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A Validation Method for the Enhanced Fuzzy Min-Max Neural Network With Application to Heart Disease Data Classification

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ABSTRACT

This study investigates early detection of heart diseases using a new Enhanced Fuzzy Min-Max (EFMM) neural network. Two key modifications are introduced: a refined EFMM contraction procedure during the learning phase to reduce data distortion and information loss, and a new weighted validation method to optimise hyperbox selection and improve prediction accuracy. The proposed model is evaluated with heart disease data sets subject to varying expansion constraints, training data sizes, and cross-validation settings. The results indicate that the proposed EFMM model consistently outperforms other models, including the original Fuzzy Min-Max (FMM), EFMM, and non-FMM classifiers such as J48, MLP, IBK, AdaBoost, and others. The test accuracy rates achieve 82.11% with 5-fold cross-validation and 81.56% with 10-fold cross-validation. Compared with other FMM variants, the proposed model achieves 80.26% accuracy, surpassing GFMM, IOL-GFMM, Kn-EFMM, and RFMM models. Using real-world heart disease data, the proposed model outperforms FMM and EFMM in terms of accuracy, sensitivity, and specificity. Statistical test results indicate the effectiveness of the proposed model in performance statistically, as compared with those FMM, EFMM, and other classifiers. The improved EFMM model is generic and applicable not only to heart disease diagnosis but also to other general data classification tasks.

1 | Introduction

Artificial neural networks (ANNs) have emerged as useful computational models for solving pattern classification problems with successful outcomes in various medical diagnosis applications [1–5]. Medical diagnosis plays a pivotal role in ensuring effective patient care and improving health outcomes. Due to their ability to model complex non-linear relationships, artificial neural networks (ANNs) have gained widespread use in diagnostic classification tasks, where precision and the ability to recognise patterns are essential. Heart disease classification, in particular, is a critical application, as cardiovascular diseases are the leading cause of death globally, which is responsible for nearly one-third of all fatalities worldwide [6]. Heart-related diseases claim approximately 17.9 million lives annually,

Abbreviations: ANNs, artificial neural networks; ECG, electrocardiogram; EFMM, enhanced Fuzzy Min-Max; FWNN, fuzzy wavelet neural network; MLP, multi-layer perceptron; PSO, particle swarm optimisation; RBF, radial basis function; SVM, support Vector Machines; TLBO, teaching learning-based optimisation.

[Correction added on 26 September 2026, after first online publication: The affiliation "Artificial Intelligence & Data Science Department, Isra University, Amman, Jordan" was included in error for author Osama Nayel Al Sayaydeh. It has been removed and replaced with "Department of Computer Science, Faculty of Information Technology, Middle East University, Amman 11831, Jordan". The author's affiliation with Tafila Technical University, Tafilah, Jordan remains unchanged].

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making them one of the most fatal health issues globally [6]. Early detection and appropriate treatment are key factors in preventing heart disease-related deaths and reducing overall mortality risks [6].

The primary goal of data classification is to develop a hypothesis that can effectively distinguish input data samples and categorises them into specific target classes or labels [7]. This process involves pre-processing the data to eliminate noise and then training a model that can identify underlying patterns in the data set [2]. Several ANNs and related machine learning models, such as support vector machines (SVM), modular granular neural networks, Fuzzy Min-Max (FMM) neural networks, convolutional neural networks (CNNs), and Geno-Fuzzy algorithm, have been developed for pattern classification, face recognition, and path planning tasks, and have shown good results [8–16].

Despite the progress in handling noisy and nonlinear data, ANNs encounter a significant challenge known as catastrophic forgetting when learning new information incrementally. Catastrophic forgetting refers to the inability of a learning system to retain previously acquired knowledge while absorbing new knowledge from incoming data samples [17]. Various ANNs have been proposed, including the FMM neural network, which combines ANN and fuzzy set theory [18], to address this issue. FMM inherits the advantages of both neural networks and fuzzy logic, allowing it to handle imprecise data effectively and support online learning with short training times [18]. Nonetheless, certain limitations in FMM, particularly related to its expansion, overlap test, contraction, and validation procedures during the prediction (i.e., test) phase, hinder its full potential [18].

In our previous work [18], an EFMM neural network was introduced to address the specific limitations of the FMM model. The improvements comprise of three main components. First, a new hyperbox expansion rule has been implemented to address the problem of overlapping hyperboxes. Second, an extended set of hyperbox overlap test rules has been incorporated to identify additional cases of overlap. Lastly, a new rule for hyperbox contraction has been introduced to effectively resolve the issue. Although EFMM demonstrated improved classification results for various data sets [18], there are rooms to further enhance its performance.

This paper introduces a new weighted validation technique aimed at improving the EFMM prediction accuracy and effectiveness for the classification of heart diseases. Specifically, our research aims to address the limitations by introducing two key modifications to the EFMM model: refining the contraction procedure during the learning stage to reduce data distortion and missing information, and proposing a new weighted validation method for the selection of winning hyperbox during prediction. The proposed model's performance is compared with that of the original FMM, EFMM, and non-FMM models using heart disease data. In summary, this study contributes towards improving the EFMM model for heart disease data classification.

The paper is organised as follows: Section 2 introduces an overview of utilising neural networks for diagnosis of heart diseases, focussing on offline models and their limitations, is presented. Recent FMM variants for analysis of medical diseases are also discussed. In Sections 3 and 4, we explain the EFMM model and conduct a comprehensive analysis of the EFMM membership function, respectively. A new weighted validation method is elucidated in Section 5. This is followed by an in-depth analysis of the computational time complexity of the resulting EFMM model in Section 6. A performance evaluation of our proposed model on heart disease-related data sets is presented in Section 7. Discussion and concluding remarks are given in Sections 8 and 9, respectively.

2 | Literature Review

Researchers have constantly improved machine learning models, including ANNs, for pattern classification [1, 2]. The learning process within ANNs can be broadly categorised into two paradigms: offline and online learning [19]. Online learning, also referred to as incremental learning, offers a viable strategy for undertaking catastrophic forgetting. In this respect, the FMM model has an adaptable boundary structure and a straightforward operational framework, which has achieved successful outcomes across diverse applications, spanning medical diagnosis, ECG beat analysis, fault detection, and face recognition [1–4]. In this section, we first discuss the application of offline ANNs in the context of heart diseases. Then, we present online learning models, specifically on FMM variants for medical disease diagnoses.

2.1 | ANNs for Diagnosis of Heart Diseases

Many experiments on diagnosing heart diseases have been conducted over the years using various data mining techniques [20, 21]. A new hybrid intelligent system was developed by Shao et al. [22] for classifying heart diseases. The first stage of this model determined the best ANN input variables using three techniques: a rough set, logistic regression, and multivariate adaptive regression splines. In the second stage, the best subset of ANN input variables was identified. The proposed model outperformed single-stage ANNs. A combination system of two multi-layer perceptron (MLP) networks and a single radial basis function (RBF) network was developed by Hadziabdic and Koker for the diagnosis of heart diseases [23]. The majority voting scheme was used to obtain the final result. Bashir et al. [24] introduced an ensemble framework that combines three classifiers—Naïve Bayes, SVM, and Decision Tree—in order to improve heart disease prediction [24]. These three classifiers were combined using the majority vote scheme to yield a high classification rate. Alneamy and Alnaish [25] presented a hybrid system that integrates teaching learning-based optimisation (TLBO) with fuzzy wavelet neural network (FWNN) to improve the accuracy of heart diseases diagnostic [25]. TLBO was used for training FWNN and updating its parameters to obtain the minimum misclassification rate. Subbulakshmi and Deepa [26] employed particle swarm optimisation (PSO) algorithm to optimise the ELM parameters, reducing its hidden layer neurons while enhancing the overall network performance [26]. Feshki and Shijani [27] developed a hybrid model consisting of feature ranking, a backpropagation neural network, and PSO to diagnose heart disease with high accuracy, and low costs and short time [27]. This method consisted of three stages: feature ranking, feature optimisation, and classification. First, the data features were ranked using the feature ranking method. Next, the ranked features were used by PSO to derive the optimum

feature set. Then, the backpropagation network was used to classify the input features [27]. Samuel et al. [28] combined an ANN and a fuzzy analytic hierarchy process (fuzzy-AHP) to improve the prediction performance of heart failure risks in patients [28]. Fuzzy-AHP was used to calculate the global weights for each feature based on single contribution [28]. The global weights were employed to train the backpropagation neural network for prediction of heart failure risks [28]. Overall, this hybrid system outperformed other conventional ANN classifiers. Another diagnostic model for heart disease was introduced in [29]. This model integrates the discretisation method with the Naïve Bayes classifier, thereby enhancing its accuracy and reliability. The approach involves gathering initial value partitions from domain experts and utilising them to generate the ultimate discretisation of numerical features. The results indicated the usefulness of using the incorporated discretisation method to achieve a good distinguishing rate of heart diseases [29]. In [30], researchers developed various feature selection techniques to optimise feature selection and machine learning algorithms for predicting heart attacks. Four feature selection methods were used and 12 classifiers from diverse categories to determine the optimal classifier and the most relevant features. Based on three performance metrics—processing time, accuracy, and Receiver Operating Characteristic (ROC) results, it was found that the SVM classifier with a linear kernel and relief feature selection method was the most effective algorithm for heart attack prediction. The algorithm achieved a classification rate of 84.81% [30]. A compilation of relevant literature findings is presented in Table 1. Although various models have been developed for heart disease diagnosis, several challenges persist, including parameter optimisation, selection of the optimum ANN hidden neurons, and mitigating the issue of catastrophic forgetting, as evidenced in Table 1. In light of these challenges, our research adopts an online learning

TABLE 1 | A summary of the literature review (Y = yes indicates that the limitation still exists, N = no limitations have been overcome).

Ref.	Model/method	Key components	Key findings	Tuning parameters	Hidden neurons	Catastrophic forgotten
[22]	Hybrid intelligent system	Rough set, logistic regression, multivariate adaptive regression splines, ANN	Outperforms single-stage ANNs in classifying heart diseases	Y	Y	Y
[23]	Combination system	Two MLP neural networks, single RBF neural network	Majority voting scheme used for final result	Y	Y	Y
[24]	Ensemble framework	Naïve Bayes, SVM, decision tree	Majority vote scheme used to enhance prediction of heart diseases	Y	N	Y
[25]	Hybrid system	TLBO, fuzzy wavelet neural network (FWNN)	TLBO used to train FWNN and improve diagnostic accuracy of heart diseases	Y	Y	Y
[26]	Hybrid algorithm	Self-regulated learning particle swarm optimisation (SRLPSO), extreme learning machine (ELM)	SRLPSO reduces hidden layer neurons for improved classification accuracy	Y	Y	Y
[27]	Hybrid model	Feature ranking, backpropagation neural network, practical swarm optimisation (PSO)	Achieves higher accuracy, lower costs, and shorter time for heart disease diagnosis	Y	Y	Y
[28]	Hybrid system	ANN, fuzzy-AHP	Outperforms other ANN classifiers in predicting heart failure risks	Y	Y	Y
[29]	Naïve Bayes classifier with supervised discretisation	Naïve Bayes classifier, supervised discretisation	Provides good distinguishing rate of heart diseases	Y	N	Y
[30]	Various feature selection techniques and machine learning algorithms	Four feature selection methods, twelve classifiers	SVM with a linear kernel and relief feature selection method identified as best for heart attack prediction	Y	N	Y

approach for solving some of the identified problems. In the subsequent subsection, FMM variants that are capable of online learning are described.

2.2 | FMM Variants for Data Classification

We study recent FMM variants within the domain of online learning frameworks. A noteworthy milestone in this pursuit was achieved by Gabrys and Bargiela [31], who introduced the General FMM model (GFMM) capable of simultaneously addressing both supervised and unsupervised problems through a principled expansion and contraction mechanism. Kim and Yang [32] contributed a Weighted FMM (WFMM) tailored for pattern classification. This model introduced a novel expansion process and membership function, thoughtfully incorporating a weight factor for each hyperbox dimension to improve robustness against noisy features within data sets. Addressing the complexity of overlapping regions, Nandedkar and Biswas proposed a novel model featuring compensatory neurons to enhance classification accuracy [33]. The method circumvented the conventional contraction procedure by leveraging the compensatory neurons to effectively address the hyperbox overlap issue.

Quteishat and Lim introduced the Modified FMM (MFMM) with rule extraction to augment the classification accuracy of FMM [34]. The modifications were useful in tackling the issue associated with large hyperboxes in the network. Specifically, a Euclidean distance measure alongside fuzzy membership function calculations were formulated for data classification. Additionally, a rule extraction algorithm was integrated, using confidence factors to prune less certain hyperboxes and extract fuzzy if-then rules [34]. The proposed improved the FMM performance with large hyperboxes and equipped it with rule extraction capability [34]. Later, a hybrid model, MFMM-GA, was conceived to further enhance the capabilities of MFMM [35]. MFMM-GA adopted a pruning strategy in its learning phase, systematically reducing the number of generated hyperboxes through a confidence-based mechanism [35]. Furthermore, the implementation of a 'don't care' procedure during the rule extraction process effectively diminishes the quantity of features (consequents) identified in the resulting if-then rules [35].

In [36], the authors introduced a Data Core FMM (DCFMM) model. It aimed to optimise the FMM accuracy by sidestepping the traditional contraction procedure. The FMM architecture was enhanced by incorporating additional neurons to efficiently classify data samples that exist within hyperboxes associated with different classes. A new membership function was formulated to enrich its criteria with parameters such as the hyperbox geometric centre, data core, and noise [36]. In an effort to tackle overlapping regions, Davtalab et al. [37] devised a multi-level FMM to harness the benefits of smaller hyperboxes for resolving this challenge. Within the hyperbox layer, the neurons were strategically divided into Overlap Box Segments (OLS) and Hyperbox Segments (HBS) [37]. Although HBSs effectively represented the created hyperboxes during the training phase, OLSs were used to identify samples that fall within the overlapped regions, thus achieving an improved performance [37]. In [18], the authors formulated an Enhanced FMM (EFMM) with a range of rules, including new expansion rules, extended overlap test rules, and novel contraction rules, to improve the classification rate [18]. A KNN rule was also introduced to address the complexity inherent to the original FMM model [38]. Specifically, EFMM-II was developed to address the issues of complexity and noise in EFMM. It achieves this by integrating the KNN rule and implementing a strategic pruning strategy during the learning phase to enhance performance [39].

Azad et al. [40] successfully integrated the Euclidean distance with the membership function, a fusion that markedly improved the FMM performance. This integration utilised the Euclidean distance to calculate the hyperbox membership values. It was found to achieve higher classification accuracy compared to FMM. In a study by Paourpanah et al. [41], FMM and Brain Storm Optimisation (BSO) were integrated to improve feature selection and enhance data classification. The BSO algorithm strategically selected most impactful features to enhance classification while concurrently minimising network complexity. In brain MRI analysis, FMM is offered as a useful tool for classifying both normal and abnormal brain tissues, as presented in [5]. Gabor filtering was employed for image denoising and enhancement, enabling accurate identification of tumour components during segmentation. Dynamic region growth, characterised by modified thresholds, augmented the Modified Region Growing method, ultimately refining the segmentation process. Edge detection and entropy-based techniques further contributed to texture removal and extraction of colour information. The FMM was then used to make a prediction based on the outcomes from Modified Region Growing and texture feature generation [5]. Table 2 provides a concise summation of FMM variants for data classification. For so many years, numerous different variants of FMM have been developed for pattern classification. Nevertheless, a prevailing constraint found is the dependency on the membership function to anticipate the output class, potentially leading to misclassification. As such, we leverage the EFMM variant in this study, in view of its useful properties [18]. A new weighted validation method is introduced to boost the EFMM capability, improving its predictive efficacy in heart disease data classification tasks.

3 | The Enhanced Fuzzy Min-Max (EFMM) Neural Network

Similar to FMM [42], EFMM is constructed using hyperbox fuzzy sets. These sets are defined as regions in an n-dimensional feature space that consist of patterns derived from data samples with class membership [18].

A hyperbox is characterised by its minimum (Min) and maximum (Max) points, which are determined by a membership function. These points create a fuzzy set known as a Min-Max fuzzy set. The collection of hyperbox fuzzy sets forms a class of fuzzy sets. In particular, a hyperbox fuzzy set, represented as B_j, can be expressed using an ordered set as follows [18]:

$$B_j = \{X, V_j,\ W_j, f(X,\ V_j,\ W_j)\}, \qquad \forall\, X \in I^n, \tag{1}$$

where V_j and W_j are the Min and Max vertices of B_j, respectively, and I^n is an n-dimensional unit cube. EFMM utilises the

TABLE 2 | FMM variants for pattern classification.

Reference	Model	Novelty of proposed model
[31]	GFMM	Simultaneous handling of supervised and unsupervised problems based on expansion and contraction principles.
[32]	WFMM	A weight factor is assigned to each dimension of a hyperbox to reduce sensitivity to noisy features.
[33]	Compensatory neuron model	Utilising compensatory neurons to improve classification accuracy in overlapping regions.
[34]	MFMM	Modified FMM with rule extraction to enhance classification accuracy. Addresses category formation issue in FMM with large hyperboxes. Utilises membership function and euclidean distance for classification.
[35]	MFMM-GA	Hybrid model combining MFMM and genetic algorithm (GA) for improvement. Divides data into training, prediction, and test sets. Employs pruning strategy with confidence factor to reduce hyperboxes. Utilises 'don't care' procedure in genetic rule extraction to reduce features.
[36]	DCFMM	Omitting contraction procedure and utilising additional neurons for classifying samples in overlapping regions.
[37]	Multi-level FMM	Handling overlapped regions by using smaller hyperboxes with overlap box segments and hyperbox segments.
[18, 38, 39]	Enhanced EFMMs	Enhancing the classification rate of the FMM and resolve issues related to complexity and noise in the KNN rule and pruning by proposing a new expansion, overlap test, and contraction rules.
[40]	EFMM with euclidean distance	Integrating euclidean distance with membership function to improve FMM performance.
[41]	FMM-BSO	Utilising brain storm optimisation to select the best features and improve the pattern classification.
[5]	Brain MRI analysis	FMM-NN assists in brain tissue classification, aided by techniques such as Gabor filtering, region growing, and edge detection.

FMM membership function in order to assess the level of membership of an input sample to a hyperbox [18]. The next step is to select a winning class based on the hyperbox that produces the highest membership value, as described in [18],

$$b_j(A_h) = \frac{1}{2n}\sum_{i=1}^{n}\left[\begin{array}{l}\max\left(0.1-\max\left(0\cdot\gamma\min\left(1\cdot a_{hi}-w_{ji}\right)\right)\right)\\ +\max\left(0.1-\max\left(0\cdot\gamma\min\left(1\cdot v_{ji}-a_{hi}\right)\right)\right)\end{array}\right], \quad (2)$$

where $A_h = (a_{h1}, a_{h2}, ..., a_{hn}) \in I^n$ is the hth input sample, $V_j = (v_{j1}, v_{j2}, ..., v_{jn})$ and $W_j = (w_{j1}, w_{j2}, ..., w_{jn})$ are the min and max points of hyperbox B_j, and γ is a sensitivity parameter. The membership function is utilised to compute the average min and max points violation with respect to each dimension of hyperbox B_j. The membership value is 1 when the input sample is contained within hyperbox B_j. When a new input data sample is given, the hyperbox that yields the highest membership value is chosen as the winner. The predicted class is then determined by the linked output of this hyperbox.

EFMM addresses several limitations of FMM while preserving its original structure. Similar to FMM, the EFMM learning algorithm consists of three phases: expansion, overlap test, and contraction. During the EFMM learning procedure, when a new input sample is provided, it searches for the hyperbox with the highest membership degree. If necessary, the hyperbox is allowed to expand to encompass the input sample. If a suitable hyperbox cannot be found, a new hyperbox is added to the network structure. When the condition mentioned in [18] is satisfied for an ordered pair $A_h \cdot d_h \in D$ (where A_h is the input sample and d_h is the index of the hyperbox fuzzy set node), hyperbox B_j is expanded to include A_h.

$$Max_n\left(W_{ji}, a_{hi}\right) - Min_n\left(v_{ji}, a_{hi}\right) \leq \Theta. \quad (3)$$

To determine whether each dimension of the j^{th} hyperbox surpasses the expansion coefficient Θ, Equation (3) is evaluated. The expansion procedure is only applied if none of the dimensions of the hyperbox exceed Θ.

Instead of summarising all the dimensions and comparing the outcome, as done in the original FMM technique, the EFMM focuses on the difference between the minimum and maximum points for each dimension in relation to Θ. If expansion is required, the hyperbox expands in a way that is similar to the FMM [18]. The equations utilised to adjust the minimum and maximum points of the hyperbox are:

$$v_{ji}^{new} = \min\left(v_{ji}^{old}, x_{hi}\right) \qquad \forall\ i = 1, 2...., n, \quad (4)$$

$$v_{ji}^{new} = \max\left(w_{ji}^{old}, x_{hi}\right) \qquad \forall\ i = 1, 2, ..., n. \quad (5)$$

Figure 1 illustrates an example of hyperboxes for a two-class problem in I^2. It is evident that hyperboxes belonging to the same class can overlap, whereas hyperboxes from different classes cannot overlap. However, this does not imply that

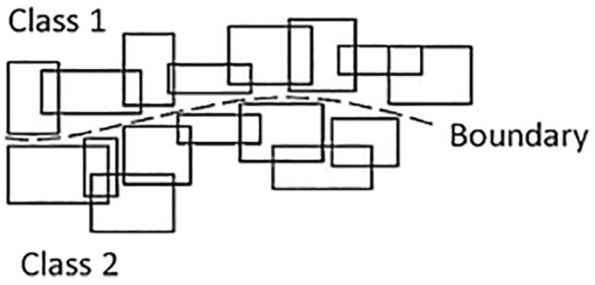


FIGURE 1 | Fuzzy hyperboxes are located along the boundaries of two classes [18].

hyperboxes from different classes cannot overlap upon expansion. The overlap of hyperboxes from two different classes introduces ambiguity and the possibility for a single sample to be assigned to two classes simultaneously.

In order to prevent the overlapping of hyperboxes from different classes, it is necessary to conduct a hyperbox overlap test. This test determines if the expansion of a hyperbox will create any regions of overlap [18]. In EFMM, new cases have been added to detect additional potential overlap scenarios compared to those in FMM [42]. It is important to note that cases (Equation 6) and (Equation 7) already exist in FMM, whereas cases (Equation 8) to (Equation 14) have been introduced in EFMM [18] to address the limitations of FMM.

$$\begin{aligned} &Case\ 1:\ v_{ji} < v_{ki} < w_{ji} < w_{ki}, \\ &\delta^{new} = \min\left(w_{ji} - v_{ki},\ \delta^{old}\right), \end{aligned} \tag{6}$$

$$Case\ 2:\ v_{ki} < v_{ji} < w_{ki} < w_{ji},\ \delta^{new} = \min\left(w_{ki} - v_{ji},\ \delta^{old}\right), \tag{7}$$

$$\begin{aligned} &Case\ 3:\ v_{ji} = v_{ki} < w_{ji} < w_{ki}, \\ &\delta^{new} = \min\left(\min\left(w_{ji} - v_{ki},\ w_{ki} - v_{ji}\right),\ \delta^{old}\right), \end{aligned} \tag{8}$$

$$\begin{aligned} &Case\ 4:\ v_{ji} < w_{ki} < w_{ji} = w_{ki}, \\ &\delta^{new} = \min\left(\min\left(w_{ji} - v_{ki},\ w_{ki} - v_{ji}\right),\ \delta^{old}\right), \end{aligned} \tag{9}$$

$$\begin{aligned} &Case\ 5:\ v_{ki} = v_{ji} < w_{ki} < w_{ji}, \\ &\delta^{new} = \min\left(\min\left(w_{ji} - v_{ki},\ w_{ki} - v_{ji}\right),\ \delta^{old}\right), \end{aligned} \tag{10}$$

$$\begin{aligned} &Case\ 6:\ v_{ki} < v_{ji} < w_{ki} = w_{ji}, \\ &\delta^{new} = \min\left(\min\left(w_{ji} - v_{ki},\ w_{ki} - v_{ji}\right), \delta^{old}\right), \end{aligned} \tag{11}$$

$$\begin{aligned} &Case\ 7:\ v_{ji} < v_{ki} \le w_{ki} < w_{ji}, \\ &\delta^{new} = \min\left(\min\left(w_{ji} - v_{ki},\ w_{ki} - v_{ji}\right),\ \delta^{old}\right), \end{aligned} \tag{12}$$

$$\begin{aligned} &Case\ 8:\ v_{ki} < v_{ji} \le w_{ji} < w_{ki}, \\ &\delta^{new} = \min\left(\min\left(w_{ji} - v_{ki},\ w_{ki} - v_{ji}\right),\ \delta^{old}\right), \end{aligned} \tag{13}$$

$$Case\ 9:\ v_{ki} = v_{ji} < w_{ki} = w_{ji}, \delta^{new} = \min\left(w_{ki} - v_{ji},\ \delta^{old}\right). \tag{14}$$

Assuming that $\delta^{old} = 1$, by carrying out a dimension-by-dimension evaluation, if $\delta^{old} - \delta^{new} < 1$, this indicates that an overlap region exists. The learning algorithm then proceeds to its final part, the contraction procedure. The contraction mechanism is developed based on the hyperbox overlap test.

Note that (Equation 15) and (Equation 16) are the existing cases in FMM, whereas (Equation 17) to (Equation 26) are new cases introduced in EFMM [18].

$$Case\ 1: v_{j'} < v_{k'} < w_{j'} < w_{k'}, \quad w_{j'}^{new} = v_{k'}^{new} = \frac{w_{j'}^{old} + v_{k'}^{old}}{2}, \tag{15}$$

$$Case\ 2: v_{k'} < v_{j'} < w_{k'} < w_{j'}, \quad w_{k'}^{new} = v_{j'}^{new} = \frac{w_{k'}^{old} + v_{j'}^{old}}{2}, \tag{16}$$

$$Case\ 3: v_{j'} = v_{k'} < w_{j'} < w_{k'}, \quad \left(v_{k'}^{new} = w_{j'}^{old}\right), \tag{17}$$

$$Case\ 4: v_{j'} < v_{k'} < w_{j'} = w_{k'}, \quad \left(w_{j'}^{new} = v_{k'}^{old}\right), \tag{18}$$

$$Case\ 5: \quad v_{k'} = v_{j'} < w_{k'} < w_{j'}, \quad \left(v_{j'}^{new} = w_{k'}^{old}\right), \tag{19}$$

$$Case\ 6: \quad v_{k'} < v_{j'} < w_{k'} = w_{j'}, \quad \left(w_{k'}^{new} = v_{j'}^{old}\right), \tag{20}$$

$$\begin{aligned} &Case\ 7(a): \ v_{j\Delta} < v_{k\Delta} \le w_{k\Delta} < w_{j\Delta} \\ &\quad and\ \left(w_{k\Delta} - v_{j\Delta}\right) < \left(w_{j\Delta} - v_{k\Delta}\right),\ v_{j\Delta}^{new} = w_{k\Delta}^{old}, \end{aligned} \tag{21}$$

$$\begin{aligned} &Case\ 7(b): \ v_{j\Delta} < v_{k\Delta} \le w_{k\Delta} < w_{j\Delta} \\ &\quad and\ \left(w_{k\Delta} - v_{j\Delta}\right) > \left(w_{j\Delta} - v_{k\Delta}\right),\ w_{j\Delta}^{new} = v_{k\Delta}^{old}, \end{aligned} \tag{22}$$

$$\begin{aligned} &Case\ 8(a): \ v_{k\Delta} < v_{j\Delta} \le w_{j\Delta} < w_{k\Delta} \\ &\quad and\ \left(w_{k\Delta} - v_{j\Delta}\right) < \left(w_{j\Delta} - v_{k\Delta}\right),\ w_{k\Delta}^{new} = v_{j\Delta}^{old}, \end{aligned} \tag{23}$$

$$\begin{aligned} &Case\ 8(b): \ v_{k\Delta} < v_{j\Delta} \le w_{j\Delta} < w_{k\Delta} \\ &\quad and\ \left(w_{k\Delta} - v_{j\Delta}\right) > \left(w_{j\Delta} - v_{k\Delta}\right),\ v_{k\Delta}^{new} = w_{j\Delta}^{old}, \end{aligned} \tag{24}$$

$$\begin{aligned} &Case\ 9(a): \ v_{j\Delta} = v_{k\Delta} < w_{k\Delta} = w_{k\Delta}, \\ &\quad w_{j\Delta}^{new} = v_{k\Delta}^{new} = \frac{w_{j\Delta}^{old} + v_{k\Delta}^{old}}{2}, \end{aligned} \tag{25}$$

$$\begin{aligned} &Case\ 9(b): \ v_{k\Delta} = v_{j\Delta} < w_{k\Delta} = w_{j\Delta}, \\ &\quad w_{k\Delta}^{new} = v_{j\Delta}^{new} = \frac{w_{k\Delta}^{old} + v_{j\Delta}^{old}}{2}. \end{aligned} \tag{26}$$

4 | Analysis of the EFMM Membership Function

Although EFMM outperforms FMM and many of its variants, as well as other machine learning models as presented in [18], further improvements to its membership function are possible, as explained in this section. An example is shown in Figure 2 to illustrate the scenario. Consider three hyperboxes that belong to two different classes in a $2 - D$ feature space, that is, hyperbox $B_1 \in C_1$ (Class 1) with $V_1 = (0.15, 0.25)$ and $W_1 = (0.3, 0.4)$, whereas B_2 and $B_3 \in C_2$ (Class 2) with $V_2 = (0.1, 0.6)$, $W_2 = (0.3, 0.75)$ and $V_3 = (0.5, 0.6)$ and $W_3 = (0.8, 0.8)$, respectively. During the test stage, a new input $A_1 = (0.4, 0.5)$ that belongs to C_2, is applied. EFMM computes the membership function of each hyperbox with respect to A_1, as indicated in Equation (2). The reason why Hyperbox B_1, which is part of C_1, is chosen as the winner is because it has the highest membership value. However, it is important to note that using only the membership function to predict the output class can result in an incorrect prediction, as demonstrated by this example. As such, there is a need to further improve EFMM during the winning hyperbox selection procedure, in order to improve its performance.

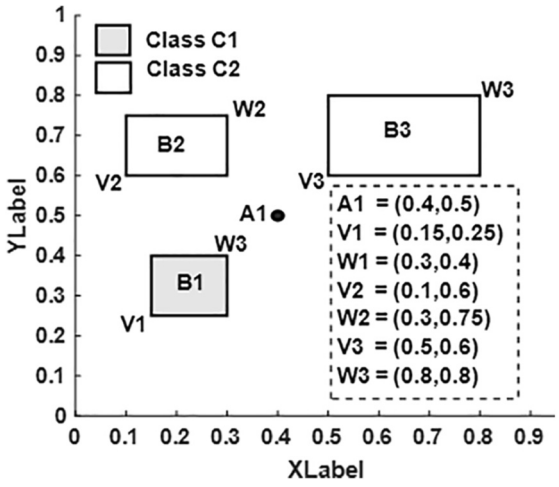


FIGURE 2 | Selection of the winning hyperbox in EFMM.

5 | The Proposed Weighted Validation Method

To address the limitation identified in the preceding section, we propose a weighted validation method to enhance the selection of the winning hyperbox in EFMM. This method consists of combining the weights of a set of hyperboxes that have the highest membership scores, and selecting the one with the highest weight as the winner. During the testing stage, we calculate the membership values of all hyperboxes based on the current input pattern. Instead of choosing the hyperbox with the highest membership value as the winner, we choose a group of top-k hyperboxes, where k is determined by the number of classes (l) plus one ($k = l + 1$). This choice of k aims to ensure flexibility and an appropriate coverage of regions associated with each class. Note that using a larger k value could introduce unrelated regions from weak hyperboxes, potentially leading to incorrect decisions. The weights for the rule dimensions are determined by selecting the highest weight from each dimension's hyperboxes within the same class. We then extract new rules based on the chosen hyperboxes. Among the selected hyperboxes belonging to the same class as the winner, we choose the one with the highest weight. In the case of a two-class problem, if the top three selected hyperboxes belong to two different classes, we extract two hyperboxes, one for each class, and then select the hyperbox with the highest fitness score from the extracted hyperboxes as the winner. To implement this method, we calculate the fitness value of each dimension separately. This is done by computing the membership value of each dimension using Equation (27).

$$d_j(T_h) = \frac{1}{2}\left[\begin{array}{l} \max\left(0, 1 - \max\left(0, \gamma\min\left(1, t_{hi} - w_{ji}\right)\right)\right) \\ +\max\left(0, 1 - \max\left(0, \gamma\min\left(1, v_{ji} - t_{hi}\right)\right)\right) \end{array}\right], \quad (27)$$

where $T_h = (t_{h1}, t_{h2}, ..., t_{hi})$ is the h_{th} test sample, $V_j = (v_{j1}, v_{j2}, ..., v_{ji})$ and $W_j = (w_{j1}, w_{j2}, ..., w_{ji})$ are the minimum and maximum points, respectively, for the selected hyperbox, and γ is the sensitivity parameter. Additionally, the membership function of the top hyperboxes can be specified using the average fitness values of all dimensions in each hyperbox, as determined by Equation (28).

$$b_j(A_h) = \frac{1}{n}\sum_{i=1}^{n}\left[d_j(T_h)\right]. \quad (28)$$

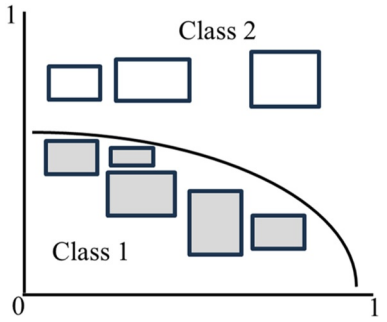


FIGURE 3 | Conceptual illustration of the decision boundary in a two-class problem, highlighting fuzzy hyperboxes located near class boundaries and the effect of the proposed weighted validation method.

Figure 3 illustrates a conceptual decision boundary for a two-class problem using the proposed weighted validation method. When fuzzy hyperboxes from different classes are located near the class boundary, the decision boundary is influenced towards the region with a higher concentration of hyperboxes belonging to the same class [1]. This behaviour demonstrates how the proposed method resolves boundary ambiguity and improves the robustness of the final classification outcome.

In summary, our proposed modifications to EFMM consist of two key components. First, during the learning stage, we apply the FMM contraction procedure when the hyperbox dimensions overlap. This reduces the number of contracted dimensions, minimising data distortion and information loss caused by the EFMM contraction process. Second, we introduce a new weighted validation method that significantly enhances the hyperbox selection process during the prediction stage. Unlike traditional methods that select the winning hyperbox based solely on the highest membership value, the weighted validation method considers a group of top k hyperboxes, where k is chosen to ensure a good coverage of class-specific regions. By assigning weights to these hyperboxes based on their membership values, the model reduces the risk of misclassification caused by overlapping hyperboxes caused by noisy data. This method not only improves the overall classification accuracy but also enhances the model stability, as shown in our experimental results. By evaluating a set of top-performing hyperboxes, instead of relying on single hyperbox with the highest membership value, the proposed model achieves more reliable and robust predictions, as demonstrated in the subsequent heart disease classification tasks.

6 | The Computational Complexity of the Proposed Method

In FMM and its variants, network complexity is frequently evaluated using the number of hyperboxes generated during the training process. Several studies, such as [37, 38] compare the network complexity using the hyperbox count. Algorithmic complexity, on the other hand, is measured using the big O notation [43, 44], which provides insight into the complexity of the computational time. The big O notation determines the worst-case computational time to describe its asymptotic behaviours.

In order to perform an analysis of the computational time needed for the training cycle of the FMM, we will define M as

the dimension of the input sample A and N as the number of hyperboxes. The FMM determines the winning hyperbox in relation to the input sample A by examining all M dimensions of each of the N hyperboxes. Likewise, for the hyperbox overlap test, the FMM verifies all M dimensions of each of the N hyperboxes. Consequently, the worst-case time complexity of the FMM can be expressed as $O(MN)$ as the values of M and N tend towards infinity. In the proposed method, the time complexity remains unchanged compared with that of FMM. However, implementing the weighted validation method during the test stage requires an additional constant amount of time. Therefore, the proposed model has a worst-case time complexity of $O(MN)$ as M and N approach infinity. Similarly, the computational time complexity of the proposed method aligns with that of FMM. However, there is an additional constant time requirement for extracting new rules. Here, the number of new rules is denoted as l and is equal to the number of classes. This extraction process involves selecting the highest weight for each dimension from hyperboxes within the same class. This represents the weighted value in each extracted rule. Consequently, the worst-case time complexity of the proposed method remains asymptotically equivalent to $O(MN)$ as M and N approach infinity.

7 | Performance Evaluation

Four case studies were conducted to assess and compare performance. The first three cases used the Statlog (heart) data from the UCI machine learning repository [45]. Specifically, the data set consists of 270 instances, each with 13 features. The fourth case study uses a real heart disease diagnosis data set from a hospital. The primary goal of each case study is as follows:

Case Study 1: To assess the impact of the new validation method and expansion coefficient on the performance of FMM, EFMM, and the proposed model, an evaluation will be conducted.

Case Study 2: To contrast the performance of the proposed model with various non-FMM models, such as J48, MLP, IBK, AdaBoost, and rough set.

Case Study 3: To compare the proposed model performance with those from various FMM variants based on different expansion coefficient Θ values with a 10-fold cross-validation method.

Case Study 4: To evaluate the proposed model efficacy in undertaking a real-world heart disease diagnosis task with statistic tests of the results.

Across these case studies, a variety of settings are employed to assess the performance. This includes diverse expansion coefficient Θ values, different training set sizes, the k-fold cross-validation method with varying values of $k = 3,\ 5,\ and\ 10$, a number of repetitions for each experiment, and pairwise comparisons using the t-statistic and p-values. The repetitive runs per each expansion coefficient setting account for variability in the model performance due to randomness in initialisation and data partitioning, especially considering the online learning nature of FMM. Repeating the experiment multiple times helps ensure that the results are not biased by single run and provides a reliable measure of the model performance under different conditions. The results from all runs are averaged to yield an overall performance indicator.

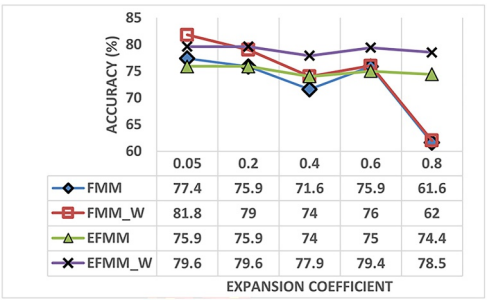
7.1 | Case Study 1

This case study evaluates the impact of the proposed modifications on FMM and EFMM by testing various expansion coefficient (Θ) values, different k-fold cross-validation settings, and a range of training set sizes using the Statlog (heart) data set. The performances of FMM, EFMM, and the modified EFMM where the dimension contraction is applied only to the smallest overlapping region and a new validation method is implemented are analysed and compared. Additionally, the sensitivity (the ability to classify positive cases correctly) and specificity (the ability to classify negative cases correctly) rates of the proposed model are assessed and compared with those from FMM and EFMM.

In the first evaluation, the data set was divided into 80% for training and 20% for testing. The expansion coefficient (Θ) increased from 0.05 to 0.8, whereas the sensitivity parameter (γ) was set to 1. The experiment was repeated 10 times for each expansion coefficient value. A total of 10 repetitions per expansion coefficient setting was made to provide a comprehensive performance assessment across different configurations. This number was chosen to balance between computational time and performance significance, ensuring that the results could represent the model performance without excessive computational overheads.

In Figure 4, it is evident that the proposed weighted validation method (W) enhances the overall average test accuracy rates of the base counterparts, namely *FMM_W* and *EFMM_W*. Notably, *FMM_W* exhibits a superior performance at $\Theta = 0.05$ when dealing with smaller hyperbox sizes. However, the test accuracy rates of *FMM* and *FMM_W* decline as the number of hyperboxes decreases, implying a reduction in the encoded knowledge within the network. In contrast, *EFMM* and *EFMM_W* maintain consistent performance across various scenarios owing to the distinctive hyperbox expansion, overlap test, and contraction procedures in EFMM to effectively minimise the misclassification rates [18].

Referring to Figure 5, the outcomes highlight the usefulness of combining the strengths of both models within a single network for improving classification performance. To achieve this, the



	0.05	0.2	0.4	0.6	0.8
FMM	77.4	75.9	71.6	75.9	61.6
FMM_W	81.8	79	74	76	62
EFMM	75.9	75.9	74	75	74.4
EFMM_W	79.6	79.6	77.9	79.4	78.5

FIGURE 4 | Impact of weighted validation method on FMM and EFMM performance with Statlog heart dataset.

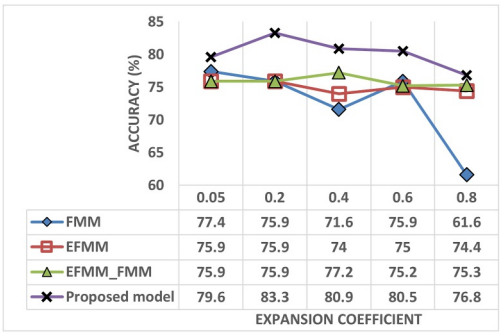


FIGURE 5 | Enhanced performance with a combined model and the proposed modifications using the Statlog heart dataset.

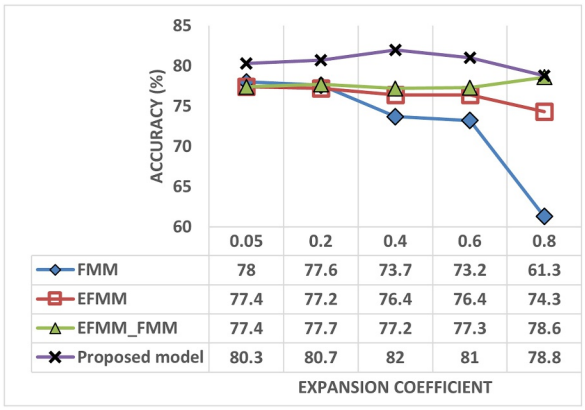


FIGURE 6 | The average accuracy rates of 5-fold cross-validation using various sizes of expansion coefficient Θ are measured.

contraction procedure in EFMM is adapted to mitigate missing information from the contracted regions, akin to that in the original FMM model. Subsequently, the new weighted validation method is applied to enhance prediction quality. The evaluation encompasses FMM, EFMM, EFMM with the FMM contraction activation function (EFMM-FMM), and the proposed model, all under consistent experimental conditions. Notice that EFMM-FMM offers enhanced stability and marginally better accuracy rates across different expansion coefficient values. Furthermore, the integration of both EFMM and FMM can successfully address the limitations and substantially improve the proposed model performance, as highlighted in Figure 5. Moreover, the second sub-experiment employed the 5-fold cross-validation technique to assess the efficacy of various models, wherein Θ was systematically increased from 0.05 to 0.8. The sensitivity parameter (γ) remained at 1 across all experiments. The experiment was conducted 10 times for each expansion coefficient setting. As such, a total of 50 times per fold was performed, resulting in 250 results in each evaluation. Figure 6 shows the average results for each Θ setting, which depict a consistent trend of accuracy. The first stage of modification, represented by the FMM contraction process, is able to improve the overall classification accuracy, while combining it with the new validation method results in a better classification performance, as shown in Figure 6. This demonstrates that the proposed model has the ability to overcome the current limitations and generate more accurate predictions compared with FMM and EFMMs. The origin of the oscillating performance is related to the number of hyperboxes created, which reflects the quality of knowledge based on better or slightly inferior performance [18]. The results suggest that the new modifications are capable of addressing the limitations in both FMM and EFMM.

Moreover, the proposed model performance is effective as compared with those of FMM and EFMM using a variety of training set sizes in the third evaluation. The training set size was varied from 50% to 80% with an increment of 10%. The remaining samples were used for testing. The experiment was repeated 10 times for each training size. The expansion coefficient, denoted as Θ, was set to 0.05 in order to maximise the performance accuracy. The sensitivity parameter, γ, was kept at its default value of 1.

As shown in Table 3, increasing the training set size from 50% to 80% reveals an oscillating accuracy trend for all models. In the case of FMM and EFMM, the worst accuracy rate is recorded at 70% of the training set size. EFMM depicts an almost comparable accuracy performance for the remaining cases as compared with those of FMM. Its best accuracy rate is at 80% of the training set size. The proposed model produces an opposite trend. The best accuracy rate is recorded at 70% of the training set size, whereas the worst accuracy rate is shown for 80% and 60% training set sizes, although it is still better than those from FMM and EFMM. Unlike FMM and EFMM, the performance of the proposed model is higher, which is attributed to the efficiency of the modifications in handling information loss. The proposed model demonstrates a superior classification performance for all categories, even though each model reveals an oscillating trend. Following this, FMM emerges as marginally less adept than EFMM in terms of average performance. The oscillation results are owing to the contraction procedure in FMM and EFMM, which can lead to information loss. To address this issue, the modified contraction procedure proposed in this study can produce more stable results, as evidenced in this evaluation.

TABLE 3 | Comparison of test accuracy among FMM, EFMM, and the proposed model, considering various training set sizes, expansion coefficient (Θ = 0.05), and sensitivity parameter (γ = 1).

Training in (%)	FMM	EFMM	The proposed model
80%	77.41	75.93	**79.63**
70%	71.61	72.84	**81.48**
60%	74.07	75.93	**79.63**
50%	74.22	75.56	**80.74**

Note: The bold values indicate the best results.

Finally, to comprehensively assess the performance of the proposed model against those of original FMM and EFMM models, we conducted evaluations using three key classification metrics: sensitivity, specificity, and accuracy. For this evaluation, we employed a single-fold approach, where 80% of the data set was utilised for training, and the remaining 20% was reserved for testing. The expansion coefficient of all models was fixed at 0.2, a value determined from previous trials, ensuring a stable performance across the board. The reported results were averages

of 10 independent runs, accounting for performance variations in different runs.

Sensitivity (also referred to as Recall) indicates the proportion of true positive (TP) predictions relative to the total number of actual positive cases, whereas **Specificity** quantifies the proportion of true negative (TN) predictions relative to the total number of actual negative cases. These two metrics, alongside accuracy, provide a comprehensive overview of the model capability to distinguish between positive (heart disease) and negative (healthy) cases. The equations for computing sensitivity and specificity are as follows [7]:

$$\text{Sensitivity} = \frac{TP}{TP + FN}, \tag{29}$$

$$\text{Specificity} = \frac{TN}{TN + FP}. \tag{30}$$

The evaluation results for the proposed model, FMM, and EFMM are summarised in Table 4. The proposed model consistently demonstrated superior performances across all metrics. Specifically, it achieved the highest accuracy of 83.33%, reflecting its overall predictive capability. Notably, the model exhibited an improved sensitivity of 77.5%, indicating its enhanced ability to correctly identify true positive cases, that is, patients with heart disease. Additionally, it attained the highest specificity of 88.67%, underscoring its strength in identifying true negative cases (healthy individuals), thereby minimising false positive predictions (i.e., false alarm rates).

The results demonstrate the significant advancements made by the proposed model over both FMM and EFMM. The consistent and good performances in terms of accuracy, sensitivity, and specificity highlights the effectiveness of the proposed modifications. This is particularly important in heart disease classification, where the balance between sensitivity (to minimise missed diagnoses) and specificity (to reduce false alarms) is critical for supporting accurate clinical decision-making processes.

7.2 | Case Study 2

This experiment seeks to assess the efficacy of the proposed model in comparison to various non-FMM models, namely J48, MLP, IBK, AdaBoost, and rough set. The evaluation was performed with the 5-fold cross-validation method. The Θ value of the proposed model was set to 0.4. Table 5a shows the average classification accuracy rates. The proposed model yields better results than those from other models.

In the second evaluation, the Fast Scalable Classifier (SLIQ), fuzzifying Gini index-based decision trees (G-FDT), bagging, Support Vector Machine, Random Forest, Navie Bayes, and decision tree were used for comparison. By following the same experimental procedure in [46–48], the 10-fold cross-validation was used, where the (Θ) value of the proposed model was set to 0.4. Table 5b shows the average accuracy rates from different models. The proposed model yields the best accuracy rate. In general, the results from all evaluations ascertain the effectiveness of the proposed model in enhancing classification accuracy. Please note that better results were reported in [48] after applying data engineering techniques to the original data. However, to ensure a fair comparison, the results from the original data set were extracted from [48] for performance comparison.

TABLE 4 | Comparison between the proposed model FMM, and EFMM in terms of accuracy, sensitivity, and specificity.

Model	Accuracy	Sensitivity	Specificity
FMM	75.19	74.17	76
EFMM	75.93	70.83	80
The proposed model	**83.33**	**77.50**	**88.67**

Note: The bold values indicate the best results.

TABLE 5 | Comparison of the proposed model's test accuracy with other classifiers using 5-fold cross-validation and 10-fold cross-validation.

Classifier	Accuracy in (%)
(a) 5-fold cross-validation	
J48	77.79
MLP	78.15
IBK	72.96
AdaBoost	80
Rough set	80.37
The proposed model	**82.11**
Model	**Accuracy in (%)**
(b) 10-fold cross-validation	
SLIQ [46]	74.33
G-FDT [46]	75.33
Bagging [47]	81.41
SVM [48]	77.4
RF [48]	70.0
NB [48]	79.25
DT [48]	79.25
The proposed model	**81.56**

Note: The bold values indicate the best results.

7.3 | Case Study 3

The experiment assessed the efficacy of the proposed model in comparison with several FMM variants, namely General FMM (GFMM), Improved Online Learning algorithm with GFMM (IOL-GFMM) [49], K-nearest neighbour with EFMM (Kn-EFMM), Redefined FMM (RFMM) [11], and Accelerated Agglomerative Learning algorithm for GFMM (ACCAGGLO-GFMM) [50]. The proposed model and its FMM variants were compared in terms of their generalisation performance using a 10-fold cross-validation. A systematic evaluation process was executed by progressively increasing the hyperbox size by 0.2. The resulting average classification rates for various coefficient values are tabulated in Table 6. The proposed model achieved the highest classification rate. The best and worst results were observed at hyperbox sizes of 0.2 and 0.8, respectively. The decrease in classification rate can be attributed to the limited

TABLE 6 | Comparison between the proposed model and FMM variants in terms of test accuracy with 10-fold cross-validation.

	Size of the hyperbox (Θ)					
Model	**0.05**	**0.2**	**0.4**	**0.6**	**0.8**	**Average**
GFMM	68.47	69.82	72.36	70.91	79.45	72.2
IOL-GFMM	68.55	69.83	73.33	71.11	76.63	71.89
Kn-EFMM	71.36	72.86	74.68	77.85	**80.34**	75.42
RFMM	71.36	72.86	73.33	75.82	76.63	74
ACCAGGLO-GFMM	68.47	69.83	71.65	73.47	75.29	71.74
The proposed model	**79.63**	**83.33**	**80.93**	**80.56**	76.85	**80.26**

Note: The bold values indicate the best results.

number of generated hyperboxes. This leads to compromised effectiveness of the extracted rules and a decline in the prediction efficiency of the proposed model. Overall, as shown in Table 6, the proposed model outperforms FMM variants for all hyperbox sizes. A *t*-test and *p*-value analysis were conducted to provide statistical validation for these findings. Table 7 presents the t-statistic and *p*-value. The *t*-test measures the significance of mean accuracy disparities between the proposed model and the compared FMM variant. The t-statistic of the proposed model is 2.27, indicating a substantial mean accuracy difference from other models. With a *p*-value of 0.027 below the significance threshold of 0.05, the null hypothesis is rejected, highlighting the significant performance in mean accuracy of the proposed model.

For the GFMM classifier, a t-statistic of 1.09 indicates a smaller mean accuracy difference. A *p*-value of 0.280 surpasses the significance threshold, precluding the null hypothesis rejection. A similar pattern can be observed for the IOL-GFMM model, with a t-statistic of 1.77 and a *p*-value of 0.083. A more pronounced difference can be observed from the results of GFMM, whereby the *p*-value indicates the possibility of chance. A moderate mean accuracy difference is indicated by the results of Kn-EFMM, that is, t-statistic of 1.95, and *p*-value of 0.057, indicating a potential significance in performance.

For RFMM, a t-statistic of 1.33 indicates a smaller mean accuracy difference, whereas its *p*-value of 0.186 suggests a chance-driven discrepancy. The t-statistic of 0.84 from ACCAGGLO-GFMM signifies a minor mean accuracy difference, and a *p*-value of 0.366 indicates a likelihood of chance. In summary, the *t*-test outcomes confirm the efficacy of the proposed model, as evidenced by its highest t-statistic and lowest *p*-value. These results underscore the consistent accuracy advantage of the proposed model over those from other models, reinforcing its effectiveness in the classification of heart disease data.

TABLE 7 | Summary of *t*-Test Results for mean accuracy comparison among classifiers.

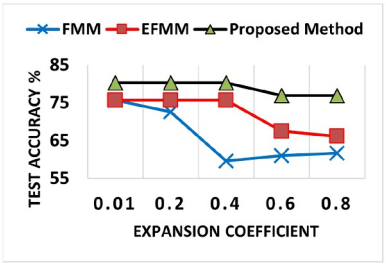
Classifier	t-statistic	*p*-value
Proposed model	2.27	0.027
GFMM	1.09	0.28
IOL-GFMM	1.77	0.083
Kn-EFMM	1.95	0.057
RFMM	1.33	0.186
ACCGLO-GFMM	0.84	0.366

7.4 | Case Study 4

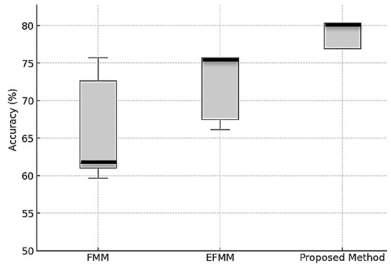
This case study involved the analysis of 118 real medical records from a hospital. The records belonged to patients who had been diagnosed with acute coronary syndrome (ACS). The study aimed to assess the efficacy of the proposed model. There were 15 patients who were not diagnosed with ACS, whereas 103 patients were diagnosed with ACS. After consulting with medical experts, a total of 16 features were extracted from each patient's record for evaluation. These features included physical symptoms (such as sweating and chest pain), background information (such as hypertension and smoking), and representations of EEG signals.

A *k*-fold cross-validation method with varying values of $k = 3, 4, and\ 5$ with a variety of training samples was performed, and the hyperbox size was varied from 0.05 to 0.8. As can be observed in Figure 7a,c,e, the proposed method consistently achieves a superior test accuracy performance, as compared with those from FMM and EFMM across all experimental setups. Notably, the proposed method demonstrates greater stability pertaining to variations in the expansion coefficient and cross-validation folds. In contrast, although EFMM outperforms FMM, it is unable to reach the level of accuracy achieved by the proposed method. Further analysis and discussion of the performance are presented, as follows.

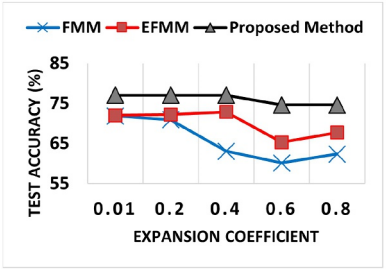
1. Impact of the Expansion Coefficient
   For all models, the expansion coefficient plays a significant role in determining test accuracy:
   - Proposed Method: Test accuracy remains relatively stable regardless of the change in expansion coefficient. For 3-fold cross-validation, it achieves a maximum accuracy rate of 80.34% for coefficient settings of 0.05 and 0.2. Similar trends are observed for 4-fold and 5-fold validation evaluations, where accuracy stays at 77.07% and 77.21%, respectively. Compared with other models, the proposed method indicates a stable accuracy performance with respect to changes in the expansion coefficient.
   - EFMM: Accuracy exhibits a better stability in performance with respect to higher expansion coefficient settings, as compared with those from FMM, particularly for settings up to 0.4. However, EFMM performance declines notably at 0.6 and 0.8 settings across all validation evaluations.



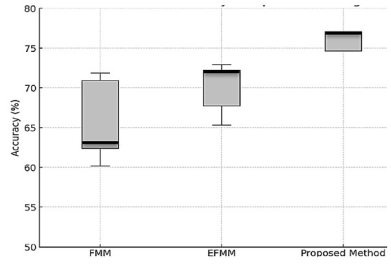
(a) Average (3-fold) test accuracy rates



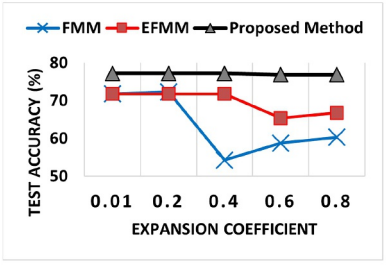
(b) 3-fold: Accuracy Comparison Among Classifiers (t-Test Analysis)



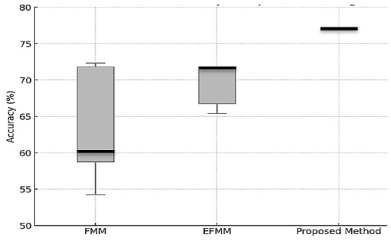
(c) Average (4-fold) test accuracy rates



(d) 4-fold: Accuracy Comparison Among Classifiers (t-Test Analysis)



(e) (Average (5-fold) test accuracy rates



(f) 5-fold: Accuracy Comparison Among Classifiers (t-Test Analysis)

FIGURE 7 | Average (3, 4, and 5-fold) test accuracy rates of FMM, EFMM, and the proposed model for diagnosis of acute coronary syndrome (ACS) patients.

- FMM: Test accuracy shows significant reduction in performance with respect to increase in the expansion coefficient, with a marked decrease beyond 0.2. For instance, in 5-fold cross-validation, accuracy drops from 72.34% (at 0.2) to 54.26% (at 0.4).

2. Effect of Cross-Validation Fold
   The number of cross-validation folds influences the overall performance of all models:
   - Proposed Method: This model maintains a consistent performance across different folds, achieving the highest stability. The accuracy rate ranges between 77.07% and 80.34%, depending on the expansion coefficient and fold size.
   - EFMM: Performance declines slightly with an increase in the number of cross-validation fold but remains more stable than that of FMM. For instance, accuracy with 5-fold cross validation at the expansion coefficient of 0.01 achieves 71.82%, as compared with 72.92% in 4-fold cross validation.
   - FMM: Although accuracy increases from 3-fold to 4-fold cross validations for certain expansion coefficient settings (e.g., 0.4 and 0.8), the overall average accuracy rate declines as the number of cross-validation fold increases. This suggests that FMM exhibits reduced generalisation with more cross-validation fold, despite some instances of improvement at specific expansion coefficient settings.
3. Statistical significance using the t-statistic and *p*-value
   Figure 7b,d,f and Table 8 illustrate a statistical comparison of accuracy pertaining to FMM, EFMM, and the proposed method across 3-fold, 4-fold, and 5-fold cross-validation. The boxplots in Figure 7b,d,f visually depict distribution of accuracy scores of each classifier under different fold settings, highlighting variations in mean accuracy, spread, and consistency. It is evident from the boxplots that the proposed method consistently achieves the highest median accuracy with narrower spreads, indicating a superior and stable performance, as compared with those of FMM and EFMM.

Table 8 presents pairwise *t*-test results (t-statistic and *p*-value) for FMM, EFMM, and the proposed method across different cross-validation settings. The proposed method significantly outperforms both FMM and EFMM in all three cross-validation settings (3-fold, 4-fold, and 5-fold) at the 5% significance level. Although EFMM achieves higher mean accuracy than that of

TABLE 8 | Pairwise paired *t*-test results for FMM, EFMM, and the proposed method under 3, 4, and 5-fold cross-validation.

Cross-Val	Pair	t-Statistic	*p*-value	Significance ($\alpha = 0.05$)
3-fold	FMM versus EFMM	−2.21	0.1	No ($p > 0.05$)
	FMM versus Proposed	−4.39	0.01	Yes ($p < 0.05$)
	EFMM versus Proposed	−5.02	0.01	Yes ($p < 0.05$)
4-fold	FMM versus EFMM	−2.55	0.07	No ($p > 0.05$)
	FMM versus Proposed	−5.25	0.006	Yes ($p < 0.01$)
	EFMM versus Proposed	−6.45	0.003	Yes ($p < 0.01$)
5-fold	FMM versus EFMM	−1.85	0.14	No ($p > 0.05$)
	FMM versus Proposed	−3.76	0.02	Yes ($p < 0.05$)
	EFMM versus Proposed	−5.64	0.005	Yes ($p < 0.01$)

FMM, none of the observed improvements meet the statistical significance level ($p < 0.05$) under 3-fold, 4-fold, or 5-fold cross-validation settings. Overall, this analysis underscores the effectiveness of the proposed method in delivering consistently higher accuracy and greater stability across a range of cross-validation settings.

In both FMM and EFMM models, larger hyperbox sizes tend to reduce classification accuracy due to increased overlap between hyperboxes of different classes. This overlap introduces class ambiguity, leading to misclassification. In FMM, hyperbox expansion is not strictly controlled, allowing overlap to increase as more data samples are incorporated. EFMM mitigates this issue by employing a contraction mechanism when overlapping regions occur. However, this often results in an excessive reduction in the hyperbox size, leading to the loss of valuable information and a consequent decline in accuracy. The proposed model refines the EFMM contraction procedure, minimising data distortion and preventing unnecessary information loss. Additionally, large hyperboxes tend to cover regions with samples from multiple classes, further complicating prediction by increasing the likelihood of incorrect classification. To address this, our model introduces a weighted validation method, where multiple hyperboxes are considered, and the one with the highest weighted score is selected as the winner, reducing the risk of misclassification. In summary, large hyperboxes in both FMM and EFMM degrade the performance due to class overlap. The proposed model improves accuracy by minimising hyperbox overlaps through refined contraction and enhancing hyperbox selection via weighted validation. This is beneficial for data classification, as shown in Figure 7. Although EFMM relies solely on the membership functions pertaining to hyperbox selection, the proposed method incorporates a weighted aggregation of k top-performing hyperboxes, leading to more accurate predictions and superior classification results, as demonstrated in this case study.

8 | Discussion

8.1 | Empirical Validation

The experimental results from four case studies provide valuable empirical validation, supported by statistical confirmation of the proposed model. The integration of a refined contraction mechanism and weighted validation consistently yields superior outcomes, as compared with those from FMM, EFMM, and several existing classifiers. As illustrated in Figures 4–7 and Tables 3–8, the proposed method not only achieves higher average accuracy scores but also exhibits an improved balance between sensitivity and specificity (Table 4). This is a critical factor in clinical decision support tools, where both false negative and false positive rates carry serious implications.

The reliability of these findings is further substantiated through statistical analyses. The paired t-tests indicate that improvements are statistically significant in most instances, with the proposed model achieving the highest t-statistic (2.27) and a *p*-value of 0.027 (Table 7). Furthermore, boxplot analyses (Figure 7) illustrate that the proposed model consistently attains higher median accuracy rates with reduced variability, reflecting enhanced stability and robustness. Collectively, these analyses provide evidential justification for the efficacy of the proposed model.

Another significant observation is the model's robustness to varying conditions. Unlike FMM (where accuracy diminishes as the expansion coefficient increases) and EFMM (which frequently suffers from excessive contraction), the proposed model consistently demonstrates superior accuracy rates across a broad spectrum of Θ values and cross-validation folds, as can be observed in Figures 5–7 and Table 8. This indicates the model's robustness to operate effectively in scenarios where data availability and partitioning may vary considerably.

8.2 | Model Limitations

Despite the promising results, several limitations of the present study must be acknowledged.

First, although the proposed model generally surpasses its counterparts, oscillations in accuracy occur when training set sizes change (Table 3), suggesting its sensitivity to data distribution. This indicates that the model's performance may vary depending on the representativeness of training samples. As such, further investigations are needed to understand the underlying causes of these fluctuations.

Second, although validation has been performed on both benchmark (UCI Statlog) and real hospital data sets, the current study lacks feedback from medical experts, who represent the target end-users. Engaging domain experts is useful to evaluate and provide feedback on the functionality and practicality of the

proposed model, thereby establishing its applicability in real-world environments for clinical decision support tasks.

Third, although most improvements are statistically significant, certain comparative studies (e.g., against Kn-EFMM in Table 8) indicate that the benefits may vary depending on specific experimental conditions, such as the choice of expansion coefficient or the number of cross-validation folds. Thus, further refinements are necessary to enhance consistency across different settings.

Fourth, the current study is limited to analysing data sets related to heart diseases only. Although the proposed model is intended to be generic, its generalisability to other domains with different data characteristics (e.g., image data, text classification, or high-dimensional sparse data) has not yet been empirically verified.

Finally, the computational complexity analysis confirms that the proposed method maintains the same asymptotic complexity as the original FMM model, that is, O(MN), as the values of M and N approach infinity. However, it should be noted that the additional constant-time operations for rule extraction and weighted validation may introduce minor overhead in real-time or resource-constrained applications.

In summary, addressing these limitations will further enhance the proposed model for real-world applications, including broader validation across multiple domains, integration of expert feedback, and further stabilisation of performance across varying training distributions.

9 | Conclusion

This paper presents useful methods to tackle the existing constraints of EFMM and improve its predictive precision in the classification of heart disease data. First, a refined contraction procedure has been introduced during the learning stage, reducing data distortion and missing information in EFMM due to overlapped areas of hyperboxes. This adjustment leads to a more stable performance with improved accuracy, as demonstrated through empirical evaluation. Second, a novel weighted validation method has been incorporated into EFMM during its prediction stage, selecting the top-performing hyperbox and ensuring flexibility and appropriate coverage of class-specific regions. The extraction of new rules based on the selected hyperboxes further refines the winning hyperbox selection procedure, leading to improved classification performance as compared with those from other models on tackling heart disease-related data sets. Additionally, a comprehensive analysis of the computational complexity of the proposed method in relation to both EFMM and original FMM models were provided. Overall, this study makes significant contributions by introducing a robust EFMM model for classifying heart disease data. This model exhibits superior accuracy and stability performance compared with existing models. The combination of the refined contraction procedure and the new weighted validation method yields stable and accurate results, leading to possible early heart disease detection for improving patient treatment and outcomes. Since this study is limited to analysing data sets related to heart diseases only, it is necessary to further evaluate the generalisability of our method to data classification in other domains. As a result, future research will be conducted to verify and validate the applicability and performance of our proposed method on a wide range of domains with different data characteristics. By addressing this limitation, we can ensure a better relevance of our proposed method to promoting the use of EFMM as an effective classification model beyond the realm of medical data sets.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are openly available in figshare at Zhang, Xinyue (2022). Statlog (Heart) Data Set. Figshare. Data set. https://doi.org/10.6084/m9.figshare.19236777.v1.

References

1. E. Alhroob, M. F. Mohammed, C. P. Lim, and H. Tao, "A Critical Review on Selected Fuzzy Min-Max Neural Networks and Their Significance and Challenges in Pattern Classification," *IEEE Access* 7 (2019): 56129–56146, https://doi.org/10.1109/access.2019.2911955.

2. O. N. Al Sayaydeh, M. F. Mohammed, and C. P. Lim, "Survey of Fuzzy Min–Max Neural Network for Pattern Classification Variants and Applications," *IEEE Transactions on Fuzzy Systems* 27, no. 4 (2018): 635–645, https://doi.org/10.1109/tfuzz.2018.2865950.

3. ÖN. Kenger and E. Özceylan, "Fuzzy Min–Max Neural Networks: A Bibliometric and Social Network Analysis," *Neural Computing & Applications* 35, no. 7 (2023): 5081–5111, https://doi.org/10.1007/s00521-023-08267-9.

4. D. Sarvamangala and R. V. Kulkarni, "Convolutional Neural Networks in Medical Image Understanding: A Survey," *Evolutionary intelligence* 15, no. 1 (2022): 1–22, https://doi.org/10.1007/s12065-020-00540-3.

5. S. L. Bangare, "Classification of Optimal Brain Tissue Using Dynamic Region Growing and Fuzzy Min-Max Neural Network in Brain Magnetic Resonance Images," *Neuroscience Informatics* 2, no. 3 (2022): 100019, https://doi.org/10.1016/j.neuri.2021.100019.

6. M. Ozcan and S. Peker, "A Classification and Regression Tree Algorithm for Heart Disease Modeling and Prediction," *Healthcare Analytics* 3 (2023): 100130, https://doi.org/10.1016/j.health.2022.100130.

7. R. O. Duda, P. E. Hart, and D. G. Stork, *Pattern Classification*. 2 ed (Wiley-Interscience, 2000).

8. L. Onyango, A. Waititu, T. Mageto, and M. Kilai, "A Hybrid Classification Model of Artificial Neural Network and Non Linear Kernel Support Vector Machine," *International Journal of Data Science and Analytics* 8 (2022): 47–58, https://doi.org/10.11648/j.ijdsa.20220802.15.

9. Y. Xu, X. Liu, X. Cao, et al., "Artificial Intelligence: A Powerful Paradigm for Scientific Research," *Innovation* 2, no. 4 (2021): 100179, https://doi.org/10.1016/j.xinn.2021.100179.

10. T. T. Khuat, D. Ruta, and B. Gabrys, "Hyperbox-Based Machine Learning Algorithms: A Comprehensive Survey," *Soft Computing* 25, no. 2 (2021): 1325–1363, https://doi.org/10.1007/s00500-020-05226-7.

11. S. K. A, A. Kumar, V. Bajaj, and G. K. Singh, "A Compact Fuzzy Min Max Network With Novel Trimming Strategy for Pattern Classification," *Knowledge-Based Systems* 246 (2022): 108620, https://doi.org/10.1016/j.knosys.2022.108620.

12. X. Luo, W. Wang, Y. Xu, et al., "A Deep Convolutional Neural Network for Diabetic Retinopathy Detection via Mining Local and Long-Range Dependence," *CAAI Transactions on Intelligence Technology* 9, no. 1 (2024): 153–166, https://doi.org/10.1049/cit2.12155.

13. M. Zeeshan Aslam, B. Raza, M. Faheem, and A. Raza, "AML-Net: Attention-Based Multi-Scale Lightweight Model for Brain Tumour Segmentation in Internet of Medical Things," *CAAI Transactions on Intelligence Technology* (2024): 1–17, https://doi.org/10.1049/cit2.12278.

14. E. Alhroob, M. F. Mohammed, O. N. Al Sayaydeh, F. Hujainah, N. Ab Ghani, and C. P. Lim, "A Flexible Enhanced Fuzzy Min-Max Neural Network for Pattern Classification," *Expert Systems with Applications* 251 (2024): 124030, https://doi.org/10.1016/j.eswa.2024.124030.

15. M. A. A. Alobaidy, Z. M. Yosif, M. S. Alsoufi, and S. Al-Ashqar, "Attendance System Based on Face Recognition Dependent on Deep Intelligent Techniques," *Revue d'Intelligence Artificielle* 38, no. 3 (2024): 1009–1016, https://doi.org/10.18280/ria.380326.

16. A. A. H. Saber, O. A. Ibrahim, and G. Alwan, "Enhancing Path Planning of Assistive Robots in Complex Environments Using Geno-Fuzzy Algorithm," *Journal Européen des Systèmes Automatisés* 56, no. 2 (2023), https://doi.org/10.18280/jesa.560215.

17. O. N. Al Sayaydeh, M. F. Mohammed, E. Alhroob, H. Tao, and C. P. Lim, "A Refined Fuzzy Min–Max Neural Network With New Learning Procedures for Pattern Classification," *IEEE Transactions on Fuzzy Systems* 28, no. 10 (2019): 2480–2494, https://doi.org/10.1109/tfuzz.2019.2939975.

18. M. F. Mohammed and C. P. Lim, "An Enhanced Fuzzy Min–Max Neural Network for Pattern Classification," *IEEE Transactions on Neural Networks and Learning Systems* 26, no. 3 (2014): 417–429, https://doi.org/10.1109/TNNLS.2014.2315214.

19. D. Sharma, A. K. Sood, P. S. Darius, E. Gundabattini, S. Darius Gnanaraj, and A. Joseph Jeyapaul, "A Study on the Online-Offline and Blended Learning Methods," *Journal of The Institution of Engineers (India): Series B* 103, no. 4 (2022): 1373–1382, https://doi.org/10.1007/s40031-022-00766-y.

20. B. Venkatalakshmi and M. Shivsankar, "Heart Disease Diagnosis Using Predictive Data Mining," *International Journal of Innovative Research in Science, Engineering and Technology* 3, no. 3 (2014): 1873–1877, https://api.semanticscholar.org/CorpusID:44216820.

21. S. Babu, E. Vivek, K. Famina, et al., "Heart Disease Diagnosis Using Data Mining Technique," in *2017 International Conference of Electronics, Communication and Aerospace Technology (ICECA)* (IEEE, 2017), 750–753, https://doi.org/10.1109/ICECA.2017.8203643.

22. Y. E. Shao, C. D. Hou, and C. C. Chiu, "Hybrid Intelligent Modeling Schemes for Heart Disease Classification," *Applied Soft Computing* 14 (2014): 47–52, https://doi.org/10.1016/j.asoc.2013.09.020.

23. K. Karaduzovic-Hadziabdic and R. Köker, "Diagnosis of Heart Disease Using a Committee Machine Neural Network," in *Proceedings of the 9th International Conference on Applied Informatics (ICAI 2014), Eszterházy Károly College, Eger, Hungary* (2015), 351–360, https://doi.org/10.14794/ICAI.9.2014.1.351.

24. S. Bashir, U. Qamar, and M. Y. Javed, "An Ensemble Based Decision Support Framework for Intelligent Heart Disease Diagnosis," in *International Conference on Information Society (i-Society 2014)* (IEEE, 2014), 259–264, https://doi.org/10.1109/i-Society.2014.7009056.

25. J. S. Majeed Alneamy and R. A. Hameed Alnaish, "Heart Disease Diagnosis Utilizing Hybrid Fuzzy Wavelet Neural Network and Teaching Learning Based Optimization Algorithm," *Advances in Artificial Neural Systems* 2014 (2014): 11, https://doi.org/10.1155/2014/796323.

26. C. Subbulakshmi, S. Deepa, "Medical Dataset Classification: A Machine Learning Paradigm Integrating Particle Swarm Optimization With Extreme Learning Machine Classifier," *Scientific World Journal* 2015 (2015), https://doi.org/10.1155/2015/418060.

27. M. G. Feshki and O. S. Shijani, "Improving the Heart Disease Diagnosis by Evolutionary Algorithm of PSO and Feed Forward Neural Network," in *2016 Artificial Intelligence and Robotics (IRANOPEN)* (Ieee, 2016), 48–53, https://doi.org/10.1109/RIOS.2016.7529489.

28. O. W. Samuel, G. M. Asogbon, A. K. Sangaiah, P. Fang, and G. Li, "An Integrated Decision Support System Based on ANN and Fuzzy_AHP for Heart Failure Risk Prediction," *Expert Systems With Applications* 68 (2017): 163–172, https://doi.org/10.1016/j.eswa.2016.10.020.

29. J. Bohacik and M. Zabovsky, "Naive Bayes for Statlog Heart Database With Consideration of Data Specifics," in *2017 IEEE 14th International Scientific Conference on Informatics* (IEEE, 2017), 35–39, https://doi.org/10.1109/INFORMATICS.2017.8327218.

30. H. Takci, "Improvement of Heart Attack Prediction by the Feature Selection Methods," *Turkish Journal of Electrical Engineering and Computer Sciences* 26, no. 1 (2018): 1–10, https://doi.org/10.3906/elk-1611-235.

31. B. Gabrys and A. Bargiela, "General Fuzzy Min-Max Neural Network for Clustering and Classification," *IEEE Transactions on Neural Networks* 11, no. 3 (2000): 769–783, https://doi.org/10.1109/72.846747.

32. H. J. Kim, T. W. Ryu, T. T. Nguyen, J. S. Lim, and S. Gupta, "A Weighted Fuzzy Min-Max Neural Network for Pattern Classification and Feature Extraction," in *International Conference on Computational Science and Its Applications* (Springer, 2004), 791–798, https://doi.org/10.1007/978-3-540-24768-5_85.

33. A. V. Nandedkar and P. K. Biswas, "A Fuzzy Min-Max Neural Network Classifier With Compensatory Neuron Architecture," *IEEE Transactions on Neural Networks* 18, no. 1 (2007): 42–54, https://doi.org/10.1109/tnn.2006.882811.

34. A. Quteishat and C. P. Lim, "A Modified Fuzzy Min–Max Neural Network With Rule Extraction and Its Application to Fault Detection and Classification," *Applied Soft Computing* 8, no. 2 (2008): 985–995, https://doi.org/10.1016/j.asoc.2007.07.013.

35. A. Quteishat, C. P. Lim, and K. S. Tan, "A Modified Fuzzy Min–Max Neural Network With a Genetic-Algorithm-Based Rule Extractor for Pattern Classification," *IEEE transactions on systems, man, and cybernetics-part a: systems and humans* 40, no. 3 (2010): 641–650, https://doi.org/10.1109/tsmca.2010.2043948.

36. H. Zhang, J. Liu, D. Ma, and Z. Wang, "Data-Core-Based Fuzzy Min–Max Neural Network for Pattern Classification," *IEEE Transactions on Neural Networks* 22, no. 12 (2011): 2339–2352, https://doi.org/10.1109/tnn.2011.2175748.

37. R. Davtalab, M. H. Dezfoulian, and M. Mansoorizadeh, "Multi-Level Fuzzy Min-Max Neural Network Classifier," *IEEE Transactions on Neural Networks and Learning Systems* 25, no. 3 (2013): 470–482, https://doi.org/10.1109/tnnls.2013.2275937.

38. M. F. Mohammed and C. P. Lim, "A New Hyperbox Selection Rule and a Pruning Strategy for the Enhanced Fuzzy Min–Max Neural Network," *Neural Networks* 86 (2017): 69–79, https://doi.org/10.1016/j.neunet.2016.10.012.

39. M. F. Mohammed and C. P. Lim, "Improving the Fuzzy Min-Max Neural Network With a K-Nearest Hyperbox Expansion Rule for Pattern Classification," *Applied Soft Computing* 52 (2017): 135–145, https://doi.org/10.1016/j.asoc.2016.12.001.

40. C. Azad, A. K. Mehta, and V. K. Jha, "Improved Data Classification Using Fuzzy Euclidean Hyperbox Classifier," in *2018 International Conference on Smart Computing and Electronic Enterprise (ICSCEE)* (IEEE, 2018), 1–6, https://doi.org/10.1109/ICSCEE.2018.8538389.

41. F. Pourpanah, C. P. Lim, X. Wang, C. J. Tan, M. Seera, and Y. Shi, "A Hybrid Model of Fuzzy Min–Max and Brain Storm Optimization for Feature Selection and Data Classification," *Neurocomputing* 333 (2019): 440–451, https://doi.org/10.1016/j.neucom.2019.01.011.

42. P. K. Simpson, "Fuzzy Min-Max Neural Networks. I. Classification," *IEEE Transactions on Neural Networks* 3, no. 5 (1992): 776–786, https://doi.org/10.1109/72.159066.

43. S. Phalke, Y. Vaidya, and S. Metkar, "Big-O Time Complexity Analysis of Algorithm," in *2022 International Conference on Signal and Information Processing (IConSIP)* (IEEE, 2022), 1–5, https://doi.org/10.1109/ICoNSIP49665.2022.10007469.

44. M. Bianchini and F. Scarselli, "On the Complexity of Neural Network Classifiers: A Comparison Between Shallow and Deep Architectures," *IEEE Transactions on Neural Networks and Learning Systems* 25, no. 8 (2014): 1553–1565, https://doi.org/10.1109/tnnls.2013.2293637.

45. 45. Lichman M., UCI machine learning repository, (2013).

46. B. Chandra and P. P. Varghese, "Fuzzifying Gini Index Based Decision Trees," *Expert Systems with Applications* 36, no. 4 (2009): 8549–8559, https://doi.org/10.1016/j.eswa.2008.10.053.

47. M. C. Tu, D. Shin, and D. Shin, "Effective Diagnosis of Heart Disease Through Bagging Approach," in *2009 2nd International Conference on Biomedical Engineering and Informatics* (IEEE, 2009), 1–4, https://doi.org/10.1109/BMEI.2009.5301650.

48. H. El Hamdaoui, S. Boujraf, N. E. H. Chaoui, B. Alami, and M. Maaroufi, "Improving Heart Disease Prediction Using Random Forest and Adaboost Algorithms," *International Journal of Online and Biomedical Engineering* 17, no. 11 (2021): 61–75, https://doi.org/10.3991/ijoe.v17i11.24781.

49. T. T. Khuat, F. Chen, and B. Gabrys, "An Improved Online Learning Algorithm for General Fuzzy Min-Max Neural Network," *arXiv preprint arXiv:2001.02391* (2020), https://doi.org/10.1109/IJCNN48605.2020.9207534.

50. B. Gabrys, "Agglomerative Learning Algorithms for General Fuzzy Min-Max Neural Network," *Journal of VLSI signal processing systems for signal, image and video technology* 32, no. 1–2 (2002): 67–82, https://doi.org/10.1023/a:1016315401940.