Phys. Rev. B* 86(1), 014417 (2012)
8. S. Tacchi, M. Madami, G. Gubbiotti, G. Carlotti, A. O. Adeyeye, S. Neusser, B. Botters, and D. Grundler, Angular dependence of magnetic normal modes in NiFe antidot lattices with different lattice symmetry, *IEEE Trans. Magn.* 46(6), 1440 (2010)
9. R. Mandal, S. Saha, D. Kumar, S. Barman, S. Pal, K. Das, A. K. Raychaudhuri, Y. Fukuma, Y. Otani, and A. Barman, Optically induced tunable magnetization dynamics in nanoscale co antidot lattices, *ACS Nano* 6(4), 3397 (2012)
10. H. A. Elsayed, Quasiperiodic photonic crystals for filtering purpose by means of the n doped semiconductor material, *Phys. Scr.* 95(6), 065504 (2020)
11. D. Lusk, I. Abdulhalim, and F. Placido, Omnidirectional reflection from Fibonacci quasi-periodic one-dimensional photonic crystal, *Opt. Commun.* 198(4–6), 273 (2001)
12. M. Hase, H. Miyazaki, M. Egashira, N. Shinya, K. M. Kojima, and S. I. Uchida, Isotropic photonic band gap and anisotropic structures in transmission spectra of two-dimensional fivefold and eightfold symmetric quasiperiodic photonic crystals, *Phys. Rev. B* 66(21), 214205 (2002)
13. A. L. Chen, Y. S. Wang, Y. F. Guo, and Z. D. Wang, Band structures of Fibonacci phononic quasicrystals, *Solid State Commun.* 145(3), 103 (2008)
14. D. Sutter-Widmer, S. Deloudi, and W. Steurer, Prediction of Bragg-scattering-induced band gaps in phononic quasicrystals, *Phys. Rev. B* 75(9), 094304 (2007)
15. M. Gei, Z. Chen, F. Bosi, and L. Morini, Phononic canonical quasicrystalline waveguides, *Appl. Phys. Lett.* 116(24), 241903 (2020)
16. D. Sutter-Widmer, S. Deloudi, and W. Steurer, Periodic average structures in phononic quasicrystals, *Philos. Mag.* 87(18–21), 3095 (2007)
17. J. Luck and D. Petritis, Phonon spectra in one-dimensional quasicrystals, *J. Stat. Phys.* 42(3–4), 289 (1986)
18. A. Jagannathan, The Fibonacci quasicrystal: Case study of hidden dimensions and multifractality, *Rev. Mod. Phys.* 93(4), 045001 (2021)
19. R. Mehta, B. Rana, and S. Saha, Magnetization dynamics in quasiperiodic magnonic crystals, *J. Phys.: Condens. Matter* 36(44), 443003 (2024)
20. C. Kong, J. Liu, and H. Xiong, Nonreciprocal microwave transmission under the joint mechanism of phase modulation and magnon Kerr nonlinearity effect, *Front. Phys. (Beijing)* 18(1), 12501 (2023)
21. T. Manago, K. Fujii, K. Kasahara, and K. Nakayama, Spin wave propagation and nonreciprocity in metallic magnonic quasi-crystals, *J. Phys. D* 55(11), 115005 (2022)
22. S. Watanabe, V. S. Bhat, K. Baumgaertl, M. Hamdi, and D. Grundler, Direct observation of multiband transport in magnonic penrose quasicrystals via broadband and phase-resolved spectroscopy, *Sci. Adv.* 7(35), eabg3771 (2021)
23. F. Wei, Y. Zhou, W. Zhang, Z. Ren, G. Chen, H. Li, G. Han, S. Yan, and S. Kang, Fractal properties and tunable flat band structures in chiral magnonic quasicrystals, *Phys. Rev. B* 112(14), 144405 (2025)
24. A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, and B. Van Waeyenberge, The design and verification of MuMax3, *AIP Adv.* 4(10), 107133 (2014)
25. V. S. Bhat, S. Watanabe, F. Kronast, et al., Spin dynamics, loop formation and cooperative reversal in artificial quasicrystals with tailored exchange coupling, *Commun. Phys.* 6, 193 (2023)
26. D. Kumar, J. W. Kłos, M. Krawczyk, and A. Barman, Magnonic band structure, complete bandgap, and collective spin wave excitation in nanoscale two-dimensional magnonic crystals, *J. Appl. Phys.* 115(4), 043917 (2014)
27. S. Neusser and D. Grundler, Magnonics: Spin waves on the nanoscale, *Adv. Mater.* 21(28), 2927 (2009)
28. S. Neusser, G. Duerr, S. Tacchi, M. Madami, M. Sokolovskyy, G. Gubbiotti, M. Krawczyk, and D. Grundler, Magnonic minibands in antidot lattices with large spin-wave propagation velocities, *Phys. Rev. B* 84(9), 094454 (2011)
29. J. W. Kłos, D. Kumar, J. Romero-Vivas, H. Fangohr, M. Franchin, M. Krawczyk, and A. Barman, Effect of magnetization pinning on the spectrum of spin waves in magnonic antidot waveguides, *Phys. Rev. B* 86(18), 184433 (2012)
30. F. Groß, M. Zelent, A. Gangwar, S. Mamica, P. Gruszecki, M. Werner, G. Schütz, M. Weigand, E. J. Goering, C. H. Back, M. Krawczyk, and J. Gräfe, Phase resolved observation of spin wave modes in antidot lattices, *Appl. Phys. Lett.* 118(23), 232403 (2021)
31. S. Tacchi, G. Duerr, J. W. Kłos, M. Madami, S. Neusser, G. Gubbiotti, G. Carlotti, M. Krawczyk, and D. Grundler, Forbidden band gaps in the spin-wave spectrum of a two-dimensional bicomponent magnonic crystal, *Phys. Rev. Lett.* 109(13), 137202 (2012)
32. J. Wolny, B. Kozakowski, and P. Repetowicz, Construction of average unit cell for penrose tiling, *J. Alloys Compd.* 342(1–2), 198 (2002)

33. A. Jagannathan and M. Duneau, Properties of the Ammann–Beenker tiling and its square periodic approximants, *Isr. J. Chem.* 64(10–11), e202300119 (2024)
34. M. O. Oktel, Strictly localized states in the octagonal Ammann–Beenker quasicrystal, *Phys. Rev. B* 104(1), 014204 (2021)
35. M. Baake, and U. Grimm, Mathematical diffraction of aperiodic structures, *Chem. Soc. Rev.* 41(20), 6821 (2012)
36. J. Wolny, B. Kozakowski, P. Kuczera, R. Strzalka, and A. Wnek, Real space structure factor for different quasicrystals, *Isr. J. Chem.* 51(11–12), 1275 (2011)
37. J. Wolny, P. Kuczera, and R. Strzalka, Periodically distributed objects with quasicrystalline diffraction pattern, *Appl. Phys. Lett.* 106(13), 131905 (2015)