



Groundwater hydrodynamics and supply sustainability in Punjab, Pakistan: A geological statistical approach using the GAMEAS algorithm based on data from 1105 boreholes

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ABSTRACT

Effective groundwater management is crucial for economic sustainable development, particularly as climate change and population growth increase the uncertainty of aquifer dynamics. Due to limited geological data, Punjab's complex hydrogeological conditions and Quaternary alluvial deposits present significant challenges for groundwater management. This study employs cost-effective numerical techniques as alternatives to traditional methods to safeguard groundwater quality, quantity, and accessibility. It introduces an edit-embedded transition frequency model that integrates regional datasets and utilizes algorithms such as GAMEAS, MCMOD, and TSIM to evaluate aquifer heterogeneity and simulate spatial variations using one-dimensional and three-dimensional Markov chains. Findings show that sand exhibits the highest self-transition (33.112 m), indicating strong stability, followed by silt, clay, and gravel, suggesting overall hydrofacies stability both horizontally and vertically. The model's predictions are largely consistent with actual material distribution, with a slight under-prediction of clay (−0.750%) and an over-prediction of sand (2.985%), which accounts for 58.77% of the aquifer material. It also highlights significant heterogeneity in the northern mountainous regions and minor variations in the south. The study emphasizes Punjab's severe water crisis, with groundwater reserves of 3502.3 BCM, declining water levels (0.38–33.62 m), and low hydraulic conductivity, urging government action on rainwater harvesting and sustainable groundwater management policies.

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1. Introduction

Groundwater is a vital natural resource for human survival, but overexploitation, pollution, and inefficient practices put it at risk (Saraswat A et al., 2023). Climate change, human activities, and technological advances affect water supply and quality, creating a water security crisis (Pan X et al., 2022; Akbar H et al., 2023). Effective groundwater

management requires understanding hydrological processes, geological characteristics, and human intervention (Haq ZU and Ashraf M, 2023). Factors such as sedimentary environment, evolutionary dynamics, and soil composition affect hydraulic parameters, so public development policies are essential for sustainable development (Arif K and Khalid P, 2023).

Pakistan is a significant water user and has an irrigation region, but it faces numerous challenges due to limited water resources and insufficient water storage capacity. It ranks third in the world in water consumption, utilizes 9% of the world's extraction volume, and irrigates 4.6% of the land (Nabi G et al., 2019). It is estimated that 27% of irrigation relies on surface water and 73% on groundwater in Punjab.

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Currently, the agricultural sector consumes 97% of the country's freshwater, contributing 18% of gross domestic products (Akbar H et al., 2022). With a population of 110×10^6 and an annual growth rate of 2.1%, Punjab faces increasing water challenges due to intensive agricultural production and frequent droughts (PBS, 2019). Water resources are distributed differently across arid, semi-arid, and humid regions, with most water available only during the rainy season. The average annual rainfall ranges from 100–2500 mm in Punjab (Yonus M et al., 2023). The area of water bodies and wetlands in Punjab is decreasing, and more than 80% of the water body pixels are classified as permanent water bodies. Climate change exacerbates these challenges, with Punjab ranked second globally in the 2019 Climate Risk Index due to severe extreme rainfall, intermittent flooding, and groundwater depletion (Naveendrakumar G et al., 2019). The Indus River Basin is the world's largest irrigation network that irrigates 16.2×10^6 hectares, but the river basin faces limitations in meeting crop water needs due to insufficient resources (Nadeem A et al., 2023). Siltation has reduced the water storage capacity of dams, leading to an increase in private tube wells (Butt AQ et al., 2022, 2023). The escalating trend in groundwater extraction has exacerbated water quality degradation and soil salinization, posing risks to future agricultural productivity (Javeed B et al., 2024). Despite efforts to address water scarcity in Punjab, factors such as inefficient water use, policy deficiencies, population growth, and overexploitation continue to pose challenges to sustainable water management in Punjab (Pan X et al., 2023; Rossignoli C et al., 2023).

Aquifer heterogeneity studies and scientific evaluation are crucial to understanding aquifer behavior, especially in the Punjab region. It provides insights into the spatial distribution of properties such as permeability, porosity, and hydraulic conductivity, which aid in sustainable extraction, resource assessment, and prediction of recharge rates (Afzal M et al., 2023). This knowledge is essential for protecting groundwater-dependent ecosystems and water quality. Accurate numerical groundwater flow and transport process models have been developed using aquifer heterogeneity studies (Gong K et al., 2023). Risk assessment and mitigation are also essential (Akbar H et al., 2022). However, regional approaches have not been used in this area. This study highlights the importance of aquifer heterogeneity, simulation models, and prediction of management scenario responses using the Transition Probabilistic Geostatistical Software (T-PROGS model). T-PROGS helps identify spatial variations in lithologic and hydrofacies units within an aquifer, which is essential for understanding groundwater flow patterns, predicting contaminant migration, and designing effective groundwater management strategies (Grana D et al., 2023). T-PROGS helps integrate geological data from boreholes and other sources to represent better subsurface hydrogeological properties (Mizutani D et al., 2017). Additionally, the software provides visualization tools that help explain and communicate model results to stakeholders and decision-makers (Dixit N et al., 2020). However, T-PROGS modeling

also has limitations. It requires extensive input data, including detailed lithological and hydrogeological data, which may be challenging to obtain in some areas. The software also demands expertise in geostatistics and hydrogeology to calibrate and interpret model results correctly (Guo J et al., 2022). Furthermore, T-PROGS modeling is computationally intensive and may require specialized software and high-performance computing resources, making it inaccessible to some researchers or organizations with limited resources (Datta D et al., 2023). Despite these limitations, T-PROGS remains a valuable tool for hydrogeological modeling that can provide insights into aquifer heterogeneity and support informed decisions in groundwater management and environmental protection (Glennie R et al., 2023).

This study aligns with the goals of SDG-6 by utilizing the GAMEAS algorithm to analyze data from 1105 boreholes, providing detailed insights into groundwater hydrodynamics. This analysis aids in effective aquifer management, preventing over-extraction and contamination, thereby ensuring a sustainable groundwater supply. The study employs a simulation model to understand aquifer heterogeneity in Punjab Province, Pakistan by utilizing the Markov chain-based model, integrated with the GAMEAS, MCMOD, and TSIM algorithms in T-PROGS software. It offers a conceptual framework for geological understanding. The model focuses on regional water resource management approaches and aims to identify institutional and technological strategies for environmental sustainability and agricultural water productivity. It accurately replicates aquifer heterogeneity and aligns the Markov chain curves with the observed transition probability curves.

2. Material and methods

2.1. Description of the study area

Punjab, the second largest province of Pakistan, encompasses an immense territory of 205344 square kilometers and is situated on the northwestern border of South Asia. Punjab is located in the subtropical region, with coordinates ranging from 28.42° to 34.02° north latitude and 69.81° to 74.23° east longitude (GSP G, 2023). The name "Punjab" comes from its location at the confluence of five rivers, which created a highly fertile area suitable for agriculture. The average altitude of Punjab ranges from (64–2350 m). The central region of Punjab is flat and lies at an altitude of 600 m above sea level. Murree Hill Station in the north is 2350 m above sea level and surrounded by dense forest (Fig. 1a). The Punjab terrain is flat and mainly comprised of farmland in river valleys. However, it also has a sandy desert called Cholistan to the southeast, hilly terrain to the south and northwest, and the Potohar Plateau to the north. The basin contains large amounts of alluvial grains from the rapidly disintegrating southern Himalayas (Bakr MU, 2023). These particles then enter the Indus Delta and the Arabian Sea. The province significantly contributes to the national economy and is home to nearly half of the country's

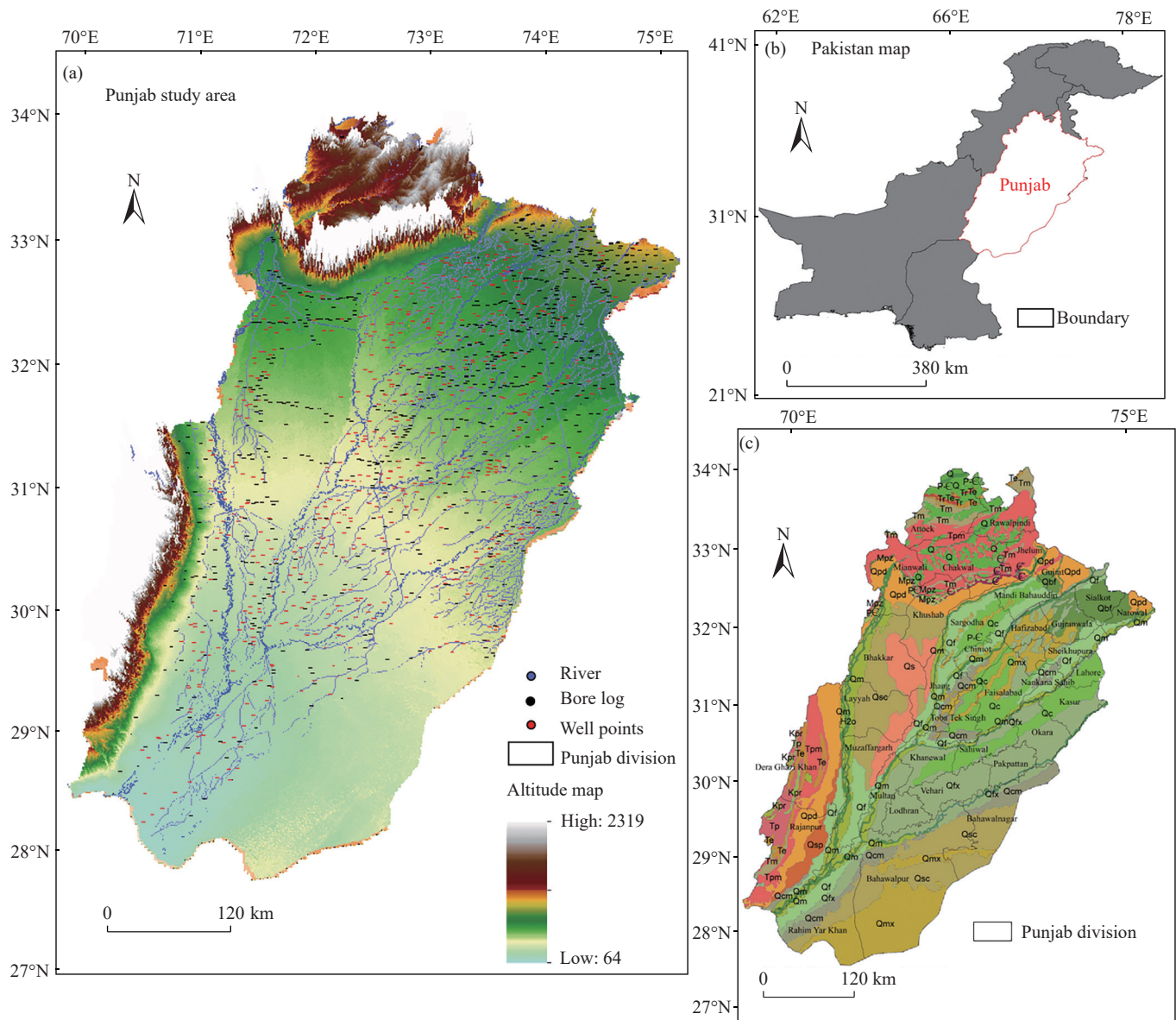


Fig. 1. a–Research area’s description map displays the spatial location of bore-log and piezometric well points; b–the Pakistan boundary map with its territories; c–the geological map shows the distribution of geology types in the study area (Source: Geology map of Punjab, Pakistan). C–Cambrian deposits; Qc–Older terrace deposits; Qmx–Deposits of extinct streams; Te–Tertiary deposits; Qs–Older eolian deposits; Qsc–Older eolian deposits; Qfx–Deposits of extinct streams; Qf–Steam deposits; PC–Precambrian deposits; Qcm–Older terrace deposits; Tm–Quaternary deposits; Kpr–Cretaceous and Jurassic; Tp–Tertiary deposits; Tpm–Quaternary deposits; Qbf, Qpd, Qsp–Piedmont related deposit; Qm–Steam deposits; Tr–Mesozoic rocks; Mpz–Early Mesozoic and Late Paleozoic; Q–Alluvium deposits (after [GSP, 2023](#)).

agricultural workforce. The province’s estimated population is 100.6×10^6 , with a population density of 460 individuals per square kilometer. The population growth rate is approximately 2.05%, with the urban population constituting roughly 32% ([PBS, 2017](#)). The climatic zones of Punjab are divided into three different types: Humid, semi-arid, and arid. The classification is based on changes in rainfall patterns, soil types, and significant land use categories. The macroclimate is dominated by drought, with precipitation ranging from 100 mm in the south to 600 mm in the northwest ([Yonus M et al., 2023](#)). From June to September, the monsoon season can end with catastrophic flooding. Temperatures vary between -2°C – 45°C , but in rare cases, temperatures can rise to 55°C in the summer and drop to -10°C in the winter, causing

mercury levels to fluctuate accordingly ([Khattak MS and Ali S, 2015](#)). Punjab is known for its efficient irrigation system, which is supported by over 3000 irrigation canals. The system enables abundant agricultural production of major crops such as wheat, rice, cotton, sugarcane, corn oilseeds, millets, vegetables, pulses, and tree fruits. However, due to economic progress and rapid urban development, Punjab faces multiple thermal challenges requiring sustainable urban management ([Razzaq A et al., 2023](#)).

2.2. Hydrogeology and aquifer characteristics of the study area

Hydrogeological variables in the natural environment determine the formation and regularity of groundwater. These

conditions also generate precise soil permeability, hydraulic conductivity, and porosity data. These phenomena pertain to water infiltration into the earth via aquifers and their interaction with the adjacent soil and rocks (Akhter G et al., 2022). Geological features, such as faults, cracks, joints, and folds, are crucial in shaping landforms and directly impact hydrogeological processes. The Punjab Plain, known as the Salt Range, hosts a highly developed Cambrian stratigraphic sequence and experiences a warm temperate paleoclimate. The eastern section of the area consists of marine sediments, while the westernmost section consists of sedimentary rocks ranging from the Precambrian Salt Ridge Formation to the Jurassic Datta Formation (Muhammad S et al., 2023a). Geological studies reveal that the Punjab region is covered with unconsolidated alluvium, covering bedrock up to 1000 feet deep and having Holocene and Pleistocene sediment plain, sourced from the Himalayas and overlain by Precambrian age rocks (Table 1 and Fig. 1c). The Pothwar plateau and southwest Punjab have separate confined groundwater aquifers, while the Northwestern, western, and southeastern Punjab aquifers are unconfined (Ali SK et al., 2021; Bakr MU, 2023). The aquifer areas encompass a varied alluvial plain consisting of clay, loam, fine sand, silt, gravel, and low levels of organic matter. Additionally, aquitards and aquicludes are composed of sandy clay and clayey soil (Afzal M et al., 2023). The hydrogeology of Punjab is affected by precipitation, and irrigation and seepage canals can contribute to the recharge of groundwater systems. The total area of underground aquifers is 16×10^6 hectares, of which 6×10^6 hectares are high-quality groundwater and 10×10^6 hectares are low-quality groundwater. Cumulative recharge amounts to 42.75×10^6 acre-feet and includes precipitation, canal infrastructure, irrigation, river recharge, and domestic and industrial wastewater. Total tubewell flow reaches 31.5×10^6

acre-feet (Zakir H et al., 2023). The Punjab region's water supply shows an overall trend of scarcity in the south and abundance in the north. The region's various terrains, including Sirowal, Kandi, Inter-montane, and Upper Siwalik, which provide irrigation water, are recharged through precipitation, leakage, canals, cropped lands, and surface water during the flood season (Malkani MS and Haroon Y, 2022). Uncontrolled and unregulated extraction has led to saltwater migration, causing salinization and waterlogging. The unregulated and spontaneous expansion of privately owned tubewells will worsen the water mixing and threaten the aquifer's long-term viability.

2.3. Data collection

Groundwater characteristics exhibit notable variations across different locations, influenced by climatic fluctuations, terrain quality, and hydrogeological variables. The borehole data utilized in this study were sourced from various databases, primarily from Pakistan's Geological Survey and the Punjab Hydropower Development Department (Khan S et al., 2003; GSP, 2023; Wapda, 2023). Aquifer heterogeneity in the Punjab province of Pakistan was assessed through available data, expert insights, and information from previous studies. Additionally, specific site data were gathered from literature reviews, published reports, and existing maps. A comprehensive survey involved collecting lithological information from 17815 samples from 1105 boreholes, as illustrated in Tables 2–3. The process involves collecting detailed data, analyzing aquifer properties, and mapping the location of each borehole. Twenty-eight geological types were identified and classified based on grain gradation and plasticity index for geostatistical modeling, with unconsolidated sediments categorized into four aqueous units

Table 1. Types of deposition along the strata and aquifer thickness in Punjab (Source: GSP G, 2023).

Geology Types	Geology code	Stratum	Aquifer Thickness/ft
Cambrian deposits	C	Cambrian sedimentary rocks	600
Older terrace deposits	Qc	Chung formation	617
Deposits of ectinct streams	Qmx	Ectinct streambed and meander belt	1500
Tertiary deposits	Te	Eocene Sedimentary rocks deposits	6000
Older eolian deposits	Qs	Eolian to barchan sand dunes	1265
Older eolian deposits	Qsc	Eolian sand deposits	100
Deposits of ectinct streams	Qfx	Flood plain ectinct stream	1010
Steam deposits	Qf	Flood plain stream deposits	915
Precambrian deposits	PC	Igneous and metamorphic rocks	450
Older terrace deposits	Qcm	Loess and flood plain deposits	890
Precambrian deposits	P-C	Metamorphic and ingeous rocks	127
Quaternary Deposits	Tm	Miocene sedimentary rocks	2500
Cretaceous and Jurassic	Kpr	Pab sandstone	180
Tertiary deposits	Tp	Paleocene sedimentary rocks	2000
Quaternary Deposits	Tpm	Pliocene, Miocene, Piedmont and subpiedmont sedimentary rocks	1500–4500
Piedmont related deposit	Qbf, Qpd, Qsp	Sheet flood and flood plain deposits	1700
Steam deposits	Qm	Streambed and meander belt deposit	1200
Mesozoic rocks	Tr	Triassic limestone, dolomite, some sandstone and shale undivided	400
Early mesozoic and late paleozoic	Mpz	Triassic, Permian and Carboniferous sedimentary rocks	100
Alluvium deposits	Q	Silt, sand and gravel deposits	600

(Clay, silt, sand, and gravel) based on hydraulic conductivity (Table 3). Fig. 2 depicts a soil map, showing various types of soil and their infiltration rates, contributing to a thorough site analysis. The study relied on essential software tools such as T-PROGS, GMS10.6, Arc GIS 10.8, and Origin Pro 2023 for data analysis and interpretation (Carle SF, 1999).

2.4. Hydrogeological and Numerical Modelling Simulation

Hydrogeological structural models are tools for studying regional groundwater systems, focusing on the alluvial strata of the study area. These formations are classified based on their physical properties (e.g., particle size, aquifer medium, and viscosity) and material properties (e.g., horizontal hydraulic conductivity, vertical anisotropy, and compressive modulus) (Awais M et al., 2022; Muhammad S et al., 2023b; Tayyab A, 2016).

The T-PROGS model represents transition probabilistic geostatistical software used to characterize spatial variations in geological properties within aquifers by analyzing the likelihood of transitions between different lithological units or hydrofacies (Ballio F and Guadagnini A, 2004). A groundwater model is a 2D and 3D model that accurately represents a groundwater system's heterogeneity, anisotropy, and instability. The T-PROGS module and Markov chain method in GMS software construct a hydrogeological structure model to facilitate stochastic calculations (Carle SF et al., 1998; de Almeida J, 2010). The alluvial strata in the study area were divided into five categories: Filling material, Clay, Silt, Sand, and Gravel. A 3D grid stratigraphic map was created using 1105 soil boreholes and divided the area into two layers, 100 rows, and 100 columns, and the map was modeled on a grid with dimensions of X-L = 745053.66 m × Y-L = 896967.25 m × Z-L = 300 m (Fig. 2b).

GMS serves as an interface to the T-PROGS software, which performs transition probability geostatistics on borehole data. The software generates N material sets on a 3D grid, each customized based on drill data and the proportions and transitions of the materials. The more complex the original observations, the more difficult and non-similar the realizations, while the less available data or relevance, the higher the uncertainty the model introduces (Eramian MG et al., 1999). T-Progs is a method for modeling categorical variables using transition probabilities and Markov chain geostatistics. It is simpler than the traditional variogram approach due to its simplicity in formulating cross-correlated variables and interpreting the results (Gray JM et al., 2016). T-Progs involves three key steps: Establishing transition probabilities from observational data, developing a Markov chain model to represent spatial variability, and applying sequential index simulation and quenching to generate geological realizations. Fig. 3 shows the flow chart of the methodology.

T-PROGS software employs a geostatistical approach with transition probabilities to describe geographic variability using 3-D Markov chains, index co-kriging equations, and a simulated annealing objective function. This approach considers asymmetric juxta-positional tendencies and incorporates geologic interpretations into cross-correlated spatial variability (Harp D et al., 2008; Papapetrou M and Theodossiou N, 2010). It is suitable for geologic data and provides a theoretical structure for understanding geological processes using concise mathematical models (He X et al., 2015; Zhang ZY et al., 2025). The software also uses embedded Markov chain analysis, which uses embedded transition frequencies defined in the vertical direction. This type of analysis is more interpretable and has a long history in earth sciences (GMS A, 2024; Xue P et al., 2022). An

Table 2. Bore-log classification based on sample collection types and water level depth in the Punjab Province.

Area Division	Total samples	Average water level depth/m	Average thickness/m	Soil type samples	Drill cuttings	Sediment samples	Rock cores
Bahawalpur	214	71.17	10.67	108	2	103	1
Dera Ghazi	1621	123.95	17.00	576	6	1026	13
Faisalabad	4063	81.55	10.04	883	75	3096	9
Gujranwala	618	66.00	15.22	325	1	292	0
Lahore	2488	93.87	13.23	1298	7	1180	3
Multan	2240	101.40	9.70	1049	10	1174	7
Rawalpindi	18	60.89	12.73	9	1	8	0
Sargodha	6573	52.02	7.85	2446	229	3834	64

Table 3. Lithological classification based on hydraulic conductivity values.

Generalized soil	Original lithology	Hydraulic conductivity/(m/s)	Permeability Class	S_y	S_s	Sample	Proportion
Clay	Clayey, clay with fine silt, Loamy soil, calcareous Clay	$10^{-8} > K > 10^{-12}$	Low	$10^{-2}-10^{-3}$	0.05–0.07	5441	30.54
Silt	Silty, Silty Clay, Loamy silt, Fine silt with sand, silty fine sand with thin Clay	$10^{-5} > K > 10^{-8}$	Medium	$10^{-3}-10^{-5}$	0.10–0.15	747	4.19
Sand	Sandy cobbles, sandy Silt, coarse and medium Sand, Loamy Sand, Sand and gravels	$10^{-3} > K > 10^{-5}$	High	$10^{-5}-10^{-7}$	0.15–0.2	9938	55.78
Gravel	kankers, Cobbles, mixture, Fragments, Pebbles, Boulder, Bed rock, Granule, Cemented, Quartzite, Slate, Shale	$10^{-2} > K > 10^{-3}$	Very High	$10^{-6}-10^{-8}$	0.2–0.28	1689	9.48

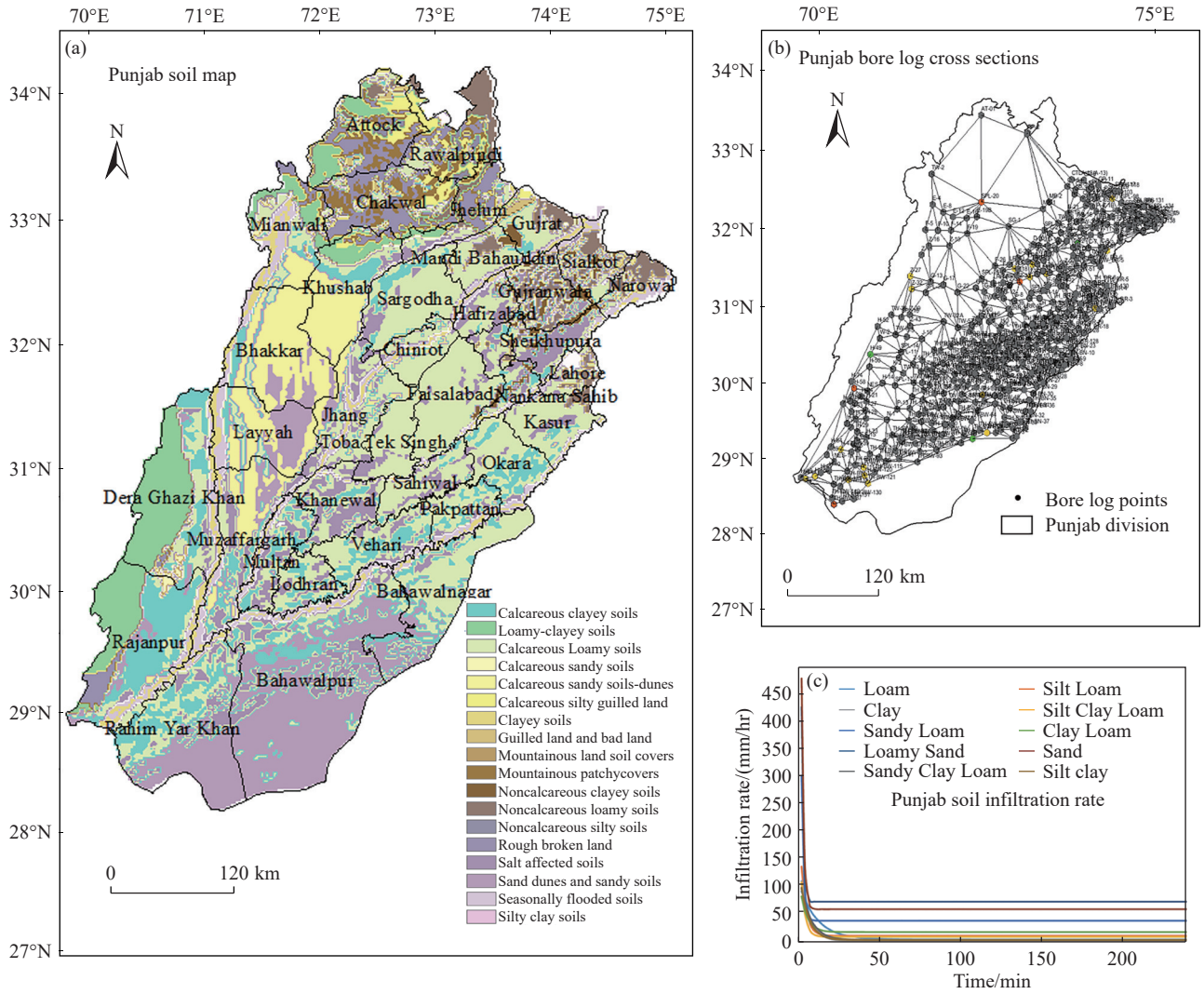


Fig. 2. Map showing a–types of soil distribution along the study area; b–spatial map of bore-log with their cross-section; c–soil infiltration rate depending on various soil types.

embedding transition frequency matrix can be formulated in which the diagonal entries represent the row and column sums corresponding to the edge frequencies at which embedding classes occur. The transition frequency can be converted into a transition rate using the additional average length information. Therefore, MCMOD can build a rate matrix based on transition frequencies in off-diagonal entries. Off-diagonal entries for background rows and columns do not need to be specified, but the average length of the background categories is required. The basic steps involve using an embedded transition frequencies algorithm that follows a model development method that consists of creating a bore log data set, defining transition probability, and developing one and three-dimensional Markov chain models through the MCMOD (He X et al., 2014; Naranjo-Fernández N et al., 2019). The triangulation algorithm assumes unique vertices in the x-y plane, and duplicate points can be removed. Transfer probability is calculated using a set of transition probability curves as a function of lag distance for each material (Hu LY and Chugunova T, 2008; Ullah J et al., 2023). The software imports borehole data and uses GAMEAS to compute and edit

embedded transition frequencies probability curves for each category, and it is defined as

$$t_{jk}(h) = \Pr\{k \text{ occurs at } X_0 | i_j(X_a); \alpha = 1, N; j = 1, \dots, K\} \approx \sum_{a=1}^N \cdot \sum_{j=1}^K i_j(X_a) \omega_{jk,a} \quad (1)$$

$$i_j(X_a) = \begin{cases} 1, & \text{if category } j \text{ occurs at } X_a \\ 0, & \text{otherwise} \end{cases} \quad j = 1, \dots, K$$

Where N is the number of data, K is the number of categories, $\omega_{jk,a}$ represents a weighting coefficient, and $i_j(X_a)$ represents the value of an indicator variable.

Eq. 1 generates 3-D Markov chain models representing spatial dependencies and determining mean length and proportions. Markov chains assume that spatial occurrences depend only on the nearest data. The mean length ratio is applied to the Markov chains in the strike and dip directions based on Walther's law (Sun Q et al., 2019) to overcome the sparse distribution of boreholes. The 3-D Markov chain model is used as a reference in Sequential indicator simulation and quenching in the TSIM simulator. The TSIM utility is

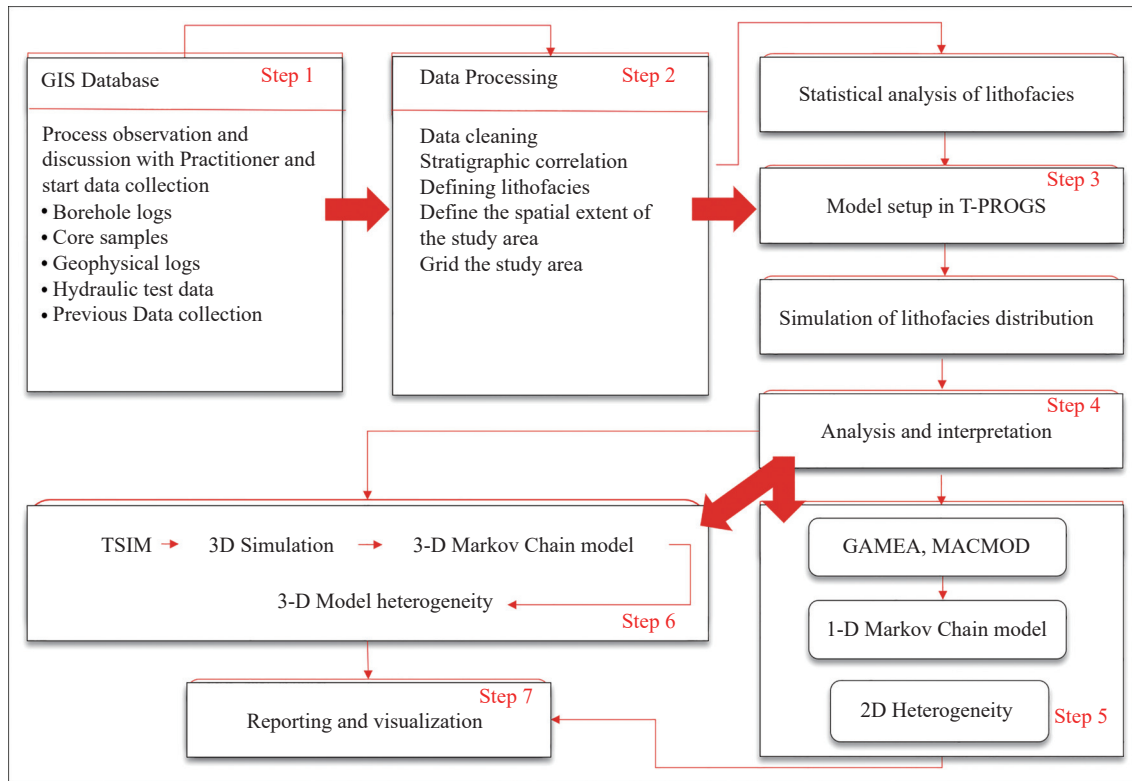


Fig. 3. Schematic diagram showing implementation of the T-PROGS model.

used to formulate indicator co-kriging equations and an objective function for simulated annealing (Carle SE, 1999; Papapetrou M and Theodossiou N, 2010). The computational formula for it is presented as (Eqs. 2, 3)

$$\begin{pmatrix} T(X_1 - X_1) & \cdots & T(X_N - X_1) \\ \vdots & \ddots & \vdots \\ T(X_1 - X_N) & \cdots & T(X_N - X_N) \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} = \begin{pmatrix} T(X_0 - X_1) \\ \vdots \\ T(X_0 - X_N) \end{pmatrix} \quad (2)$$

$$\min_0 = \sum_{i=1}^m \cdot \sum_{j=1}^K \cdot \sum_{k=1}^K [t_{jk}(h_i) \text{sim} - t_{jk}(h_i) \text{mod}]^2 \quad (3)$$

Where $T(X_i - X_j)$ represents the covariance or semi-variance between two points X_i and X_j . In contrast, W_{ijk} represents a weighting coefficient along the x, y, and z directions.

Quenching is applied, and boreholes are used to define soils and hydrogeologic units, which can group materials into one hydrogeologic unit. The T-PROGS Materials dialogue allows users to choose materials from all or one hydrogeologic unit. The materials list includes material associated with boreholes and a background material automatically adjusted to balance equations (GMS A, 2024; Gong K et al., 2023; Illman WA et al., 2010). The azimuth orientation determines depositional trends in strike or dip directions, aligning with horizontal flow in the aquifer. The T-PROGS algorithm has a maximum of five materials to keep

data processing and user interface simple.

3. Results and discussion

3.1. Three-dimensional Embedded Markov chain

Subsurface stratigraphy challenges material classification and hinders heterogeneity evaluation. Adjusting the grid spacing to align with layers requires careful consideration of user-defined vertical thickness. However, due to thin units, this approach is challenging. One proposed solution involves increasing the number of grid cells while decreasing the range in the z-direction (Chandna A and Srinivasan S, 2023). The current study used transfer probabilities of a 5000 m interval as parameters and elliptically interpolated terms in the transfer probability matrix in the vertical direction. The results showed that the average thickness of hydrofacies units reflected by the simulation in the vertical direction was more specifically represented. Gravel material represented the rock foundation, the lowest within the borehole accessible range. The calculated vertical transfer probability showed that the transfer from hydrofacies units to gravel was trivial, implying minimal transition from other sediments to solidification in the upward vertical direction. The horizontal Markov chain and parameters, such as average extension length, showed no significant difference in transfer probability in the x-direction from the y-direction. This study chooses sand as the background material due to its predominant formation in the aquifers. Fig. 4 represents the uncertainty distribution of this model, indicating the possibility of soil material in each cell of the aquifer's first layer of the discretization network. The

edit-embedded transition frequencies method in T-PROGS was chosen to generate 1-D and 3-D Markov chains, providing a conceptual and accurate process. The diagonal values for each pair of geological materials in the vertical direction coincide with the vertical mean lengths of “Clay,” “Silt,” “Sand,” and “Gravel,” with “Sand” having a 33.12-meter mean length. For more details, see Tables 4–5.

It is noteworthy that two sets of materials exhibit distinct transition probabilities. The transition probability value for the clay-gravel combination is 0.047, but the gravel-clay pair has a value of 0.051. Since $0.051 > 0.047$, one can assume that it is less likely to move from clay to gravel rather than from gravel to clay in the z-direction. The initial four plots (Fig. 4) demonstrate a satisfactory correlation in the clay, silt, sand, and gravel material combinations utilizing the “edit embedded transition frequencies” technique. The self-transitional and off-diagonal curves have a probability of 1.0 and 0, respectively. The flat part of the curves represents the average fraction of the material. The mean proportion of the material is defined by the point where a tangent line intersects the horizontal axis. The highest transition probabilities are found in the same material pairs, particularly in sand with gravel.

The same observations apply to all 16 plots (Fig. 4). Generally, the curves are perfectly matched between the first and all four material sets, indicating that the data are consistent. Table 5 represents each material’s mean lens length values and shows that sand with silt material takes the highest value of 33.112. The background material can be described as filling the remaining areas not occupied by other units. In this scenario, the sand is chosen as the background material, filling in unoccupied areas with a 0.5578 proportion.

3.2. 2D and 3D stochastically simulated hydrofacies

Grid plots are essential in T-PROGS modeling, as they discretize the study area into interconnected units. It serves as a computational framework for simulating groundwater models, allowing calculations of hydraulic properties. The current model’s mesh diagram accurately represents the aquifer’s physical characteristics, which the future may use for numerical solutions. It helps evaluate solutions, identify problems, optimize computational efficiency, and facilitate stakeholder communication by clearly representing the model domain (Fig. 5).

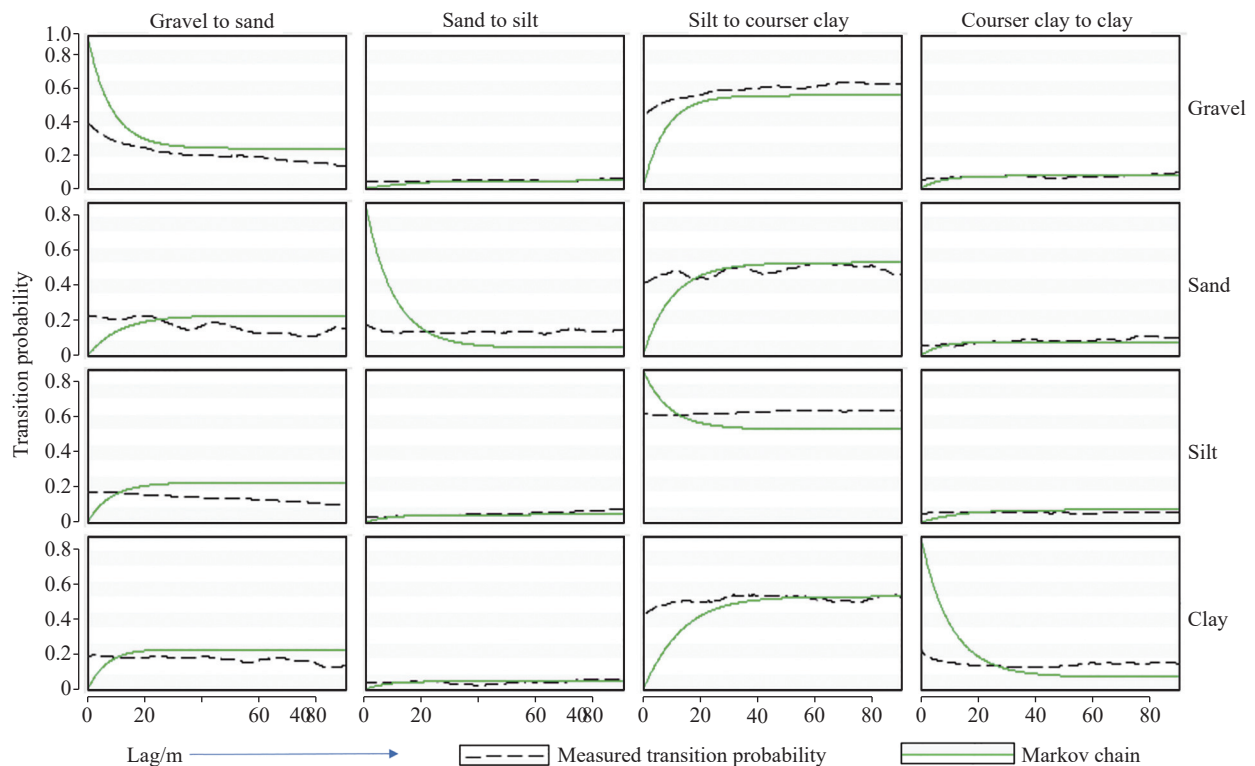


Fig. 4. Relationship between the Markov chain and transition probability curves created using the edit-embedded transition frequencies probability technique.

Table 4. Calculated edit embedded transition frequencies among hydrofacies in the X and Y direction.

Material	X-direction				Y-direction			
	Clay	Silt	Sand	Gravel	Clay	Silt	Sand	Gravel
Clay	15.985	0.009	0.324	0.047	14.985	0.009	0.326	0.048
Silt	0.018	15.140	0.048	0.011	0.019	15.130	0.048	0.011
Sand	0.311	0.058	33.112	0.058	0.312	0.057	33.112	0.058
Gravel	0.051	0.010	0.054	16.536	0.052	0.011	0.054	16.556

In this study, after obtaining the Markov chain, the T-PROGS model uses two-dimensional (2D) and three-dimensional (3D) stochastic simulations of hydrofacies to represent the spatial distribution of geological materials within the aquifer. In 2D simulations, the hydrofacies are represented on a 2D plane (usually the horizontal plane), focusing on the X - Y plane and neglecting vertical changes. It provides insights into lateral changes in the hydrofacies within the aquifer. It helps understand how different geological materials are distributed horizontally across the study area. These simulations are less computationally demanding and more visualizable than 3D simulations. However, they may oversimplify the vertical structure of the aquifer and may not capture the full complexity of the hydrofacies distribution. In contrast to 3D simulations, hydrofacies are represented in all three dimensions: Length, width, and height. It means that changes in the hydrofacies are considered in horizontal and vertical planes, allowing for a

more accurate representation of aquifer heterogeneity. The 3D simulation details how the hydrofacies change laterally and longitudinally within the aquifer along the X -axis, Y -axis, and Z -axis. It allows for a better understanding of the complex geology of the aquifer and its impact on groundwater modeling. These simulations are more computationally intensive and require higher-resolution data than 2D simulations. However, they provide a more realistic representation of the aquifer system and are more suitable for studies requiring a detailed understanding of aquifer heterogeneity. The result also shows that borehole data uncertainty is not the dominant factor in overall uncertainty, with a decreasing slope observed with decreasing borehole quality. However, the study's findings suggest that higher borehole quality may represent more heterogeneity, but the impact of borehole data uncertainty remains inconclusive. The study also explores the effect of varying sand medium permeability on accumulated subsidence, significantly affecting subsidence up to 58.76%. The representative results of the model are shown in Fig. 6, which shows the 2D and 3D views of mesh diagrams.

Table 5. Vertical Markov chain model as input and output material percentage contribution of the hydrofacies.

Material	Input/%	Probability/%	Error/%	Lens Length/m
Clay	30.54	29.790	-0.750	15.985
Silt	4.19	4.193	0.003	16.537
Sand	55.78	58.765	2.985	33.112
Gravel	9.48	7.242	-2.238	15.140

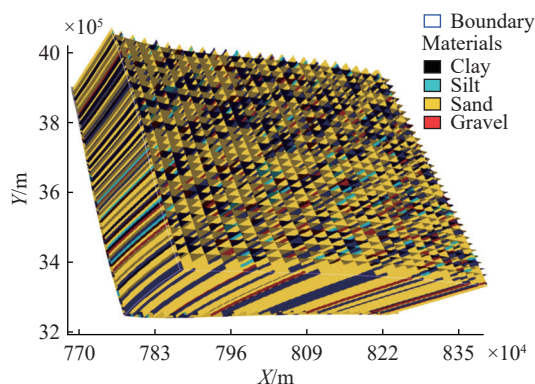


Fig. 5. Fence diagram representing hydrofacies in the study area.

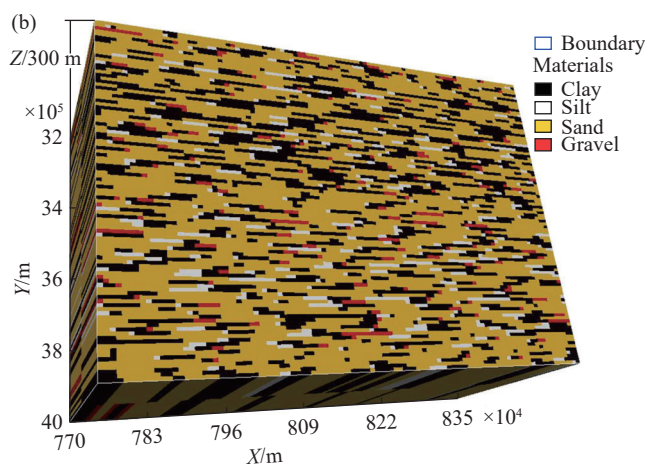
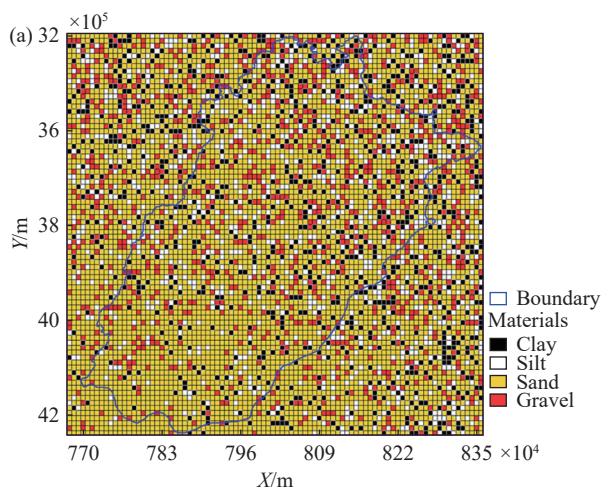


Fig. 6. a–2D model view representing the distribution of hydrofacies units; b–macro diagram showing the 3D model view.

mountainous region. The hydraulic conductivity map in the study area ranges from 1.3×10^{-3} – 32.80×10^{-3} m/sec indicating a generally high potential for groundwater storage and movement (Afzal M et al., 2023). Nevertheless, the presence of silt-clay interlayers diminishes hydraulic connections, resulting in places where convertible behavior occurs. Fluctuations in water levels result in the formation of unconfined and confined aquifers, which are affected by changes in hydrostatic pressure. Groundwater level changes in the study area result from a complex interplay of natural processes, human activities, and climatic factors. Water level maps indicate varying water table depths across the study area, with higher levels observed in the northern and lower levels in the southern regions. The water level map ranges from 0.38–33.62 m (Afzal M et al., 2023). Fig. 7a represents the water level map, and Fig. 7b represents the hydraulic conductivity map in the study area. 3D modeling uses altitude variation along the X-axis, Y-axis, and Z-axis to represent changes in elevation or height within a three-dimensional space. Each axis corresponds to a specific direction of movement within the model, allowing for the representation of terrain features, surface topography, and elevation gradients. The X-axis and Y-axis represent changes in elevation in the east-west and north-south direction, respectively. The Z-axis represents changes in elevation or height in the vertical direction, adding depth to the model. However, this approach is crucial for accurately depicting surface topography, landforms, and elevation gradients in 3D models. The altitude variation along the Y-axis is 64–2319 m, and 300 m along the Z-axis in the study area. Fig. 8 shows the model's polar diagram.

3.3.2. Groundwater volume estimation based on boreholes

The study provides a comprehensive understanding of groundwater levels and aquifer properties across the 1105 boreholes in the study area of 205344 km². The data shows significant spatial variability, with depths ranging from 0.38 m below ground level in the northern parts to 33.61 m in the southern parts. The average calculated value from the water table fluctuation method is 0.78 m. The average hydraulic conductivity and porosity values were 0.0163×10^{-5} – 3.780×10^{-5} m/day respectively, indicating a generally high potential for groundwater storage and movement. The saturated thickness varied between 0.22–211.3 m, with an average thickness of 10.66 m (Arshad A et al., 2020; Afzal M et al., 2023). The specific yield values ranged between 0.05–0.28, with an average of 0.16 (Tayyab A, 2016; Awais M et al., 2022). Using the specific yield and the average saturated thickness, the estimated groundwater volume for the area is approximately 350.23×10^9 m³. The basic formula is as below (Eqs.4–6)

$$\text{Groundwater Volume (V)} = \text{Area} \times \text{Average saturated Thickness} \times \text{Specific Yield} \quad (4)$$

$$\text{Groundwater Volume} = 205344 \times 10^6 \times 10.66 \times 0.16 = 3502.3 \text{ (BCM)}$$

$$\text{Total Recharge (R)} = \text{Area} \times \text{average Change in water table height} \times \text{Specific Yield} \quad (5)$$

$$R = 205344 \times 10^6 \times 0.78 \times 0.16 = 25626931200 \text{ m}^3$$

$$\text{Net Recharge} = \text{Total Recharge} - \text{Total Discharge} \quad (6)$$

$$\text{Net Recharge} = 25626931200 - 18800000000 = 6826931200 = 6826.9 \text{ MCM}$$

The assessment aligns with previous regional studies, confirming the substantial groundwater reserves available in the area. The higher water table levels in the northern region are likely due to its proximity to significant recharge sources, such as rivers and higher precipitation zones. The calculated specific yield and saturated thickness indicate a strong capacity for groundwater storage, particularly in areas with high hydraulic conductivity and porosity. However, sustainable management practices are necessary to prevent over-extraction, especially in the southern regions where water tables are lower. Future research should focus on continuous monitoring to capture seasonal and long-term changes in groundwater levels, enhancing our understanding and management of this vital resource.

3.4. Discussion on the accuracy of 2D and 3D model results

T-PROGS modeling relies heavily on the accuracy of two-dimensional (2D) and three-dimensional (3D) hydrofacies representations for accurate aquifer characterization. While 2D simulations offer conceptual clarity and computational efficiency, they may overlook vertical variations, resulting in oversimplified representations of aquifer heterogeneity. On the other hand, 3D simulations provide a detailed depiction of aquifer heterogeneity and complex flow patterns but require significant computational resources and extensive data. The hydrogeological structure model generated using one-dimensional (1D) and 3D Markov chain models and decision information demonstrates that hydrofacies units are well connected and effectively capture regional heterogeneity. The reflected sedimentary environments in both 2D and 3D geological models show similar characteristics, indicating that these models can accurately represent the sedimentary characteristics of strata. However, several limitations impact the accuracy of these models. Although not very detailed, the grid profile of this simulation results in an acceptable error range. The simulations accurately reflect historical depositional characteristics and external geology in the vertical direction, predicting crustal uplift and saltwater withdrawal due to tectonic movements. They also predict sedimentary processes and sedimentation rates in the horizontal direction. Notably, clay and silt occupy a large

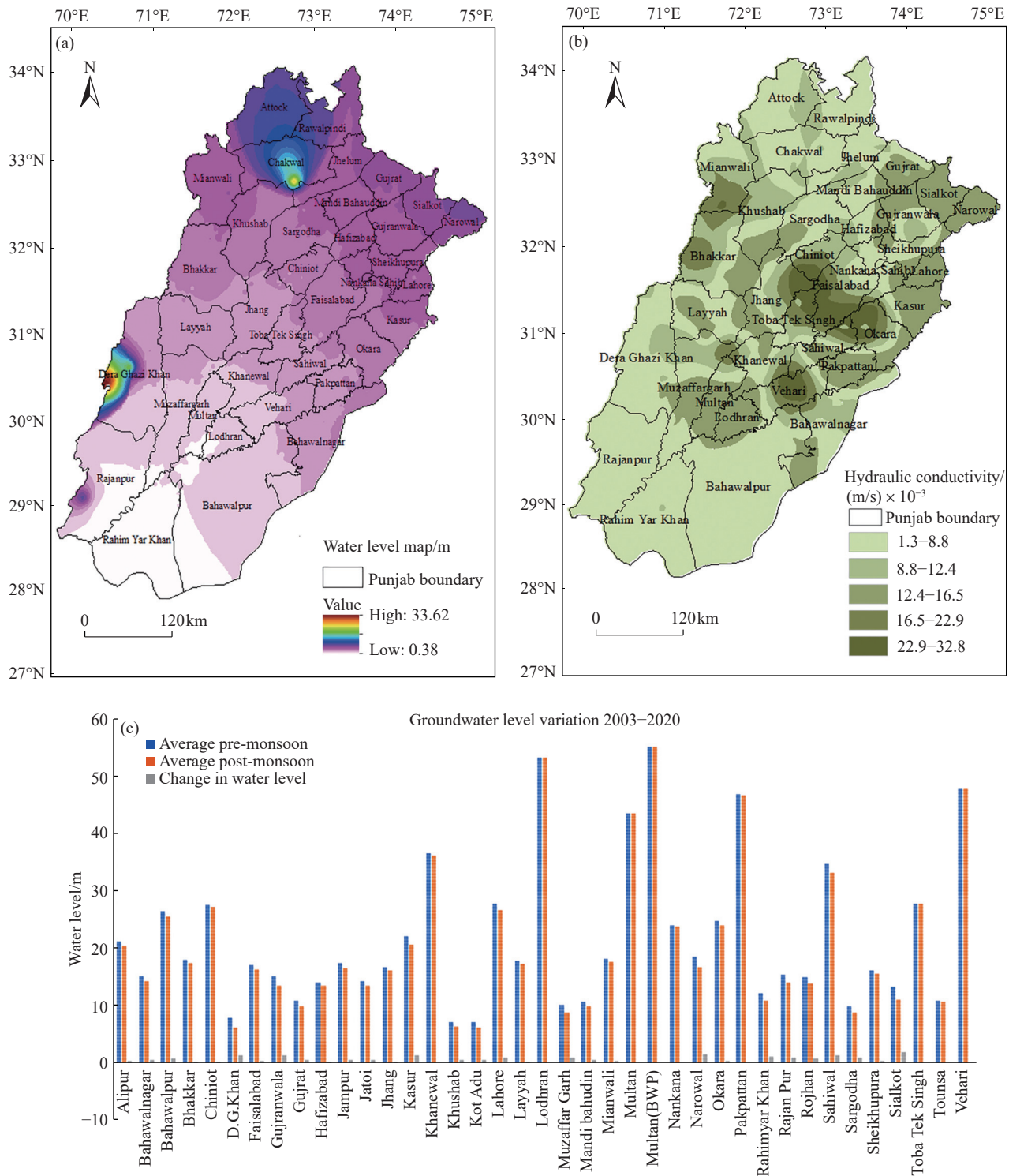


Fig. 7. a–Water level map in the study area; b–hydraulic conductivity map of Punjab; c–map showing water level fluctuation from 2003–2020 in the Punjab.

proportion in the upper section of the model, while sand gradually becomes the dominant aquifer structure. In the northeastern hilly region of Punjab, the model indicates a thick layer of solid rock and loose clay sediments, with silt and gravel distribution influenced by river impacts. Conversely, the central-south plain area, covered with sand and gravel, experiences lateral sediment accumulation due to river influences, resulting in variations in rock layers and water-bearing units' distribution.

Despite the strengths of these models, several factors introduce uncertainty. The accuracy of simulations depends significantly on the thoroughness and precision of the input data. However, it is not feasible to characterize every specific aspect of a formation precisely. In regions with less data availability, such as the northern and southern areas, the simulation uncertainty is higher compared to the eastern and central regions. Stochastic simulations, which generate models with equal probabilities, help address this uncertainty

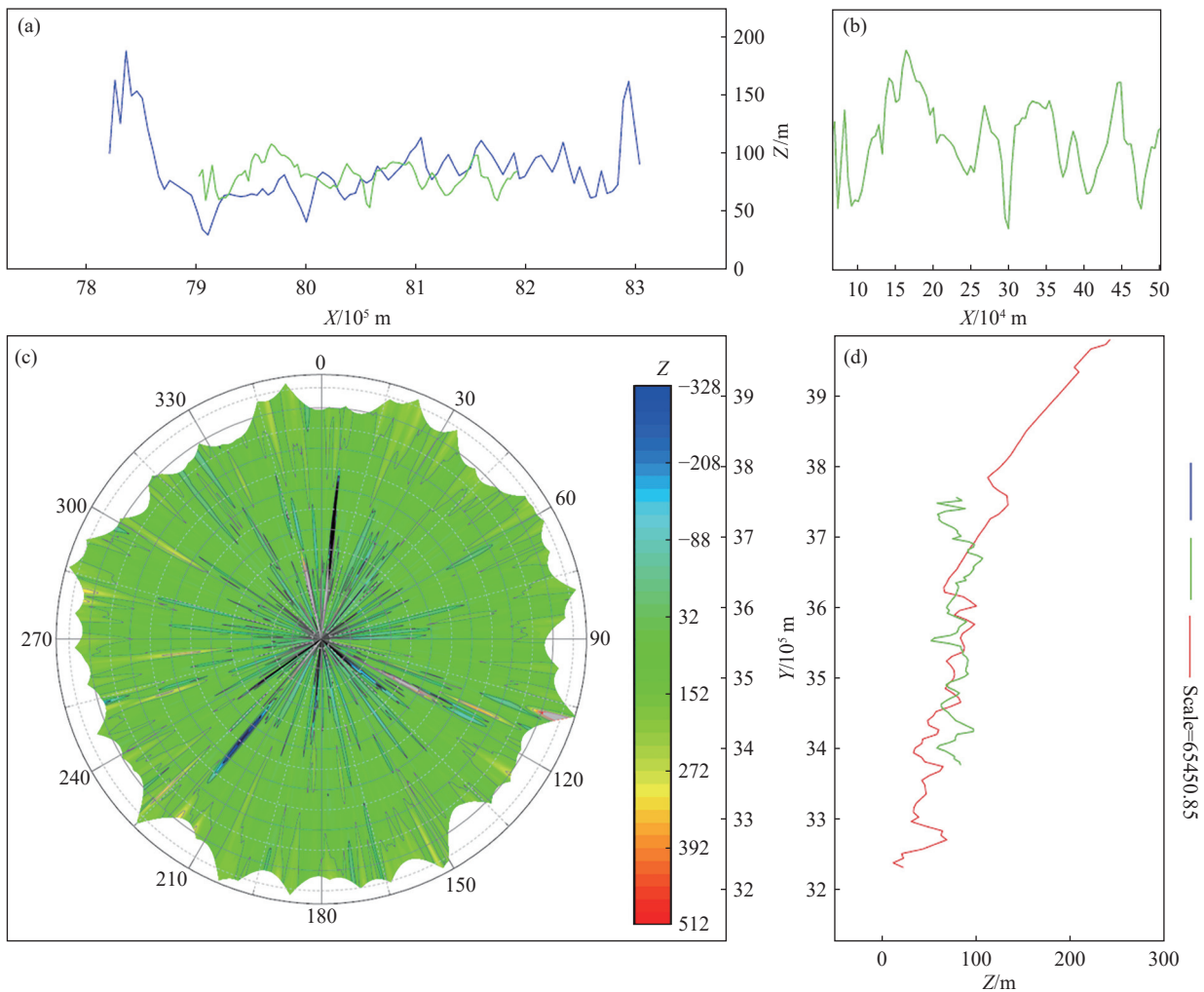


Fig. 8. a–Altitude variation along the X -axis; b–arbitrary line; c–polar contour map; d–altitude variation along the Y -axis.

but have limitations in accurately recreating small-scale features or intricate spatial structures. Subsequent investigations should prioritize analyzing multi-interleaved source site data, non-smooth geostatistical techniques, and machine learning (Lukjan A and Chalermyanont T, 2017; Marini M et al., 2018). The accuracy and reliability of the T-PROGS model were validated through comparison with measured data and historical sedimentary characteristics. However, further validation using independent datasets and additional field measurements could strengthen the model's credibility. Sensitivity analysis involves varying model parameters to assess their impact on the output. It can also provide insights into the model's robustness and highlight areas where more precise data is required. Future research should integrate more recent and broader datasets to overcome these limitations, possibly including real-time monitoring data to capture groundwater hydrodynamics. Applying and comparing hydrological models can also improve the understanding of groundwater. The impact of climate change on Punjab's groundwater resources should also be studied. Combining remote sensing data with powerful machine learning can improve the accuracy and resolution of the models. The research should also address the

socioeconomic impacts of groundwater use and management to create a more comprehensive approach to sustainable groundwater management. While the current 2D and 3D models provide valuable insights into aquifer heterogeneity, continuous improvement and validation are needed. By addressing the limitations and incorporating more comprehensive data and advanced methodologies, future studies can enhance the accuracy and reliability of hydrogeological models, contributing to more effective groundwater resource management.

3.5. Limitations of study

This study has several limitations, including a limited dataset and reliance on historical borehole data. Data from 1105 boreholes may not fully capture spatial variability across the region and may not reflect recent changes in groundwater use and management practices. In addition, the GAMEAS algorithm, while robust, has inherent limitations in accurately modeling complex groundwater hydrodynamics, particularly in heterogeneous aquifers. Accurately describing small-scale features and resolving uncertainties associated with limited data availability remain significant challenges. Regional models also require extensive input data, geostatistical and

hydrogeological expertise, and computationally intensive software. Future research should address these limitations by integrating more recent and vast datasets, possibly including real-time monitoring data to capture the hydrodynamics of groundwater systems. In addition, applying and comparing different hydrological models can provide a more comprehensive understanding of subsurface conditions. It would also be valuable to investigate the impacts of climate change on groundwater resources in Punjab. Exploring the integration of remote sensing data and advanced machine learning techniques could improve the accuracy and resolution of the models. Furthermore, expanding the scope of the study to include socioeconomic influences on groundwater use and management practices would provide a more comprehensive approach to sustainable groundwater management. Despite these limitations, this study has the potential to establish a fundamental reference point for future groundwater investigations and be valuable to potential customers, groundwater managers, sustainable management, and other research and resource purposes. By improving our understanding of aquifers' physical characteristics and hydraulic properties, this research contributes to the efficient use and protection of groundwater resources, ensuring long-term availability and quality for agricultural, industrial, and domestic uses.

3.6. Comparison of model results with published studies

To validate the reliability of model results, a comprehensive comparison will be made using the Markov chain methodology with findings from previous studies that employed relative modeling techniques. For instance, [Arif K and Khalid P \(2023\)](#) research focused on hydro-stratigraphy and hydro-geophysical techniques to identify fresh and saline aquifer boundaries by utilizing data from geological surveys and borehole samples to assess groundwater characteristics. Their findings corroborated the significance of aquifer heterogeneity in understanding groundwater behavior. It aligns with the current study's emphasis on aquifer heterogeneity and its implications for groundwater management using T-PROGS modeling. Similarly, [Khan S et al. \(2008\)](#) provided insights into the hydrogeological properties and groundwater potential of quaternary aquifers in the Indus basin, emphasizing the importance of accurate data collection and analysis for effective groundwater management. The current study's utilization of borehole data and geostatistical modeling techniques, as demonstrated in previous research, underscores the reliability of this approach in characterizing aquifer heterogeneity and assessing groundwater resources. Additionally, [Wapda \(2023\)](#) studies have contributed valuable information on the Punjab province's groundwater dynamics and aquifer properties. By integrating data from various sources and employing advanced modeling tools, these studies have enhanced our understanding of groundwater systems and supported evidence-based decision-making in water resource

management. Besides these, many other studies that include ([Langousis A et al., 2018](#); [Glennie R et al., 2023](#); [Gong K et al., 2023](#); [Grana D et al., 2023](#)) highlight the importance of T-PROGS modeling using the Markov chain. Hence, by referencing and building upon the findings of these previous studies, we reinforce the credibility and reliability of this study's results and analysis derived from T-PROGS modeling. This approach ensures that our research is grounded in established scientific knowledge and contributes to the broader understanding of groundwater dynamics and management practices in the Punjab region.

3.7. Application of T-progs Modelling

Transition probability geostatistical modeling is a powerful method for various geospatial applications, providing valuable insights into spatial processes where transitions between different states are critical. One prominent application is geological modeling, which helps characterize subsurface formations and predict spatial changes in lithology or rock properties. By modeling transition probabilities between different geological units such as sandstone, shale, and limestone, geologists can generate more accurate 3D models of the subsurface, thereby aiding mineral exploration and reservoir characterization in the oil and gas industry. Additionally, transition probability geostatistical models are applied in environmental science to study land cover change, vegetation dynamics, and habitat suitability. By analyzing the probability that different land cover types change over time, researchers can assess the impact of human activities, climate change, or natural disturbances on ecosystems and biodiversity. Additionally, this approach applies to hydrology and water resources management, helping to model spatial variations in groundwater contamination, nutrient transport, or flood risk. By incorporating transition probabilities into hydrological models, scientists and engineers can improve the accuracy of predictions and make more informed decisions for water allocation, pollution control, and flood prevention strategies. Overall, transition probability geostatistical modeling is a versatile tool for understanding and predicting spatial processes in geoscience and environmental applications, enabling more informed decision-making and resource management.

4. Conclusions

The research aims to promote sustainable water management practices using T-PROGS and groundwater level fluctuation techniques. It explores groundwater volume, comprehends aquifer heterogeneity and spatial variability, and identifies areas of concern for future prospects. The following conclusions can be drawn:

(i) The predicted model integrates regional datasets using GAMEAS, MCMOD, and TSIM algorithms to analyze aquifer properties. Sand material shows the highest self-transition (33.112 m), indicating strong stability, followed by silt, clay, and gravel materials. Overall, minimal transitions

suggest high stability of hydrofacies both horizontally and vertically. The results showed that the model's predictions, compared with actual material percentages, included error margins and average lens lengths. The model slightly under-predicted clay content by -0.750% with an average length of 15.985 m, accurately predicted silt with a 0.003% error, and over-predicted sand by 2.985% with a 33.112 m length. The model's predictions were fairly accurate within a reasonable error range.

(ii) Fine sand has the highest volume and thickness, accounting for nearly 58.77% of the aquifer material, and the alluvial environment is composed of nearly 55.78 highly porous sedimentary materials. Significant heterogeneity is observed in the northern areas near the mountains, with slight variations in the southern part. The model accurately predicts groundwater dynamics, sedimentation processes, and aquifer responses to tectonic movements and fluvial influences. The accuracy and reliability of the T-PROGS model were validated by comparing it with measured data and historical sedimentary characteristics.

(iii) Punjab is facing a severe water shortage, with groundwater reserves of 3502.3 BCM and a net recharge of 6826.9 MCM. The depth-to-water table ranges from 0.38–33.62 m, showing a significant decline from historical averages with an annual decrease of 0.5–1 m, highlighting a declining trend in water availability. Additionally, hydraulic conductivity values range from 1.3×10^{-3} – 32.80×10^{-3} m/s, indicating low permeability and limited groundwater replenishment.

(iv) Internal geological structures, climate variations, deforestation, and human activities affect aquifer deformation. To address this, the government should invest in rainwater harvesting infrastructure, implement strong groundwater policies, and promote sustainable practices. Key measures include advanced irrigation, extraction regulations, and on-site surveys to validate findings. These actions will help address challenges with current water level and conductivity data findings and improve future research methods.

CRedit authorship contribution statement

Muhammad Afzal and Tie Liu conceived of the presented idea. Muhammad Afzal carried out the experiment. All authors discussed the results and contributed to the final manuscript.

Declaration of competing interest

The authors declare no conflicts of interest.

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References

- Afzal M, Liu T, Butt AQ, Nadeem AA, Ali S, Pan X. 2023. Geospatial assessment of managed aquifer recharge potential sites in punjab, Pakistan. *Remote Sensing*, 15(16), 3988. doi: [10.3390/rs15163988](https://doi.org/10.3390/rs15163988).
- Akbar H, Nilsalab P, Silalertruksa T, Gheewala SH. 2023. An inclusive approach for integrated systems: Incorporation of climate in the water-food-energy-land nexus index. *Sustainable Production and Consumption*, 39, 42–52. doi: [10.1016/j.spc.2023.05.005](https://doi.org/10.1016/j.spc.2023.05.005).
- Akbar H, Nilsalab P, Silalertruksa T, Gheewala SH. 2022. Comprehensive review of groundwater scarcity, stress and sustainability index-based assessment. *Groundwater for Sustainable Development*, 18, 100782. doi: [10.1016/j.gsd.2022.100782](https://doi.org/10.1016/j.gsd.2022.100782).
- Akhter G, Ge Y, Hasan M, Shang Y. 2022. Estimation of hydrogeological parameters by using pumping, laboratory data, surface resistivity and Thiessen technique in lower Bari doab (Indus basin), Pakistan. *Applied Sciences*, 12(6), 3055. doi: [10.3390/app12063055](https://doi.org/10.3390/app12063055).
- Ali SK, Janjuhah HT, Shahzad SM, Kontakiotis G, Saleem MH, Khan U, Zarkogiannis SD, Makri P, Antonarakou A. 2021. Depositional sedimentary facies, stratigraphic control, paleoecological constraints, and paleogeographic reconstruction of Late Permian chhidru formation (western salt range, Pakistan). *Journal of Marine Science and Engineering*, 9(12), 1372. doi: [10.3390/jmse9121372](https://doi.org/10.3390/jmse9121372).
- Arif K, Khalid P. 2023. Hydrostratigraphy and hydrogeophysical studies to delineate fresh and saline aquifer boundaries in Lesser Cholistan of Pakistan. *PLoS One*, 18(10), e0292035. doi: [10.1371/journal.pone.0292035](https://doi.org/10.1371/journal.pone.0292035).
- Arshad A, Zhang Z, Zhang W, Dilawar A. 2020. Mapping favorable groundwater potential recharge zones using a GIS-based analytical hierarchical process and probability frequency ratio model: A case study from an agro-urban region of Pakistan. *Geoscience Frontiers*, 11, 1805–1819. doi: [10.1016/j.gsf.2019.12.013](https://doi.org/10.1016/j.gsf.2019.12.013).
- Awais M, Arshad M, Ahmad SR, Nazeer A, Waqas MM, Aziz R, Shakoor A, Rizwan M, Chaudhary JN, Mehmood Q, Ahmad M. 2022. Simulation of groundwater flow dynamics under different stresses using MODFLOW in rechna doab, Pakistan. *Sustainability*, 15(1), 661. doi: [10.3390/su15010661](https://doi.org/10.3390/su15010661).
- Bakr MU. 2023. Geological Map of Pakistan (East and West Pakistan). ESDAC —European Commission, Joint research center European soil data. URL <https://esdac.jrc.ec.europa.eu/content/geological-map-pakistan-east-and-west-pakistan> 2024–01–25.
- Ballio F, Guadagnini A. 2004. Convergence assessment of numerical Monte Carlo simulations in groundwater hydrology. *Water Resources Research*, 40(4), e2003wr002876. doi: [10.1029/2003wr002876](https://doi.org/10.1029/2003wr002876).
- Butt AQ, Shangguan D, Afzal M, Haq F, Pervaiz A, Ali T. 2022. Assessment for Design Discharge and Layout Planning of Hydropower Potential of Attabad Lake, Gilgit-Baltistan, Pakistan. Conference: (ICHWR-22). URL; https://www.researchgate.net/publication/369989775_Assessment_for_Design_Discharge_and_Layout_Planning_of_Hydropower_Potential_of_Attabad_Lake_Gilgit-Baltistan_Pakistan.
- Butt AQ, Shangguan D, Waseem M, Haq FU, Ding Y, Mukhtar MA, Afzal M, Muhammad A. 2023. Ascertainment of hydropower potential sites using location search algorithm in hunza river basin,

- Pakistan. *Water*, 15(16), 2929. doi: [10.3390/w15162929](https://doi.org/10.3390/w15162929).
- Carle SF. 1999. T-PROGS: Transition Probability Geostatistical Software. Davis, California Press, Version 2.1.
- Carle SF, Labolle EM, Weissmann GS, Brocklin DV, Fogg GE. 1998. Conditional simulation of hydrofacies architecture: A transition probability/Markov approach. *Hydrogeologic Models of Sedimentary Aquifers*. Tulsa, Oklahoma, U. S. A: SEPM Society for Sedimentary Geology, 147–170. doi: [10.2110/sepmcheg.01.147](https://doi.org/10.2110/sepmcheg.01.147).
- Chandna A, Srinivasan S. 2023. Modeling natural fracture networks and data assimilation using multipoint geostatistics and machine learning-based geomechanical inferences. *Amsterdam, Elsevier*, 6, 57–82. doi: [10.1016/b978-0-323-99593-1.00007-0](https://doi.org/10.1016/b978-0-323-99593-1.00007-0).
- Datta D, Singh G, Singh SK, Jenamani M, Routray A. 2023. Application of multivariate change detection in automated lithofacies classification from well-log data in a nonstationary subsurface. *Journal of Applied Geophysics*, 215, 105094. doi: [10.1016/j.jappgeo.2023.105094](https://doi.org/10.1016/j.jappgeo.2023.105094).
- de Almeida JA. 2010. Stochastic simulation methods for characterization of lithoclasses in carbonate reservoirs. *Earth-Science Reviews*, 101(3–4), 250–270. doi: [10.1016/j.earscirev.2010.05.002](https://doi.org/10.1016/j.earscirev.2010.05.002).
- Dixit N, McColgan P, Kusler K. 2020. Machine learning-based probabilistic lithofacies prediction from conventional well logs: A case from the umiat oil field of Alaska. *Energies*, 13(18), 4862. doi: [10.3390/en13184862](https://doi.org/10.3390/en13184862).
- Eramian MG, Schincariol RA, Mansinha L, Stockwell RG. 1999. Generation of aquifer heterogeneity maps using two-dimensional spectral texture segmentation techniques. *Mathematical Geology*, 31(3), 327–348. doi: [10.1023/A:1007578305616](https://doi.org/10.1023/A:1007578305616).
- Glennie R, Adam T, Leos-Barajas V, Michelot T, Photopoulou T, McClintock BT. 2023. Hidden Markov models: Pitfalls and opportunities in ecology. *Methods in Ecology and Evolution*, 14(1), 43–56. doi: [10.1111/2041-210x.13801](https://doi.org/10.1111/2041-210x.13801).
- GMS A. 2024. GMS 10.8 Tutorials | Aquaveo. com. Groundwater modelling system. URL: <https://www.aquaveo.com/software/gms-learning-tutorials> (2024–01–26).
- Gong K, Wen Z, Li Q, Zhu Q. 2023. Geostatistical simulations of the spatial variability of hydraulic conductivity in an alluvial-marine sedimentary system in Beihai City, China. *Journal of Hydrology*, 620, 129528. doi: [10.1016/j.jhydrol.2023.129528](https://doi.org/10.1016/j.jhydrol.2023.129528).
- Grana D, de Figueiredo L, Mosegaard K. 2023. Markov chain Monte Carlo for seismic facies classification. *Geophysics*, 88(3), M131–M143. doi: [10.1190/geo2022-0442.1](https://doi.org/10.1190/geo2022-0442.1).
- Gray JM, Bishop TFA, Wilford JR. 2016. Lithology and soil relationships for soil modelling and mapping. *Catena*, 147, 429–440. doi: [10.1016/j.catena.2016.07.045](https://doi.org/10.1016/j.catena.2016.07.045).
- GSP G. 2023. Geological Survey of Punjab Pakistan. URL: <https://gsp.gov.pk/geology-of-pakistan/> (2024–01–26).
- Guo J, Wang Z, Li C, Li F, Jessell MW, Wu L, Wang J. 2022. Multiple-point geostatistics-based three-dimensional automatic geological modeling and uncertainty analysis for borehole data. *Natural Resources Research*, 31(5), 2347–2367. doi: [10.1007/s11053-022-10071-6](https://doi.org/10.1007/s11053-022-10071-6).
- Haq ZU, Ashraf M. 2023. From Groundwater Monitoring to Management: A Case Study from Central Punjab, Pakistan. *Pakistan Council of Research in Water Resources (PCRWR) Islamabad, Press*, 45.
- Harp DR, Dai Z, Wolfsberg AV, Vrugt JA, Robinson BA, Vesselinov VV. 2008. Aquifer structure identification using stochastic inversion. *Geophysical Research Letters*, 35(8), L08404. <https://doi.org/10.1029/2008GL033585>.
- He X, Højberg A, Jørgensen F, Refsgaard J. 2015. Assessing hydrological model predictive uncertainty using stochastically generated geological models. *Hydrological Processes*, 29, 4293–4311. doi: [10.1002/hyp.10488](https://doi.org/10.1002/hyp.10488).
- He X, Koch J, Sonnenborg TO, Jørgensen F, Schamper C, Christian Refsgaard J. 2014. Transition probability-based stochastic geological modeling using airborne geophysical data and borehole data. *Water Resources Research*, 50(4), 3147–3169. doi: [10.1002/2013wr014593](https://doi.org/10.1002/2013wr014593).
- Hu LY, Chugunova T. 2008. Multiple-point geostatistics for modeling subsurface heterogeneity: A comprehensive review. *Water Resources Research*, 44(11), e2008wr006993. doi: [10.1029/2008wr006993](https://doi.org/10.1029/2008wr006993).
- Illman WA, Zhu J, Craig AJ, Yin D. 2010. Comparison of aquifer characterization approaches through steady state groundwater model validation: A controlled laboratory sandbox study. *Water Resources Research*, 46(4), e2009wr007745. doi: [10.1029/2009wr007745](https://doi.org/10.1029/2009wr007745).
- Javeed B, Huang D, Shangguan D, Ahsan Mukhtar M, Sajjad W, Banerjee A, Yang Q, Butt AQ. 2024. Assessing the effectiveness of National Park's policies and laws in promoting biodiversity conservation and ecological development in Pakistan. *Frontiers in Environmental Science*, 11, 1333650. doi: [10.3389/fenvs.2023.1333650](https://doi.org/10.3389/fenvs.2023.1333650).
- Khan S, Rana T, Gabriel HF, Ullah MK. 2008. Hydrogeologic assessment of escalating groundwater exploitation in the Indus Basin, Pakistan. *Hydrogeology Journal*, 16(8), 1635–1654. doi: [10.1007/s10040-008-0336-8](https://doi.org/10.1007/s10040-008-0336-8).
- Khan S, Rana T, Ullah K, Christen E, Nafees M. 2003. Investigating conjunctive water management options using a dynamic surface-groundwater modelling approach: A case study of Rechna Doab. *Transport*. URL: https://www.researchgate.net/publication/237775472_Conjunctive_water_management_in_the_Rechna_Doab_An_overview_of_resources_and_issues.
- Khattak MS, Ali S. 2015. Assessment of temperature and rainfall trends in Punjab province of Pakistan for the period 1961–2014. *Journal of Himalayan Earth Sciences* 48, 42. doi: [1792584624/se-2](https://doi.org/10.1792584624/se-2).
- Langousis A, Kaleris V, Kokosi A, Mamounakis G. 2018. Markov based transition probability geostatistics in groundwater applications: Assumptions and limitations. *Stochastic Environmental Research and Risk Assessment*, 32(7), 2129–2146. doi: [10.1007/s00477-017-1504-y](https://doi.org/10.1007/s00477-017-1504-y).
- Lukjan A, Chalermyanont T. 2017. Assessment of alluvial aquifer heterogeneity and development of stochastic hydrofacies models for the Hat Yai Basin in Southern Thailand. *Environmental Earth Sciences*, 76(8), 316. doi: [10.1007/s12665-017-6637-2](https://doi.org/10.1007/s12665-017-6637-2).
- Malkani MS, Haroon Y. 2022. Lithostratigraphy, structure, geological history, economic geology and paleontology of mari bugti hills and surrounding areas of balochistan, south punjab and North Sindh (Pakistan). *Open Journal of Geology*, 12(1), 13–56. doi: [10.4236/ojg.2022.121002](https://doi.org/10.4236/ojg.2022.121002).
- Marini M, Felletti F, Beretta GP, Terrenghi J. 2018. Three geostatistical methods for hydrofacies simulation ranked using a large borehole lithology dataset from the Venice hinterland (NE Italy). *Water*, 10(7), 844. doi: [10.3390/w10070844](https://doi.org/10.3390/w10070844).
- Mizutani D, Lethanh N, Adey BT, Kaito K. 2017. Improving the estimation of Markov transition probabilities using mechanistic-empirical models. *Frontiers in Built Environment*, 3, 58. doi: [10.3389/fbuil.2017.00058](https://doi.org/10.3389/fbuil.2017.00058).
- Muhammad S, Ehsan MI, Khalid P, Sheikh A. 2023a. Hydrogeophysical modeling and physio-chemical analysis of quaternary aquifer in central part of Bari Doab, Punjab, Pakistan. *Modeling Earth Systems and Environment*, 9(2), 1427–1443. doi: [10.1007/s40808-022-01552-x](https://doi.org/10.1007/s40808-022-01552-x).
- Muhammad S, Khalid P, Ehsan MI, Qureshi J, Farooq S. 2023b. Evaluation of aquifer parameters through integrated approach of geophysical investigations, pumping test analysis and Dar-Zarrouk parameters in the central part of Bari Doab, Punjab, Pakistan. *Environmental Monitoring and Assessment*, 195(12), 1435. doi: [10.1007/s10661-023-12049-0](https://doi.org/10.1007/s10661-023-12049-0).
- Nabi G, Ali M, Khan S, Kumar S. 2019. The crisis of water shortage and pollution in Pakistan: Risk to public health, biodiversity, and ecosystem. *Environmental Science and Pollution Research*, 26(11), 10443–10445. doi: [10.1007/s11356-019-04483-w](https://doi.org/10.1007/s11356-019-04483-w).

- Nadeem A, Zha Y, Shi L, Zafar Z, Ali S, Zhang Y, Altaf AR, Afzal M, Zubair M, Fernández J. 2023. SAFER-ET based assessment of irrigation patterns and impacts on groundwater use in the central Punjab, Pakistan. *Agricultural Water Management*, 289, 108545. doi: [10.1016/j.agwat.2023.108545](https://doi.org/10.1016/j.agwat.2023.108545).
- Naranjo-Fernández N, Guardiola-Albert C, Montero-González E. 2019. Applying 3D geostatistical simulation to improve the groundwater management modelling of sedimentary aquifers: The case of doñana (southwest Spain). *Water*, 11(1), 39. doi: [10.3390/w11010039](https://doi.org/10.3390/w11010039).
- Naveendrakumar G, Vithanage M, Kwon HH, Chandrasekara SSK, Iqbal MCM, Pathmarajah S, Fernando WCDK, Obeysekera J. 2019. South Asian perspective on temperature and rainfall extremes: A review. *Atmospheric Research*, 225, 110–120. doi: [10.1016/j.atmosres.2019.03.021](https://doi.org/10.1016/j.atmosres.2019.03.021).
- Pan X, Wang W, Liu T, Akmalov S, De Maeyer P, Van de Voorde T. 2022. Integrated modeling to assess the impact of climate change on the groundwater and surface water in the South Aral Sea area. *Journal of Hydrology*, 614, 128641. doi: [10.1016/j.jhydrol.2022.128641](https://doi.org/10.1016/j.jhydrol.2022.128641).
- Pan X, Wang W, Liu T, Bao A, Chen X, Akmalov S, De Maeyer P, Van de Voorde T. 2023. Modeling the effects of improved irrigation methods in a groundwater system: A case study from the amu darya delta, Uzbekistan. *Journal of Hydrology*, 625, 129987. doi: [10.1016/j.jhydrol.2023.129987](https://doi.org/10.1016/j.jhydrol.2023.129987).
- Papapetrou M, Theodossiou N. 2010. Stochastic analysis of the geological structure of groundwater aquifers. doi: [228025e1n=en](https://doi.org/10.1016/j.jhydrol.2010.03.021)
- PBS. 2019. Pakistan Bureau of Statistics. URL <https://www.pbs.gov.pk/publication/pakistan-statistical-year-book-2019>(2024–01–25).
- PBS. 2017. Pakistan Bureau of Statistics. URL <https://www.pbs.gov.pk/>(2023–04–13).
- Razzaq A, Liu H, Xiao M, Mehmood K, Shahzad MA, Zhou Y. 2023. Analyzing past and future trends in Pakistan's groundwater irrigation development: Implications for environmental sustainability and food security. *Environmental Science and Pollution Research*, 30(12), 35413–35429. doi: [10.1007/s11356-022-24736-5](https://doi.org/10.1007/s11356-022-24736-5).
- Rossignoli CM, Obi C, Ali SA, Ullah N, Khalid S, Hafeez M, Shah SMH. 2023. Production system and challenges of saline aquaculture in Punjab and Sindh provinces of Pakistan. *Frontiers in Aquaculture*, 2, 1302571. doi: [10.3389/faq.2023.1302571](https://doi.org/10.3389/faq.2023.1302571).
- Saraswat A, Nath T, Omeka ME, Unigwe CO, Anyanwu IE, Ugar SI, Latore A, Raza MB, Behera B, Adhikary PP, Scopa A, AbdelRahman MAE. 2023. Irrigation suitability and health risk assessment of groundwater resources in the firozabad industrial area of north-central India: An integrated indexical, statistical, and geospatial approach. *Frontiers in Environmental Science*, 11, 1116220. doi: [10.3389/fenvs.2023.1116220](https://doi.org/10.3389/fenvs.2023.1116220).
- Ullah J, Li H, Ashraf U, Ehsan M, Muhammad A. 2023. A multidisciplinary approach to facies evaluation at regional level using well log analysis, machine learning, and statistical methods. *Geomechanics and Geophysics for Geo-Energy and Geo-Resources*, 9(1), 152. doi: [10.1007/s40948-023-00689-y](https://doi.org/10.1007/s40948-023-00689-y).
- Tayyab A. 2016. Pakistan: Institutional Transformation of the Punjab Irrigation Department to a Water Resources Department. doi: [386818/pakistan/1351305/](https://doi.org/10.3389/fenvs.2023.1116220).
- Sun Q, Shao J, Wang Y, Ma T. 2019. Research on appropriate borehole density for establishing reliable geological model based on quantitative uncertainty analysis. *Arabian Journal of Geosciences*, 12(13), 410. doi: [10.1007/s12517-019-4533-7](https://doi.org/10.1007/s12517-019-4533-7).
- Wapda. 2023. Water and Power Development Authority. URL <https://www.wapda.gov.pk/>(2024–01–26).
- Xue P, Wen Z, Park E, Jakada H, Zhao D, Liang X. 2022. Geostatistical analysis and hydrofacies simulation for estimating the spatial variability of hydraulic conductivity in the Jiangnan Plain, Central China. *Hydrogeology Journal*, 30(4), 1135–1155. doi: [10.1007/s10040-022-02495-4](https://doi.org/10.1007/s10040-022-02495-4).
- Yonus M, Jan B, Khan H, Nawaz F, Ali M. 2023. Study the seasonal trend analysis and probability distribution functions of rainfall for atmospheric region of Pakistan. *MethodsX*, 10, 102058. doi: [10.1016/j.mex.2023.102058](https://doi.org/10.1016/j.mex.2023.102058).
- Zakir-Hassan G, Akhtar S, Shabir G, Hassan FR, Ashraf H, Sultan M. 2023. Water budget study for groundwater recharge in Indus River Basin, Punjab (Pakistan). *H2Open Journal*, 6(3), 449–462. doi: [10.2166/h2oj.2023.027](https://doi.org/10.2166/h2oj.2023.027).
- Zhang ZY, Xu D, Gong CC, Ran B, Wang XK, Zhang WY, Pan JZ. 2025. Finite analytic method for simulating water flow using water content-based Richards' equation. *Journal of Groundwater Science and Engineering*, 13(2): 147–155. doi: [10.26599/JGSE.2025.9280045](https://doi.org/10.26599/JGSE.2025.9280045).