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# 土木工程环境振动控制中波屏障研究进展

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**摘 要**:振动无处不在,振动控制技术伴随着各个领域科技的发展也在不断发展.在土木、交通与环境等领域,随着轨道交通的快速发展,环境振动带来的危害日趋严重,为满足人们对居住环境舒适度的要求,特别是为满足精密仪器工作或高精尖产品生产对微振环境的需求,各种振动控制措施也在不断开发.土木工程中常用的振动控制策略主要有三种:减弱振源激励、阻断振动传播路径和直接保护受振体.本文针对路径控制中常用的排桩波屏障和空沟(填充沟)波屏障,以及波屏障所埋置的土体,完成了四项工作.首先,做了两个分类:一是依据不同设计理论,将波屏障进一步区分为传统波屏障和周期波屏障;二是根据由简单到复杂的简化方法,将土体简化为单相土体、饱和土体和非饱和土体三种简化模型.其次,对单相土体、饱和土体和非饱和土体中传统波屏障的研究进展分别作了简要回顾.再次,结合笔者近20年关于周期波屏障的研究经验,分别就周期波屏障在单相土体、饱和土体和非饱和土体中的研究进展做了较全面和系统的综述.最后,给出总结和展望.本文聚焦于土木工程中波屏障研究进展,特别是周期波屏障研究进展,可为从事环境振动控制研究者提供参考.

**关键词**:环境振动控制;体波;表面波;波屏障;周期结构;周期波屏障

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# Advances in wave barriers for ambient vibration control in civil engineering

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**Abstract**: Vibrations are ubiquitous, and vibration control technologies continue to evolve alongside scientific and technological advancements across various fields. In the fields of civil engineering, transportation and environment engineering, the rapid development of rail transit has exacerbated the adverse effects of ambient vibrations. To satisfy demands for comfortable living environments and, more importantly, the stringent micro-vibration requirements for operating precision instruments and manufacturing high-tech products, various vibration control measures are being actively developed. Three primary vibration control strategies are commonly employed in civil engineering: attenuating the excitation at the source, blocking the vibration transmission path, and directly protecting the target structure. Focusing on pile barriers and open (or in-filled) trench barriers frequently used for path control,

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as well as the surrounding soil in which these barriers are embedded, this paper presents four main contributions. First, two classification schemes are introduced: wave barriers are categorized into traditional and periodic wave barriers based on their underlying design theories; and the embedding soil is modeled with increasing complexity as single-phase, saturated, or unsaturated soil. Second, a brief review of the research progress on traditional wave barriers in single-phase, saturated, and unsaturated soils is provided. Third, based on the author's nearly 20 years of research experience with periodic wave barriers, a comprehensive and systematic review of their development in single-phase, saturated, and unsaturated soils is presented. Furthermore, there is an urgent need for research on periodic wave barriers under moving load excitations. Finally, a summary and future outlook are provided. By focusing on the advancement of wave barriers in civil engineering, particularly periodic wave barriers, this paper serves as a valuable reference for researchers engaged in ambient vibration control.

**Keywords**: ambient vibration control; bulk waves; surface waves; wave barriers; periodic structures; periodic wave barrier

自有人类活动以来,环境振动就一直伴随着人们的日常生活,特别是随着轨道交通的快速发展,环境振动带来的危害日趋严重,环境振动已被列为世界七大环境公害之一.文献[1]指出,交通运营引起的环境振动以弹性波的形式在土体中传播,主频大多在10~100 Hz,这些振动不仅严重妨碍周边建筑中精密仪器的使用,还会影响周边居民的身心健康.如,北京地铁4号线近距离经过北京大学物理试验基地,导致精密仪器无法正常使用.文献[2]指出,位于美国曼哈顿的爱丽丝·塔利音乐厅由于靠近地铁,内部声环境受到影响,不得不进行隔振改造.另外,人体各部位的固有频率主要在1~80 Hz,与环境振动的主频范围高度重合,文献[3]的研究表明,长期处于振动环境会引起人体器官的不适甚至引发疾病;而文献[4]指出,即将全面投入使用的北京高能同步辐射光源(High Energy Photon Source,HEPS),根据大科学试验装置的运行要求,地面振动位移均方根有效值需控制在25 nm以下,远超一般土木工程结构对振动控制的要求.我国轨道交通发展迅速,文献[5]表明,截至2025年底,我国铁路运营里程达16.5万km,其中高铁运营里程超5.04万km,城市轨道交通运营里程达到1.31万km.快速发展的交通在给人们出行带来便利的同时,环境振动带来的危害日趋严重,常用振动控制技术已难以满足工程需求,环境振动控制向着更精准、更精细化方向发展.

为降低环境振动带来的危害,人们已开发了多种振动控制措施.如文献[6]采用减振扣件、弹性垫层等以减弱振源能量向外输出,文献[7-9]通过在波的传播路径上设置波屏障以减弱波的传播,文献[10]在结构底部设置隔振支座以减少振动向上部结构传输等.其中,波屏障由于施工方便、经济性能好等优点,深受研究者和工程技术人员青睐,已在工程中得到广泛应用.从整体性特征看,波屏障可以分为连续和非连续型2种.连续型波屏障包括隔振墙、隔振空沟和填充沟等形式;非连续型波屏障包括排桩、排孔和板桩等.本文主要介绍排桩、空沟和填充沟等波屏障的研究进展,其他振动控制技术的研究进展不再赘述.

## 1 单相土体中传统波屏障研究进展

土体通常是由固体、液体和气体等不同组份构成的一种非常复杂的多相介质,研究波在多相介质中的传播本身就是一件非常困难的事,波屏障的置入进一步加大了问题求解的难度,为此,有必要对土体进行简化.目前对土体已有多种简化模型,其中最简单的一种就是忽略液相和气相的存在而将土体简化为单相介质.在单相土体假设下,已有许多针对单排桩、单个空沟、单个填充沟以及未考虑周期特性设计的多排波屏障(统称为传统波屏障)的研究成果.

### 1.1 排桩波屏障减振

20世纪70年代,文献[11-12]基于大量实验,提出利用排桩调控振动传播的设想,并给出了设计准则.依据该设计准则,排桩的直径需不小于被屏蔽波长的1/6,桩间净距需不大于被屏蔽波长的1/4.这样,在调控低频振动时,桩直径需要达到3~5 m,难以在工程中实施.文献[13-14]分析了单排桩引起的波在平面内的散射,并对减振效果进行三维数值模拟,讨论了桩径、桩长和桩间距等参数对减振效果的影响.文献[15-19]基于多重散射理论,研究了任意排布的双排和多排桩屏障对P、SV、SH波的减振性能,结果表明,影响减振效果的主要因素为桩间

距,桩间距为3倍桩半径时减振效果较好.文献[20-21]基于波函数展开法和Graf加法定理,得到了单排柔性实心桩和刚性空心管桩对平面纵波和横波散射系数的理论解.此外,文献[22]在平面应变模型假设基础上,研究了刚性空心管桩和圆形实心桩屏障对P波和SH波的屏蔽效果,并探讨了桩的剪切模量和入射波频率等因素对屏蔽效果的影响.

文献[9,23-25]给出单排桩对Rayleigh波散射问题的理论解,并在此基础上进一步研究了土体分层以及桩多排布置对Rayleigh波减振效果的影响,发现增加桩的排数有利于提高减振效果,而多排桩的排间距对减振效果影响较小.文献[26]对钢管桩、混凝土实心桩和空心桩的减振效果分别进行了数值分析,发现钢管桩的减振效果优于混凝土实心桩,混凝土空心桩刚度过小时无减振作用.文献[27-28]采用虚土桩模型,研究了土的剪切模量、桩的弹性模量和截面形状对排桩减振性能的影响.文献[29-30]采用振动理论,研究了层状土中单个实心桩或管桩以及横观各向同性土中群桩的垂直振动.文献[31-33]在对排桩屏障对点荷载以及线荷载诱发的表面波的衰减效果研究中发现,屏障后振动得到明显衰减,但在桩前存在振动放大现象,两种现象均得到现场试验和数值模拟的证实.此外,文献[34]和文献[35]采用波函数展开等方法,分别研究了层状土中单排桩和双排桩的减振性能,发现分层面附近会出现振动放大现象.

文献[36-37]先是对桩列和孔列的减振效果进行了三维数值分析,由于所需单元数量巨大,计算相当费时,文献[38]又提出将一排桩等价为连续屏障的设想,即将非连续排桩等效为连续墙进行分析,以此来提高计算效率.将排桩视为Euler-Bernoulli梁,土体视为半无限大、均匀的各向同性线性黏弹性介质,文献[39-40]构建了三维土体-多排桩有限元数值分析模型,该模型具有计算速度快、内存需求低且精度较高的特点.文献[41]将排桩视为Timoshenko梁,土体底部直接与基岩相连,构建了排桩减振的数值分析模型,研究发现,当排桩长度与土体厚度相当且排桩直接与基岩相连时,排桩具有较好的减振效果.文献[42-43]研究匀质土中桩屏障对体波的隔离,探讨了桩间距、入射波频率和桩刚度等参数对减振效果的影响,发现刚性桩比柔性桩更有效.文献[44]分别建立混凝土墙和一排混凝土桩的有限元数值分析模型,数值模拟结果表明,振动的衰减效果分别为70%和50%.

文献[45]将布置于高架桥沿线的管桩视为波屏障,进行了大量现场测试和数值模拟研究,结果表明,距振源25 m处的地表及地下振动响应衰减均可达到5 dB.文献[46-49]开发了一种蜂窝型波屏障(Wave Impeding Block,WIB),通过大量实验与数值模拟研究发现,相较于传统排桩,WIB对3~5 Hz范围内的低频水平激励,可取得更好的减振效果.文献[50]针对WIB和空沟对竖向减振的有效性进行对比研究,结果表明,空沟的衰减效果优于WIB.文献[51]将废弃轮胎集料回填于单排桩屏障中,并对减振效果进行数值模拟研究,结果显示当屏障深度与表面波波长相当时,振动衰减达11 dB.文献[52]采用间接边界元法,对二维多排桩屏障的平面波减振性能进行了分析.文献[53]进一步结合快速多极子-间接边界元法,研究了三维多排桩屏障对表面波的衰减.研究结果表明,当桩长超过表面波波长的1倍时,屏障能够取得较好的减振效果.文献[54]针对排桩屏障对交通荷载引起的地面振动衰减进行数值模拟,发现增大桩体长度、提高桩身弹性模量以及优化空心率等措施,均可显著提升排桩的减振性能.

### 1.2　空沟和填充沟波屏障减振

文献[55]对空沟的减振性能进行了大量的实验研究,结果表明,空沟深度不小于0.6倍瑞利波波长时,可取得75%或更高的减振效果.文献[56]对空沟和泡沫填充沟的减振性能进行全尺寸实验和数值模拟,考虑了两者的尺寸以及激励位置对衰减效果的影响,得出泡沫填充沟对地震动的衰减效果更明显的结论.文献[57]采用大比例尺实验,对比了空沟、碎石填充沟和排桩的减振效果.文献[58-59]对空沟和填充沟的减振性能进行了大量的场地测试和数值模拟研究,其二维非线性分析可较好地模拟土体和波屏障的相互作用.文献[60-61]对空沟、空沟内分别填满水或土工泡沫几种工况的减振性能进行了场地测试,发现相比充水沟,增加激励频率或沟深度可提高空沟(或填充了土工泡沫)的减振效果,而3种工况的减振效果都随着测点与沟槽间距离的增加而降低.文献[62]比较了空沟和填充了粉煤灰、土工泡沫和水的沟槽的减振性能,得出空沟减振效果最好,而后依次是填充有水、土工泡沫、粉煤灰的填充沟的结论.

文献[63-64]采用边界元法,在时域和频域内研究空沟和填充沟的减振性能,提出空沟和填充沟的设计准则,指出空沟的减振效果要优于填充沟,此结论比文献[56]的结论更可信;另外,通过合理改变空沟或填充沟的深度、长度、宽度以及距振源的距离,

还可以获得更好的减振效果.文献[65]将有限元法与遗传算法相结合,对空沟和填充沟屏障的设置位置和深度进行了优化分析,明确双排布置比单排布置可提高20%的减振能力.文献[66]研究Rayleigh波在填充沟和土体界面上的反射系数和传输系数,结果表明,随着填充材料密度和弹性模量的增加,反射系数增大,但传输系数减小.

文献[67]提出了一种在埋置波屏障顶部嵌入一个延伸板的结构,研究发现这种结构可以进一步增强填充沟的减振效果.文献[68]对传统的矩形隔振沟进行了改进,提出一种无需支护并有较大深度的倾斜弯曲空沟,这种结构的减振效果是传统隔振沟的5倍以上.文献[69]采用边界元法(Boundary Element Method, BEM)对矩形空沟和填充沟的减振性能进行研究,并据此提出了屏障设计准则.文献[70]用空沟和填充沟对列车引发振动进行控制,发现填充沟的深度和宽度是影响减振效果的重要参数,填充材料对减振效果也有显著影响,柔性材料的减振效果更优.文献[71]在利用填充沟对交通引发振动的控制研究中,发现填充沟深度的增加可极大程度提升填充沟的减振效果.文献[72]采用平面有限元模型,研究了成层土和均质土2种不同工况下,空沟和柔性材料填充沟对地铁诱发振动的减振效果.文献[73]通过三维数值分析,研究由水和泡沫材料填充的填充沟屏障对地表振动的控制,结果显示泡沫填充沟的效果较好.文献[74-76]对半空间中空沟和柔性材料填充沟的减振性能进行了大量的数值模拟,结果表明,沟槽与激励源的距离对屏障的减振效果没有明显影响,意味着无论沟槽的位置如何,振动幅度的减小几乎不受影响,这为工程应用提供了一定参考.

以上研究将土体假设为单相介质,虽可大大降低分析的难度,但在沿海等地下水位较高地区,基于单相土体假定得到的分析结果与实测结果有不小差距.因此,需要采用饱和土体模型进一步研究波屏障的减振机理.

## 2 饱和土体中传统波屏障研究进展

当土体内的孔隙被水完全填充时形成饱和土,文献[77-78]建立了描述饱和土体本构关系的多孔弹性介质理论,基于该理论,已有大量利用波屏障调控体波和表面波传播的研究成果.考虑到土体的稳定性问题,饱和土体中大多采用桩屏障调控波的传播,虽有一些采用连续型波屏障的成果,但相对较少.

### 2.1 对体波的调控

文献[7,79-80]基于Biot理论并借助Fourier-Bessel级数展开方法,成功解决了饱和土中利用单排实心桩调控体波的分析问题,并通过三维数值分析,进一步阐明了饱和土的渗透系数和桩土模量比对S波衰减的影响规律.文献[81]基于Biot波动理论,在平面应变假设和忽略固相、液相惯性附加质量前提下,通过波函数解法给出波垂直入射饱和土中无限长圆柱体(柱形空腔和刚性桩)的解析解.文献[82]建立了适用于饱和土中任意几何形态孔洞对弹性波散射问题的复变函数解法,并给出了数值分析结果.此外,文献[83-84]还采用Fourier变换法、Hankel变换法以及虚拟桩法计算了移动荷载作用下饱和土中单桩的动力响应,发现移动荷载会引起桩身的负摩阻力.文献[85-87]基于波函数展开法,在频域内推导出饱和土半空间中无限长圆柱形孔洞对平面P、SV波散射问题的解析解,并通过参数化分析研究了入射波频率及入射角度对孔洞表面应力集中因子的影响.

在平面应变假设下,文献[88-89]将快纵波P1波和SV波在饱和土中的传播转化为平面内弹性波的多重散射问题,评估了单排空心管桩的减振效果.将排桩简化为Euler-Bernoulli梁、土体为饱和土或黏弹性介质,文献[90]通过构建土体-多排桩结构体系的三维边界元-有限元耦合数值模型,研究P1波和SV波作用下系统的动力响应.文献[91-92]采用边界元-有限元耦合模型,分析层状饱和土中排桩在竖向以及水平荷载作用下的瞬态响应和长期位移响应,通过参数分析得到了土体的分层特性、排桩尺寸及桩间距对位移响应的影响规律.文献[93]完成了饱和土中单排孔近场减振效果的现场测试和三维边界元数值模拟,研究结果表明,增大孔洞半径和深度,减小相邻孔的净间距,都可以提高减振效果.文献[94]将入射波和散射波引起的饱和土的位移幅值与仅入射SV波引起的位移幅值之比作为归一化参数,比较了屏障含一排、两排和三排桩时的调控能力.文献[95]针对任意排数的桩屏障,基于Helmholtz分解定理导出了液相和固相间的复波数和振幅比,进而研究了饱和土的渗透性和排间距对减振效果的影响.文献[96]基于复变函数理论研究了饱和多孔介质中圆形空腔对弹性波的散射问题,讨论了液体渗透性、剪切模量和弹性波类型(快、慢纵波和横波)等参数对波场响应的影响,认为剪切模量对孔隙水压力和有效应力的影响大于液体渗透性产生的影响,快纵波和横波对径向位移响应的影响远大

于慢纵波.文献[97-98]建立边界元群桩模型则深化了对桩-土动力相互作用的认知.文献[99]采用高精度的间接边界积分方程分析了二维不连续桩群屏障在宽频带内对平面P波和SV波的减振效果,结果表明:对于较软土壤,减振效果更好;与P波相比,SV波对桩间距更敏感;对于低频波应采用多排桩,但对于高频波,超过三排桩不会显著改善减振效果.

文献[100]针对平面谐波入射情况,给出了饱和土中含任意形状屏障时的一种数值解,当屏障为球形夹杂物时,该解与已有分析解一致,验证了数值解法的可靠性.文献[101]构建了饱和土中置入夹水混凝土复合隔振屏障的计算模型,给出P波入射下位移振幅衰减系数的解析解,结果表明,当入射角大于14°时,屏障的隔振效果随入射角的增大总体向好.文献[102]采用2.5D间接边界元法,研究移动荷载作用下空沟对层状饱和土体中波的调控作用,讨论了土体物理参数和沟槽几何参数对隔振效果的影响.数值分析结果表明,空沟在层状饱和土中的隔振效果比在均质饱和土中要好,而在均质饱和土中的隔振效果又优于在相应的单相土中.文献[103]研究饱和多孔介质波屏障水平埋置于饱和土体中时对3种体波的衰减,发现随着入射角的增加,P2波和SV波的反射振幅比先增加后减小,而P1波的变化正好相反.

将分别用含液饱和多孔材料制作的波阻板和用单相固体制作的波阻板水平埋置于单相土中,文献[104]采用Fourier级数展开法研究了这两种波阻板的减振性能,得出前者比后者不仅可设计性好、减振效果也好的结论.文献[105]针对功能梯度材料制成的WIB,采用坐标变换法得到移动荷载作用下饱和地基的动力控制方程,进而研究了该类屏障的减振效果,结果表明,WIB的减振效果随着梯度指数的增加而增加.文献[106]则通过有限元模拟和协方差矩阵自适应演化方法,研究利用波屏障控制地下交通荷载引起的饱和土地基中振动衰减问题,探讨了屏障高度、宽度以及位置等多个参数对衰减效果的影响规律.

### 2.2　对表面波的调控

文献[7]将桩看作Euler-Bernoulli梁,基于Biot理论研究了一排刚性桩在饱和土半空间中对Rayleigh波的调控,结果表明,土的渗透率对Rayleigh波幅值的降低作用巨大.文献[107-108]利用虚拟桩法和传递矩阵法,建立了饱和土中单排桩、多排桩的第二类Fredholm积分方程,通过数值积分分析了排桩对Rayleigh波的调控能力,得出刚性桩比柔性桩调控效果好的结论.此外,文献[109]提出一个半解析模型来研究移动荷载作用下多孔弹性半空间中一排桩屏障的屏蔽效率.文献[110-111]借助半解析边界元方法,分别推导了含双排桩时分析Rayleigh波散射的三维边界元方程和空沟对Rayleigh波散射的二维边界元方程,研究了远场的求解问题,得出排间净距以及空沟宽度对调控效果影响不大、桩间净距和空沟深度起控制作用的结论.文献[112]基于多重散射理论研究了双排桩屏障对弹性波的衰减效果,结果表明,双排桩屏障减振效果优于单排桩,桩净间距为3倍桩半径时减振效果最好.

文献[113]构建了考虑土骨架和流体间耦合效应的饱和多孔弹性半空间模型,研究沟槽对列车诱发振动的隔离效果,结果表明:当列车速度高于临界速度时,采用单相介质模型会低估沟槽的屏蔽效率约25%～35%.文献[114]构建了土-WIB体系的三维边界元模型,研究WIB在不同参数下的减振效果,并与单相土工况进行了对比,结果显示:在各类等效直径与剪切模量工况下,WIB在饱和土中的减振性能优于在单相土,但饱和土中WIB减振效果对埋置深度的敏感性显著高于单相土.将半解析边界元法和薄层法相结合,文献[115]研究了饱和土中空沟的减振性能,结果表明,当空沟深度小于Rayleigh波波长时,加深空沟有利于提高减振效果,而增加空沟宽度对减振效果影响不大.

文献[116]提出一种用于分析土-结构相互作用的数值分析方法,进而研究了一种薄壁波屏障全部埋置或部分埋置于饱和土中时的减振性能.文献[117-118]采用协方差矩阵自适应进化策略优化波屏障,对比研究了单相土体和饱和土体中空沟以及填充沟屏障的减振性能,发现屏障在饱和土体中比在单相土体中减振效果要好,在保持屏障总体积不变情况下,增加排的数量会获得更好的减振效果.文献[119]研究上覆单相土层时沟槽对饱和土体中表面波的调控行为,探讨了沟槽形状、沟中不同水位深度等因素对减振效果的影响,并考虑了沟槽多排布置情况,发现多数水位下降低土体渗透性不利于多排沟槽的减振.

为简化分析,上述研究工作都是在单相土体或饱和土体假设下进行的,未考虑气相的影响.文献[120-125]研究结果表明,土体中气相的存在会改变波的传播特征,特别是饱和度指数等对波的传播影响非常显著.因此,学者们引入更精细的非饱和土模型研究饱和度等参数对波屏障减振性能的影响.

## 3 非饱和土体中传统波屏障研究进展

考虑土体中固相、液相和气相三相介质的相互作用,采用非饱和土模型研究波动问题比采用饱和土模型要困难许多,因而相关成果很有限.究其原因,主要是非饱和土中基质吸力改变了土体的本构关系、强度特性、渗透特性以及应力状态.此外,在一些有关均匀非饱和土和层状非饱和土动力响应的研究中,文献[126-129]发现不同饱和度下波的传播特性、衰减特性均不一致,文献[130-132]发现波的传播特性发生改变时,会进一步改变土的动力特性.

### 3.1 饱和度的影响

考虑土骨架的黏性,文献[133]建立三维管桩-非饱和土模型,探讨SH波作用下系统的动力响应,结果表明:桩和土在SH波作用下发生强烈的相互作用,饱和度的变化对管桩的水平动力响应影响相对较小,但土骨架的黏性影响较大.文献[134]将桩简化为梁,建立Rayleigh波作用下研究非饱和土-桩耦合作用的分析模型,给出了桩水平位移、旋转角、弯矩和剪力的解析解;为表征桩的减振特性,通过桩头响应定义动力相互作用因子,结果表明:当饱和度大于0.95时,动力相互作用因子对激励频率的变化总是很敏感.而对于多根桩,文献[135]将桩假设为铁木辛柯梁,先分别采用边界元法和有限元法计算了黏弹性非饱和土以及桩的动力响应,而后通过桩土界面的位移协调和应力连续条件,将边界元法和有限元法结果耦合进而获得系统的动力响应,最后指出,饱和度的增加可以提高桩的减振效果,且这种影响在长桩情况下更为明显.

### 3.2 其他参数的影响

为了探究在不同入射波作用下非饱和土-桩系统的动力响应,文献[136-139]基于Helmholtz矢量分解定理,研究了非饱和土体中双层波屏障对P波和S波的调控,发现波阻抗比等材料参数对波的透射/反射振幅比具有显著影响,而饱和度对另两个纵波(P2波和P3波)的幅值率影响巨大,研究还推广到了非饱和层状土体.用黏弹性多相介质描述非饱和土体、用一维铁木辛柯或欧拉梁理论简化桩,文献[140-142]研究了水平或竖向简谐荷载作用下部分或完全埋置于非饱和土体中单根桩的动力响应,以及有关参数对Rayleigh波引起的桩顶水平动力响应的影响,结果表明,由于非饱和土体中毛细压力的存在和多相之间的耦合,Rayleigh波的传播特性发生了明显变化,进而显著影响了桩顶的动力响应.文献[143]在研究非饱和粉质黏土中桩的抗震性能时,采用名为"动态桩分析代码"的独立有限元计算机代码,发现了非饱和土的基质吸力对桩的抗震性能有着显著的影响,尤其是在土壤逐渐变干的过渡区域.文献[144]利用双重Fourier变换和Galerkin法,推导了单节点5个自由度的非饱和土以土骨架位移$\boldsymbol{u}_{\mathrm{b}}$、流体和气体压力$p_{\mathrm{l}}$和$p_{\mathrm{g}}$为场变量的有限元表达式,进而研究了单排桩屏障的减振性能,发现桩间距对地表响应影响较大.随后,文献[145]又推导出非饱和土频域-波数域内的2.5维有限元表达式,计算非饱和地基-隧道系统在移动简谐荷载作用下的动力响应,并指出距离隧道越远,桩的减振效果越好.

上述关于传统波屏障的研究,不管是基于单相土体假定、饱和土体假定,还是基于非饱和土体假定,虽然得出了一些有意义的结论,然而大量数值模拟和试验结果表明,传统波屏障在某些情况下减振效果较好,但在其他情况下减振效果不明显甚至会放大振动,这一现象一直无法得到合理解释.究其原因,主要是传统波屏障的减振机理难以揭示,传统波屏障设计无法考虑衰减域这一关键因素.另外,为提高减振效果,工程中经常采用多排形式布置波屏障.在多排布置情况下,由于界面引起的多重反射和透射,使得分析非常复杂,因而相关研究进展极其缓慢,成果很少,特别是多排布置时形成的周期波屏障对波的调控机理,更是缺乏认识,波屏障设计理论有待完善.受固体物理学中声子晶体结构能够产生"频率带隙"启发,笔者在文献[2,146-147]和文献[8,148]中分别提出了周期基础和周期波屏障设计理念,由此为土木工程结构基础隔震和环境振动控制研究开辟了一个新方向,相关研究已取得极大进展.

## 4 周期波屏障研究进展

工程中使用的多排波屏障,无论是多排桩还是多排沟槽,绝大多数都是周期排布的,因此可引入周期结构理论分析其对波的调控能力.自周期波屏障设计理念提出以来,笔者及团队成员围绕周期波屏障频散关系特别是复频散关系求解方法、单相土体和饱和土体中各种周期波屏障关于体波和表面波衰减域的确定等开展了较系统和深入的研究,文献[8,149-156]不仅揭示了波屏障的减振机理、丰富了波屏障设计理论,文献[157]还直接从能量分布出发,提出了识别表面波模态的一种新准则,极大地提高了周期结构中表面波模态识别的可靠度,该准则已

被广大研究者所采用.另外,文献[158]还通过在本构方程中直接引入材料阻尼项,提出了一种材料阻尼层边界技术,极大地方便了单相土和饱和土数值分析中无限域的局域化.按照先易后难的研究习惯,周期波屏障研究可分为在单相土、饱和土和非饱和土中三个阶段.目前,周期波屏障已成为环境振动控制领域国内外学者的研究热点之一,成果颇丰.

### 4.1　单相土体中周期波屏障性能研究

周期波屏障的研究最早在单相土体中开展.按照固体物理学中“频率带隙”的产生机制,周期波屏障大致可以分为布拉格散射型、局域共振型和组合型;而按照振源类型,又可以分为固定源激励和移动荷载激励下周期波屏障两类.

#### 4.1.1　布拉格散射型

文献[159]在空旷的场地上周期性地打了几排孔洞,经检测发现,该类孔洞可以减弱地震引起的表面波的传播.文献[160]指出,该理念早在2013年就被文献[8,148]提出,文献[159]成果虽是文献[8,148]成果的特例,但仍被学界认为是超屏障工作的里程碑和奠基石.采用混凝土、EPS泡沫塑料填充沟和PVC管桩等作为波屏障,文献[161-164]就周期波屏障对表面波的隔离效果进行系统的理论分析、数值模拟和实验研究,结果表明,在表面波的衰减域内,屏障的减振能力随着屏障层数或排数的增加而增强,屏障的减振能力由其周期特性所控制.借助COMSOL数值分析软件中的偏微分方程模块,文献[165]先是在平面应变假设下求得了周期桩屏障对体波的衰减域,而后又建立二维和三维数值分析模型,通过频域分析验证了屏障的有效性.对于周期桩屏障分析中采用平面应变假定的合理性问题,文献[166]先期完成了2D模型和3D模型对比分析,数值结果表明,随着桩长度的增加,基于3D模型所得结果收敛于平面应变假设下的2D结果,验证了平面应变假设的合理性.此外,文献[167]采用弱形式求积单元法,考虑了初应力对周期屏障衰减域的影响,结果表明,初应力虽然不改变最大衰减系数,但可以显著改变衰减域的位置和宽带,进而影响波屏障的减振效果.文献[168]提出一种由各向异性超材料构成的导波屏障,用于调控出平面弹性波的传播,完成的频域和时域数值模拟结果表明,此导波屏障能有效引导弹性波沿特定方向传播,而其减振能力受径向与周向折射率之比等参数的影响,增大折射率比值可提升减振效果.

文献[169-170]成功求解了两种周期填充沟屏障的表面波衰减域,数值模拟和试验结果表明,周期波屏障可用于调控列车以及地震引起的表面波.此外,文献[171]还开发了周期性空心钢沟槽屏障,该屏障同时展现出布拉格型和局域共振型衰减域.文献[172-174]借助虚拟桩法并结合边界元法,提出基于Floquet变换的一种波数域边界元法,进而求得周期排桩屏障在波数域中和空间中的解答,讨论了排数和土的弹性模量等参数对波屏障减振效果的影响规律,得出增大土的弹性模量将提高屏障所屏蔽的弹性波波长,进而减弱周期桩屏障的减振效果.在保持填充率不变情况下,文献[175-176]求解了4种不同截面形状的周期填充桩屏障的衰减域,频域和时域数值模拟结果验证了该类波屏障对弹性波具有明显的调控效果,而且由于部分泄漏面波转化为体波,面波衰减域得到拓宽.文献[177]提出了一种分析层状土体中周期波屏障减振效果的有效方法,完成的大量模型计算和参数影响分析表明,将周期波屏障置于轨道下方比置于轨道旁边可以取得更好的减振效果.文献[178-179]研究周期桩屏障对城市轨道交通引起的振动的抑制作用,数值模拟和实测结果均表明,靠近振源布置3排14 m长的桩屏障,桩后15 m范围内的振动至少可降低5 dB,周期桩屏障展现出很好的减振效果.文献[180-181]给出理想周期桩屏障中平面波散射问题的解析表达式,据此可显著改进计算效率.文献[182]研究由混凝土形成的超材料对铁路诱发的地面振动的减振作用,通过建立列车-轨道-土壤3D耦合模型,分析了其在时域和频域内的减振效果.文献[183]以层状土体中周期屏障打到基岩为前提,发现在30 Hz以下可以形成很宽的衰减域,进而该类屏障可以大幅抑制低频面波的传播.文献[184]在周期沟槽中填充超材料来改进周期波屏障的动力特性,对不同振源距离、入射角度和沟槽数目下屏障的减振性能进行了数值模拟和试验研究,实测的传递谱衰减范围与数值分析确定的衰减域完全一致.为节省在土体中开展全尺度周期孔洞屏障试验所需的时间和资金,文献[185]采用铝试件进行了缩尺试验,考虑Rayleigh波不同入射角和不同孔洞排数对减振效果的影响.文献[186-188]对周期桩屏障和周期填充沟屏障减振的有效性进行了对比研究,并从特征方程求解方法、频散关系绘制、衰减域确定、频域和时域响应数值模拟等角度,对周期结构在地震动隔离和环境振动控制方面的研究进展进行了简要综述.文献[189]研究了周期排桩对地震表面波的隔离,通过数值分析探讨了几何参数对低频衰减域的影响.文献[190]通过对原设计方案中的支护桩和止水帷幕进行适当改

进,使其形成较宽的面波衰减域,从而以极其微小的成本成功解决了某临高铁站工程结构振动控制难题.

景观造型和树木形成的周期特性也有减振功能.文献[191]沿地表面周期性地摆放一些质量块或景观造型,进而形成周期波屏障,数值模拟和试验测试均证实该周期波屏障对调控表面波传播是有效的.文献[192]在地面周期性布置薄板形成波屏障,通过求解该屏障关于Rayleigh波的频散关系,发现由于屏障将水平传播的Rayleigh波转换为向下传播的体波,进而可以减弱地面的动力响应.文献[193-194]沿平行于铁路轨道方向在土体中周期性埋置混凝土块形成周期波屏障,采用2.5D有限元法研究了屏障的减振性能,大量参数分析结果可为工程设计提供简化依据.文献[195-196]直接将自然成长的树林看作周期性波屏障,通过建立三维数值分析模型,得到面波的衰减域;进一步的参数分析则表明,较粗的树干且较小的树距有利于在较低频段产生较宽的衰减域,这对于环境振动控制是有利的.

周期波屏障的优化设计近年来也受到不少关注.文献[197-200]将有限元法与遗传算法相结合,以平面波和出平面波的衰减域宽度和衰减指数为目标,对桩屏障的构型进行了拓扑优化,通过频域和时域分析验证了优化屏障的有效性.文献[201-202]以衰减域宽度为优化目标,采用遗传算法优化波屏障构型,对层状土体中3种优化后的周期屏障进行了频域和时域响应数值模拟,验证了该屏障的有效性.文献[203-205]提出基于机器深度学习的体波周期波屏障拓扑优化设计方法.此外,文献[206]基于遗传算法构建了单目标和多目标表面波周期波屏障优化设计框架.文献[207]开发了一种称为双向深度学习的网络模型,以期提高三组份周期桩屏障的设计效率.然而,上述优化的屏障,由于构型复杂,加之成本高和不易施工等因素,尚难应用于实际工程.

4.1.2 局域共振型

局域共振型波屏障可用于隔离体波.在管桩内填充软或硬材料,文献[208]采用平面波展开法和有限元法研究了周期性填充桩的频散特性,发现在中低频范围内该类周期排桩可以形成衰减域,但形成衰减域的机理不同,填充软的材料是由于发生局域共振,而填充硬的材料是由于布拉格散射型.文献[209-212]开发三组份局域共振周期波屏障,研究了典型单元的几何和材料参数对屏障衰减域的影响,频域和时域分析表明屏障的衰减域能够对应地铁运营引起环境振动的主频段.文献[213-215]从减轻地震动对上部结构作用的角度出发,通过数值模拟和模型试验,研究了周期基础特别是局域共振型周期基础的动力特性,表明该类周期屏障不仅可以用于隔离地震动,还可用于隔离环境振动带来的危害.文献[216]通过复频散分析,研究了桩屏障和沟槽屏障的减振性能,结果表明,周期桩屏障由于能够产生局域共振,因而在低频范围可以取得更好的减振效果.文献[217]采用平面波展开法求解了一种立方晶格中含球形夹杂物的三维周期性超材料,讨论了几何参数对带隙形成的影响.用含重金属核心外包软质材料的球形内含物替代标准骨料,文献[218]设计了一种混凝土超材料,对其动力特性进行了数值研究.文献[219]提出一种适用于局域共振超结构的带隙估算方法,并探讨了谐振器数量的影响.文献[220-221]设计一种周期性波阻块,研究其在均匀单相土和成层单相土中的衰减域特性及材料和几何参数对减振效果的影响,结果表明,通过适当选取参数,波阻块可以突破土壤截止频率的限制,实现对目标频率振动的有效隔离.

局域共振型波屏障也可以用于隔离面波.把橡胶包裹混凝土桩梯度埋置于土体中,文献[222-223]研究屏障对Rayleigh波及伪表面波的吸收特性,发现采用2 m的典型单元可在20 Hz以下形成较宽的面波衰减域.文献[224]从理论分析、数值模拟和试验研究三方面,系统研究水平浅埋于土体中由橡胶包裹混凝土柱形成的波屏障对Rayleigh波的减振性能,结果表明,填充材料阻尼和局域共振引起的能量耗散抑制了Rayleigh波的传播.文献[216,225]通过复频散分析,除研究了桩屏障对体波的调控性能外,还研究了周期桩屏障对Rayleigh波的隔离,结果表明,凋落波的产生和局域共振是形成衰减域的主要原因.最近,文献[226-227]采用土工泡沫和负泊松比材料,开发了几种在低频范围内可以形成较宽衰减域的周期波屏障,数值和试验结果验证了屏障具有明显的调控表面波的功能.文献[228]基于大量数值计算,提出一种能够显著提高周期屏障衰减域宽度的双级配设计方法,其中涉及对基质材料弹性模量的梯度设计.文献[229]在空心钢管中填充软质橡胶构建了大尺寸周期波屏障,三维数值模拟结果显示优化后的波屏障可在10 Hz以下产生衰减域,进而可以有效减弱低频表面波的传播.文献[230-232]用空心混凝土管桩和橡胶支座支撑的钢柱芯体构建周期排桩,通过理论分析、数值模拟和试验测试研究了该屏障的减振性能,指出采用米级尺度桩体,可控制层状土体中波长为10~100 m的表面波

传播.文献[233-235]提出2种新的典型单元构型,数值模拟和试验测试表明,采用该单元制作的周期波屏障具有低频宽带减振特性.

4.1.3　移动荷载下周期波屏障研究

上述关于周期波屏障的研究,主要是在固定源激励和垂直入射假设下进行的.虽然有些研究提到移动列车,但尚未从本质上考虑列车移动对波屏障设计的影响.荷载在土体上移动时会在土体中激起波的传播,此波称为主导波.由于Doppler效应,主导波的频率与移动荷载的频率有时会出现较大差异.因此,从本质上讲,应依据主导波的频率而不是移动荷载的频率来设计周期波屏障.此外,荷载移动还使得入射波方向不断改变,因此波屏障设计需要考虑斜入射问题.

文献[236]采用Floquet分解法,通过计算单元在空间-波数域的传递函数求得周期结构在移动载荷下的响应,并建立了该解与利用傅里叶变换得到的结构响应的等价性.将移动荷载速度线与周期波屏障频散特性相结合,文献[237]针对出平面振动,成功确定了移动荷载作用下周期桩屏障的衰减域,结果表明,波屏障的衰减域不仅与其自身的频散特性有关,还与荷载的移动速度和频率相关.随后,借助空间傅里叶变换方法,文献[238]又分别研究了出平面和平面应变条件下周期填充沟槽屏障对移动荷载引起振动的隔离问题,结果表明,移动荷载下周期波屏障的隔振性能主要由衰减域-荷载移动方向的波数关系图决定.基于平面应变假设,文献[239]首先给出表面移动力荷载作用下半平面体动力响应的理论解答,在此基础上进一步研究了周期波屏障对地下结构的振动隔离特性.此外,文献[240]对利用周期排桩和周期空沟波屏障隔离半空间中移动荷载引起的表面波传播进行了系统的理论分析、数值模拟和试验研究,3种结果取得很好的一致性.而对于移动荷载引起的成层土体中表面波的隔离,文献[241]则基于等效介质理论,提出非常简单的衰减域近似确定方法,该方法克服了成层土体中剪切波速难确定的难题,其合理性得到数值模拟和试验研究的证实.文献[242]通过定义目标频率范围的覆盖率,建立了考虑移动荷载影响的周期桩屏障优化框架.

### 4.2　饱和土体中周期波屏障性能研究

饱和土体中可以存在快纵波P1波、慢纵波P2波和剪切波S波3种不同类型的波.文献[243-244]研究饱和土中周期桩屏障对体波的隔离,考虑了初应力对波屏障减振性能的影响;通过二维和三维数值分析,揭示了初应力对频率响应的影响机理,桩长不太长时初应力可近似取桩顶应力值.此外,文献[245-247]将周期桩屏障的研究拓展到管桩和填充桩,求解了复频特性,揭示了单元构型、几何及材料参数等对波屏障减振效果的影响.文献[248]将饱和土中多排管桩体系简化为无限周期排布结构,采用波函数展开法求解了典型单元,显著降低了计算复杂度.研究表明,管桩屏障在饱和土中对平面SV波的减振性能优于在单相土中,且当管桩内外径比取值为0.6时可实现较优的减振效果.文献[249]采用数值方法,分别研究了土工泡沫周期波屏障对单相土体和饱和土体中P波和SV波的调控,通过频域和时域数值模拟和大量参数分析,得出轻质泡沫屏障衰减域较宽、调控效果更好的结论.此外,文献[250]针对饱和土体中周期桩屏障对体波的隔离问题,以衰减域宽度为优化目标,采用遗传算法优化了波屏障构型,并对优化屏障的减振性能进行了频域和时域响应数值模拟,数值模拟结果验证了该屏障的有效性.文献[251]研究了饱和土体中周期桩屏障对剪切波的调控,并采用遗传算法和有限元法对波屏障构型进行了拓扑优化.

在考虑初应力情况下,文献[156]在研究成层单相土中周期填充沟和周期排桩对Rayleigh波隔离基础上,进一步研究了成层饱和土中这两种屏障对Rayleigh波的隔离,同样发现初应力的存在会使波屏障的衰减域向低频移动.针对两种不同单元构型,文献[252]研究了周期桩屏障对半空间中Rayleigh波的隔离,并对波屏障设计流程提出建议.文献[253]采用Kelvin模型描述饱和土的黏性,研究了周期填充沟屏障对表面波的隔离,发现土的黏性虽然有利于波的衰减,但在衰减域内,黏性的贡献很小,而在衰减域外,黏性对衰减起主导作用.

需要指出的是,由于P1波传播速度快、能量耗散小,目前尚未找到合适的周期波屏障构型对其形成衰减域.

### 4.3　非饱和土体中周期波屏障性能研究

研究非饱和土体中的波动问题本身就是一件困难的事,原因在于非饱和土体中的基质吸力改变了土体的本构关系,最直观的结果是非饱和土体比饱和土体中多存在一种纵波模态.若再考虑多排情况下屏障中界面引起的波的反射、透射等现象,求解将变得非常困难.因此,关于多排屏障或周期屏障对非饱和土体中弹性波调控行为的研究成果极其稀少.但是,非饱和土中快纵波(P1波)的波速相比于饱和土中则显著降低,这为非饱和土中利用周期波

屏障减振带来了曙光.

文献[254]采用Galerkin法,给出了非饱和土体2.5D有限元表达式,比较了非饱和土体中单排桩与双排桩的减振性能.文献[255-256]针对双层及多层波屏障的减振效果开展系统研究,重点研究了波屏障材料参数对减振性能的影响规律,结果表明,增加波屏障层数虽能提高减振效果,但界面增多导致分析中未知量数目激增,求解变得更加复杂,因而影响到多层波屏障在工程中的应用.

为解决上述问题,文献[257-260]从周期结构理论出发,通过构建适用于非饱和土体频散关系的数值求解方法,研究了利用周期屏障调控非饱和土体中体波和表面波的可行性.在调控体波时,文献[257]首先采用Bragg散射型周期波屏障,同时打开了P1波与S波的衰减域,这在饱和土体中难以实现.考虑到非饱和土体中饱和度对波屏障的衰减域有显著影响,以及衰减域的频率范围相对较高、难以实现对低频波动的有效调控,文献[258]又设计了一种局域共振型周期填充桩屏障,打开了P1波和S波的低频衰减域,同时还考虑了填充材料的黏性,拓宽了减振带宽.在调控表面波时,文献[259]设计了周期填充沟屏障,发现非饱和土体中存在一个散射型衰减域和伪表面波衰减域,前者由于界面上Bragg散射所致,而后者源于部分表面波能量向底部泄漏.由于两个衰减域的频率范围互不重叠,共同实现了表面波的宽频减振.除采用周期填充沟屏障隔离表面波外,文献[260]还研究了利于超表面调控低频表面波的可行性.首先构造了一个超表面,而后建立了描述非饱和土体中低频表面波与超表面相互作用的解析公式,发现超表面在耦合频率处可产生一个与入射波场相位相反的散射波场,通过波场干涉叠加抵消入射波能量.基于该发现,通过布置多个不同固有频率的谐振器,实现超表面的宽频减振,数值模拟结果验证了该方案的可行性.

## 5 结论与展望

关于周期波屏障调控弹性波研究,可得如下结论:

1)基于周期结构理论分析和设计多排波屏障,可以避开波的多次反射和透射等问题,机理清晰.

2)在固定源激励下,周期波屏障调控单相土体中弹性波已有许多成果,但在饱和土体中的成果仍较少,在非饱和土体中的成果则更为稀少.

3)在移动荷载激励下,利用周期波屏障调控弹性波的研究,无论是在单相土体、饱和土体还是在非饱和土体中,成果更罕见.

尽管周期波屏障已成为土木工程环境振动控制研究热点议题之一,并已取得一定进展,但还有许多工作有待开展:

1)周期波屏障的减振性能主要由其衰减域所控制,衰减域的确定又主要依赖频散关系,尤其是复频散关系.然而,求解典型单元的频散关系并不是简单的事,特别是对构型复杂的单元,还涉及模态难识别等问题.另外,空间单元的计算量往往很庞大.开发可靠的模态识别方法和高效的频散计算方法都是非常有意义的工作.

2)Doppler效应导致土体中主导波的频率与移动荷载的频率不再相同,有时会出现较大差异.移动荷载下利于周期波屏障减振,首先就是要确定主导波的频率,而后才能设计波屏障.但目前关于移动荷载下周期波屏障的研究,主要局限于出平面问题和平面应变情况.另外,在对波屏障减振效果的数值分析中,由于计算条件的限制,移动荷载的速度还比较低,无法真正模拟高速列车荷载情况.

3)周期波屏障的研究成果目前主要是在单相土体中取得的,饱和土体特别是非饱和土体,由于控制方程包含了更多的场变量和材料参数,分析更为复杂,尤其是周期波屏障在三维空间中的数值模拟需要庞大的计算量,因而大多研究还只是在平面应变假设下完成的,其他情况也值得关注.

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