DOI: 10.19884/j.1672-5220.202503005

Prescribed-Time Group-Bipartite Consensus for Second-Order Nonlinear Multi-Agent Systems

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Abstract: The problem of prescribed-time group-bipartite leader-follower consensus for second-order nonlinear multi-agent systems (MASs) is investigated over a signed communication topology. First, by selecting a time-varying function as the control gain, we propose a novel control protocol for the case where each sub-topology is structurally balanced. By introducing an additional gain parameter into the controller, we simplify the sufficient conditions for the stability of the closed-loop system, and the calculations of parameters are more straightforward. Second, by incorporating leader dynamics into the controller, we relax the topological constraints and only require the overall topology of the MAS to be structurally balanced, and propose a novel consensus protocol based on neighbor information and leader dynamics. Based on matrix theory and Lyapunov stability theory, we derive sufficient conditions for achieving consensus. Finally, two numerical simulation examples are provided to validate the effectiveness of the two control protocols.

Keywords: group-bipartite consensus; multi-agent system (MAS); prescribed-time; signed graph
CLC number: O231; TP18 **Document code**: A
Article ID: 1672-5220(2026)04-0133-09

0 Introduction

Distributed cooperative control of multi-agent systems (MASs) has been extensively applied in many fields, such as aircraft formation control[1], sensor networks[2], and group trajectory simulation[3]. Consensus is the most fundamental issue in cooperative control, and its focus is on designing protocols to ensure that the states of all agents converge to a common value. At present, a large number of studies have investigated the consensus problem from different perspectives, such as the leaderless consensus[4-5] and leader-follower consensus[6-7].

However, most of the existing studies are based on cooperative networks, where the communication weights between agents are nonnegative. In fact, agents may also have competitive relationships, such as social networks[8] and trust networks[9]. Altafini[10] creatively proposed the concept of bipartite consensus on a signed graph, where positive and negative weights represent cooperative and competitive relations, respectively. Subsequently, growing research interest has been directed toward bipartite consensus[11-13]. Group consensus is also an important clustering behavior that can be achieved on a signed graph, where agents are partitioned into multiple subgroups, and different subgroups reach distinct consensus states[14-16]. Although bipartite consensus and group consensus share some similarities, their requirements for topology differ significantly. Specifically, to achieve bipartite consensus, the structural balance condition of the communication topology is essential. The topology for group consensus typically requires the inter-group condition or possesses an acyclic partition structure. Moreover, agents within the same subgroup still only have cooperative relationships. Liu et al.[17] combined group consensus and bipartite consensus to further propose the concept of group-bipartite consensus, where agents within the same subgroup can also have competitive relationships. In Ref. [18], the group-bipartite consensus of heterogeneous MASs was investigated, where the global topology was only required to satisfy the structural balance condition.

The consensus discussed in the aforementioned literature pertains to asymptotic stability, where convergence is achieved as time approaches infinity. However, in practical engineering applications, the system must reach consensus within a finite time. Lu et al.[19] studied the finite-time group-bipartite consensus of second-order nonlinear MASs by using the homogeneous dilation method. Zhou et al.[20] investigated the finite-time lag-consensus problems of second-order multi-agent networks that have inherent nonlinear dynamics and bounded external disturbances. Hao et al.[21] explored the fixed-time group consensus of nonlinear MASs via pinning control. For finite-time consensus and fixed-time consensus, the upper bound of the

Received date: 2025-03-05
Foundation item: National Natural Science Foundation of China (No. 12071370)
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Citation: Zhao Y, Kang J L. Prescribed-time group-bipartite consensus for second-order nonlinear multi-agent systems [J]. *Journal of Donghua University (English Edition)*, 2026, 43(4): 133-141.

convergence time depends on the initial values of the system. Although the upper bound of the convergence time for fixed-time consensus does not depend on the initial values of the system, it typically depends on system parameters. By contrast, the settling time of prescribed-time consensus is a predefined constant, which makes it more practical. Ren et al. [22] discussed the prescribed-time lag consensus of second-order nonlinear MASs, and extended the results to the cluster lag consensus problem. In Ref. [23], a novel time-varying function-based control scheme was developed to achieve prescribed-time bipartite consensus. Zhao et al. [24] investigated the appointed-time and attack-free bipartite synchronization problem for generic linear MASs and proposed an innovative prescribed-time bipartite consensus protocol. Lv et al.[25] studied the prescribed-time leaderless and leader-follower consensus for single-integrator MASs over both fixed and switching topologies.

Inspired by the above studies, we investigate the prescribed-time group-bipartite consensus for second-order nonlinear MASs over a directed signed graph. Compared with the existing results, the contributions of this study are as follows.

1) Different from the finite-time group-bipartite consensus shown in Ref. [19], we studied the prescribed-time group-bipartite consensus. For two different topological structures, we designed two distinct control protocols, each enabling the system to achieve consensus within a prescribed time.

2) Compared with the linear systems studied in Refs. [17] and [18], we considered second-order nonlinear MASs, which were more general. Moreover, as special cases, the prescribed-time group and bipartite consensus of second-order nonlinear MASs could also be addressed by the control protocol in this study.

3) Compared with Ref. [22], we adjusted the method of parameter selection, thereby reducing the computational burden.

The rest of this paper is organized as follows. Preliminaries on the graph theory and the statement of the related consensus control problem are described in Section 1. The main proof and corresponding results are presented in Section 2. Numerical simulation examples are shown in Section 3. Conclusions are drawn in Section 4.

Notations: $\mathbf{R}^n$ denotes an n dimensional Euclidean space; $\mathbf{R}^{n\times m}$ is an $n \times m$ dimensional matrix space; $\mathbf{1}_n \in \mathbf{R}^n$ represents an n-dimensional column vector with all elements equal to 1; $\boldsymbol{M}^{\mathrm{T}}$ is the transpose of matrix $\boldsymbol{M}$; for a matrix $\boldsymbol{M} \in \mathbf{R}^n$, $\lambda_{\min}(\boldsymbol{M})$ and $\lambda_{\max}(\boldsymbol{M})$ are the smallest and the largest eigenvalues of $\boldsymbol{M}$, respectively; $\mathrm{diag}(a_1, a_2, \cdots, a_n)$ denotes a diagonal matrix; $\mathrm{blkdiag}(\boldsymbol{A}_1, \boldsymbol{A}_2, \cdots, \boldsymbol{A}_n)$ is a block diagonal matrix; for matrix $\boldsymbol{M}$, $\boldsymbol{M} > 0$ means that $\boldsymbol{M}$ is positive definite, and $\|\boldsymbol{M}\|$ denotes the Euclidean norm of matrix $\boldsymbol{M}$; $\mathrm{sgn}(\cdot)$ is a sign function, $\mathrm{sgn}(x)=1$, if $x > 0$, $\mathrm{sgn}(x)=-1$, if $x < 0$ and $\mathrm{sgn}(x) = 0$, if $x = 0$; $\exp(\cdot)$ refers to the exponential function.

1 Preliminaries and Problem Formulation

1.1 Preliminaries

For a given MAS, suppose that the communication topology between agents can be represented by a directed graph $\mathcal{G}=(\mathcal{V}, \varepsilon, \boldsymbol{A})$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the vertex set and $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. $\boldsymbol{A} = [a_{ij}]_{N\times N}$ denotes the weighted adjacency matrix of $\mathcal{G}$. If there exists an edge $(j, i) \in \varepsilon$, the element $a_{ij} \neq 0$; otherwise, $a_{ij} = 0$. A directed path from i to j in $\mathcal{G}$ is a sequence of edges $(i, i_1), (i_1, i_2), \cdots, (i_k, j)$, where $i_1, i_2, \cdots, i_k$ are distinct nodes. If there exists a directed path between any two vertices, then the graph $\mathcal{G}$ is said to be strongly connected.

Definition 1[10] A connected signed graph $\mathcal{G}=(\mathcal{V}, \varepsilon, \boldsymbol{A})$ is structurally balanced if the node set $\mathcal{V}$ can be partitioned into two subsets as $\{\mathcal{V}_1, \mathcal{V}_2\}$ such that $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$. For any two nodes i and j, $a_{ij} \geqslant 0$, if i and j are from the same set $\mathcal{V}_k$, $k = 1, 2$; otherwise, $a_{ij} \leqslant 0$.

Lemma 1[10] Suppose that the connected signed graph $\mathcal{G}=(\mathcal{V}, \varepsilon, \boldsymbol{A})$ is structurally balanced with the partition $\{\mathcal{V}_1, \mathcal{V}_2\}$ according to Definition 1. Then there exists a diagonal matrix $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \cdots, d_N)$, $d_j = 1$, if $j \in \mathcal{V}_1$ and $d_j = -1$, if $j \in \mathcal{V}_2$, such that $\boldsymbol{DAD}$ has all nonnegative entries.

In this paper, the graph $\overline{\mathcal{G}}=(\overline{\mathcal{V}}, \overline{\varepsilon}, \overline{\boldsymbol{A}})$ is adopted to describe the communication topology among N followers. Suppose that the node set $\overline{\mathcal{V}} = \{1, 2, \cdots, N\}$ can be partitioned into different subsets $\{\overline{\mathcal{V}}_1, \overline{\mathcal{V}}_2, \cdots, \overline{\mathcal{V}}_m\}$ such that $\cup_{i=1}^{m} \overline{\mathcal{V}}_i = \overline{\mathcal{V}}$ and $\overline{\mathcal{V}}_p \cap \overline{\mathcal{V}}_q = \emptyset$ for $p \neq q$. Without loss of generality, each $\overline{\mathcal{V}}_i$ is indexed as $\overline{\mathcal{V}}_i = \{M_{i-1} + 1, M_{i-1} + 2, \cdots, M_i\}$, where $M_i = \sum_{k=1}^{i} N_k$, $i = 1, 2, \cdots, m$, $M_0 = 0$. Here, N_k denotes the number of nodes contained in the subset $\overline{\mathcal{V}}_k$ for each $k = 1, 2, \cdots, m$. By the completeness of the partition, we naturally have $\sum_{k=1}^{m} N_k = N$. Let $\mathcal{G}_i$ denote the underlying topology of the node subsets $\overline{\mathcal{V}}_i$, $i = 1, 2, \cdots, m$. Then the adjacency matrix $\overline{\boldsymbol{A}}$ can be rewritten in a block matrix form as

$$\overline{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \cdots & \boldsymbol{A}_{1m} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \cdots & \boldsymbol{A}_{2m} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{A}_{m1} & \boldsymbol{A}_{m2} & \cdots & \boldsymbol{A}_{mm} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{A}_{ii} \in \mathbf{R}^{N_i\times N_i}$ is the adjacency matrix of $\mathcal{G}_i$ and $\boldsymbol{A}_{ij} \in \mathbf{R}^{N_i\times N_j}$ represents the communication weights between agents in $\overline{\mathcal{V}}_i$ and $\overline{\mathcal{V}}_j$. For leader-follower consensus, suppose that each group has one leader, and

for convenience, the leader of $\overline{\mathcal{V}}_i$ is labeled with index $N+i$. Denote the leader set as $\mathcal{V}_{\mathrm{L}}=\{N+1, N+2, \cdots, N+m\}$. Let $\bar{i} \in \mathcal{V}_{\mathrm{L}}$ denote the leader of the group to which agent i belongs, i. e., $\forall i \in \overline{\mathcal{V}}, k \in \{1, 2, \cdots, m\}$, if $i \in \overline{\mathcal{V}}_k$, then $\bar{i}=N+k$. The graph $\overline{\mathcal{G}}_i$ is the underlying topology of the node set $\overline{\mathcal{V}}_i \cup \{N+i\}$, $i=1, 2, \cdots, m$.

The neighbor sets of agent i are defined as $N_i^{\mathrm{s}}=\{j \mid (j, i) \in \varepsilon, \bar{i}=\bar{j}\}$ and $N_i^{\mathrm{d}}=\{j \mid (j, i) \in \varepsilon, \bar{i} \neq \bar{j}\}$. N_i^{s} represents the set of neighbors of agent i where all agents in this set belong to the same subgroup as agent i, while N_i^{d} represents the set of neighbors that belong to subgroups different from those of agent i.

Define $\boldsymbol{B}=\mathrm{blkdiag}(\boldsymbol{B}_1, \boldsymbol{B}_2, \cdots, \boldsymbol{B}_m)$, $\boldsymbol{B}_i=\mathrm{diag}(b_{M_{i-1}+1}, b_{M_{i-1}+2}, \cdots, b_{M_i})$, $i=1, 2, \cdots, m$. If the agent i can receive the information from the leader $\bar{i}$, then $b_i>0$; otherwise, $b_i=0$.

Without loss of generality, we take $t=0$ s as the initial time instant. A time-varying function used for the controller throughout this paper is defined as

$$\varphi(t,T)=\begin{cases}\mu(t, T), & t \in [0, T),\\ 0, & t \in [T, +\infty),\end{cases} \tag{2}$$

where $T>0$ is a prescribed time. For $t \in [0, T)$, $\mu(t, T)$ and its primitive function $\bar{\mu}(t, T)$ satisfy the following conditions:

1) $\mu(t, T)$ and $\bar{\mu}(t, T)$ monotonically increase;

2) $0<\mu(0, T)<+\infty$ and $0<\bar{\mu}(0, T)<+\infty$;

3) $\mu(t, T) \to +\infty$ and $\bar{\mu}(t, T) \to +\infty$ as $t \to T$;

4) $\lim\limits_{t \to T} \mu^2(t, T)\exp(-k\bar{\mu}(t, T))<+\infty, \forall k>0$.

We choose $\mu(t,T)=\theta/(T-t)$ and $\bar{\mu}(t,T)=-\theta\ln(T-t)+C$, where $C>0$ is chosen sufficiently large to maintain $\bar{\mu}(0,T)>0$ and $\theta>2$ is a constant. It is straightforward to verify that $\mu(t, T)$ and $\bar{\mu}(t,T)$ satisfy the above conditions. Correspondingly,

$$\varphi(t, T)=\begin{cases}\theta/(T-t), & t \in [0, T),\\ 0, & t \in [T, +\infty).\end{cases} \tag{3}$$

Lemma 2[25] Consider the following system

$$\dot{x}(t)=g(t, x(t)), \tag{4}$$

where $x(t) \in \mathbf{R}^m$ is the state for the system; $g: [0, \infty) \times \mathbf{R}^m \mapsto \mathbf{R}^m$ is a nonlinear vector bounded in time. Let $V(t, x(t))$ be a Lyapunov function of system (4) such that $V(t, 0)=0$, $V(t, x(t))>0$, $\forall x(t) \neq 0$. If $\dot{V}$ satisfies

$$\dot{V} \leqslant -k\varphi(t)V+bV, \tag{5}$$

where $b \in \mathbf{R}$, $k>0$, and $\varphi(t)$ is defined in Eq. (3), then there exist the following characteristics:

$$\begin{cases}V \leqslant \exp(-k\bar{\mu}(t, T)+k\bar{\mu}(0, T)+bt), & t \in [0, T),\\ V \equiv 0, & t \in [T, +\infty).\end{cases} \tag{6}$$

Lemma 3[26] Let $\boldsymbol{M} \in \mathbf{R}^{n \times n}$ be a nonsingular M-matrix, then there exists a diagonal matrix $\boldsymbol{W}>\boldsymbol{0}$ such that $\boldsymbol{M}^{\mathrm{T}}\boldsymbol{W}+\boldsymbol{W}\boldsymbol{M}>\boldsymbol{0}$.

Lemma 4[27] Let $\lambda_i(\boldsymbol{A})$, $\lambda_i(\boldsymbol{B})$, and $\lambda_i(\boldsymbol{A}+\boldsymbol{B})$ denote the eigenvalues of the symmetric matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{A}+\boldsymbol{B}$, respectively, arranged in ascending order such that

$$\lambda_1(\cdot) \leqslant \lambda_2(\cdot) \leqslant \cdots \leqslant \lambda_n(\cdot). \tag{7}$$

Then, for each $k=1, 2, \cdots, n$, the following inequality holds:

$$\lambda_k(\boldsymbol{A})+\lambda_1(\boldsymbol{B}) \leqslant \lambda_k(\boldsymbol{A}+\boldsymbol{B}) \leqslant \lambda_k(\boldsymbol{A})+\lambda_n(\boldsymbol{B}). \tag{8}$$

1.2 Problem formulation

Consider the following nonlinear system, the dynamics of followers can be represented as

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t)+f(t, x_i(t), v_i(t)), & i \in \overline{\mathcal{V}},\end{cases} \tag{9}$$

where $x_i(t)$, $v_i(t)$, $u_i(t) \in \mathbf{R}^n$ are the position, velocity, and control input of the agent i at time t. $f(t, x_i(t), v_i(t)): [0, +\infty) \times \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^n$ describes the nonlinear characteristics of the system and satisfies Assumption 1 to be given later, the leaders' dynamic equation is described as

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=f(t, x_i(t), v_i(t)), & i \in \mathcal{V}_{\mathrm{L}}.\end{cases} \tag{10}$$

Assumption 1 For any $x_1, x_2, y_1, y_2 \in \mathbf{R}^n$, $-f(t, x_1, y_1)=f(t, -x_1, -y_1)$, and there always exist $r_1, r_2>0$ such that $\|f(t, x_1, x_2)-f(t, y_1, y_2)\| \leqslant r_1\|x_1-x_2\|+r_2\|y_1-y_2\|$.

Remark 1 For simplicity, in this paper, we only consider the one-dimensional case, i. e., $n=1$. Therefore, the absolute value is used instead of the norm in Assumption 1. The higher-dimensional case can be extended through the Kronecker product as well.

Assumption 2 Each sub-topology $\mathcal{G}_i$ is structurally balanced and strongly connected, and at least one follower in $\overline{\mathcal{V}}_i$ can receive the information from the leader $N+i$.

Under Assumption 2, for each sub-topology $\mathcal{G}_i$ ($i=1, 2, \cdots, m$), the node subset $\overline{\mathcal{V}}_i$ of $\mathcal{G}_i$ can be partitioned into two subsets $\overline{\mathcal{V}}_{i1}$ and $\overline{\mathcal{V}}_{i2}$, such that $\overline{\mathcal{V}}_{i1} \cup \overline{\mathcal{V}}_{i2}=\overline{\mathcal{V}}_i$ and $\overline{\mathcal{V}}_{i1} \cap \overline{\mathcal{V}}_{i2}=\varnothing$. From Lemma 1, there exists a diagonal matrix $\boldsymbol{D}_i=\mathrm{diag}(d_{M_{i-1}+1}, d_{M_{i-1}+2}, \cdots, d_{M_i})$, $d_k=1$, if $k \in \overline{\mathcal{V}}_{i1}$ and $d_k=-1$ if $k \in \overline{\mathcal{V}}_{i2}$, $k \in \{M_{i-1}+1, M_{i-1}+2, \cdots, M_i\}$, such that $\boldsymbol{D}_i\boldsymbol{A}_{ii}\boldsymbol{D}_i$ has all nonnegative entries.

Definition 2 The MASs (9) and (10) are considered to achieve prescribed-time group-bipartite consensus if, for any initial states and a prescribed time $T>0$, the following conditions are satisfied

$$\begin{cases}\lim\limits_{t\to T}\|x_i(t)+x_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k1},\\ \lim\limits_{t\to T}\|x_i(t)-x_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k2},\\ \lim\limits_{t\to T}\|v_i(t)+v_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k1},\\ \lim\limits_{t\to T}\|v_i(t)-v_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k2},\\ \|x_i(t)+x_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k1},\ t\geqslant T,\\ \|x_i(t)-x_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k2},\ t\geqslant T,\\ \|v_i(t)+v_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k1},\ t\geqslant T,\\ \|v_i(t)-v_{\tilde{i}}(t)\|=0,\ \forall i\in\overline{\mathcal{V}}_{k2},\ t\geqslant T,\end{cases}\tag{11}$$

where $i\in\overline{\mathcal{V}}_k$, $k\in\{1,2,\cdots,m\}$.

2 Main Results

2.1 Case 1

In this section, the problem of prescribed-time group-bipartite consensus of MASs is discussed under the constraint that each sub-topology $\mathcal{G}_i$, $i=1,2,\cdots,m$ is structurally balanced.

The Laplacian matrix of $\overline{\mathcal{G}}$ is defined as $\boldsymbol{L}=[l_{ij}]_{N\times N}$, where

$$l_{ij}=\begin{cases}-a_{ij}, & i\neq j,\\ \sum\limits_{k\in N_i^{\mathrm{s}}}|a_{ik}|+\sum\limits_{k\in N_i^{\mathrm{d}}}a_{ik}d_k, & i=j.\end{cases}\tag{12}$$

According to group partitioning, the Laplacian matrix can also be rewritten in the form of a block matrix

$$\boldsymbol{L}=\begin{bmatrix}\boldsymbol{L}_{11}&\boldsymbol{L}_{12}&\cdots&\boldsymbol{L}_{1m}\\ \boldsymbol{L}_{21}&\boldsymbol{L}_{22}&\cdots&\boldsymbol{L}_{2m}\\ \vdots&\vdots&&\vdots\\ \boldsymbol{L}_{m1}&\boldsymbol{L}_{m2}&\cdots&\boldsymbol{L}_{mm}\end{bmatrix},\ \boldsymbol{L}_{ij}\in\mathbf{R}^{N_i\times N_j}.\tag{13}$$

Assumption 3 $\sum\limits_{j\in\overline{\mathcal{V}_k}}a_{ij}d_j=0$, $\forall i\in\overline{\mathcal{V}}\setminus\overline{\mathcal{V}}_k$, $k\in\{1,2,\cdots,m\}$.

Lemma 5 Under Assumptions 2 and 3, $\forall i\in\{1,2,\cdots,m\}$, there exists a diagonal matrix

$$\boldsymbol{P}_i=\mathrm{diag}(p_{M_{i-1}+1},p_{M_{i-1}+2},\cdots,p_{M_i})>0,\tag{14}$$

such that $(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)>0$.

Proof If Assumptions 2 and 3 hold, $\forall i\in\{1,2,\cdots,m\}$, $\boldsymbol{D}_i(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)\boldsymbol{D}_i$ is a nonsingular M-matrix, then from Lemma 3, there exists a positive diagonal matrix $\boldsymbol{P}_i=\mathrm{diag}(p_{M_{i-1}+1},p_{M_{i-1}+2},\cdots,p_{M_i})$, such that

$$\begin{aligned}&\boldsymbol{D}_i(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{D}_i\boldsymbol{P}_i+\boldsymbol{P}_i\boldsymbol{D}_i(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)\boldsymbol{D}_i=\\ &\boldsymbol{D}_i[(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)]\boldsymbol{D}_i>0.\end{aligned}\tag{15}$$

From the definition of matrix $\boldsymbol{D}_i$ and the matrix similarity, it can be obtained

$$(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i(\boldsymbol{L}_{ii}+\boldsymbol{B}_i)>0.\tag{16}$$

The prescribed-time control protocol for MASs (9) and (10) is designed by

$$\begin{aligned}u_i=&-a\varphi(t)^2[c_i\sum_{j\in N_i^{\mathrm{s}}}a_{ij}(\mathrm{sgn}(a_{ij})x_i-x_j)+\sum_{j\in N_i^{\mathrm{d}}}a_{ij}(d_jx_i-x_j)+\\ &c_ib_i(x_i-d_ix_{\tilde{i}})]-a\varphi(t)[c_i\sum_{j\in N_i^{\mathrm{s}}}a_{ij}(\mathrm{sgn}(a_{ij})v_i-v_j)+\\ &\sum_{j\in N_i^{\mathrm{d}}}a_{ij}(d_jv_i-v_j)+c_ib_i(v_i-d_iv_{\tilde{i}})],\end{aligned}\tag{17}$$

where $a>1$ and $c_i>0$, and $i=1,2,\cdots,m$ are parameters to be designed later. For simplicity, define the following three auxiliary error variables: e_{xi} denotes the position error of agent i, e_{vi} denotes its velocity error, and e_{f_i} denotes the nonlinear dynamic error, i.e.,

$$\begin{cases}e_{xi}=x_i-d_ix_{\tilde{i}},\\ e_{vi}=v_i-d_iv_{\tilde{i}},\\ e_{f_i}=f(x_i,v_i)-d_if(x_{\tilde{i}},v_{\tilde{i}}),\ i\in\overline{\mathcal{V}},\end{cases}\tag{18}$$

and $\boldsymbol{e}_x=(e_{x1},e_{x2},\cdots,e_{xN})^{\mathrm{T}}$, $\boldsymbol{e}_v=(e_{v1},e_{v2},\cdots,e_{vN})^{\mathrm{T}}$, $\boldsymbol{e}_f=(e_{f_1},e_{f_2},\cdots,e_{f_N})^{\mathrm{T}}$.

Define $\boldsymbol{U}=(u_1,u_2,\cdots,u_N)^{\mathrm{T}}$, and under Assumption 3, $\boldsymbol{U}$ can be written in compact form

$$\boldsymbol{U}=-a\varphi^2(t)\boldsymbol{H}\boldsymbol{e}_x-a\varphi(t)\boldsymbol{H}\boldsymbol{e}_v,\tag{19}$$

where

$$\boldsymbol{H}=\begin{bmatrix}c_1(\boldsymbol{L}_{11}+\boldsymbol{B}_1)&\boldsymbol{L}_{12}&\cdots&\boldsymbol{L}_{1m}\\ \boldsymbol{L}_{21}&c_2(\boldsymbol{L}_{22}+\boldsymbol{B}_2)&\cdots&\boldsymbol{L}_{2m}\\ \vdots&\vdots&&\vdots\\ \boldsymbol{L}_{m1}&\boldsymbol{L}_{m2}&\cdots&c_m(\boldsymbol{L}_{mm}+\boldsymbol{B}_m)\end{bmatrix}.\tag{20}$$

Consider the following state transformation

$$\xi_i=\varphi(t)e_{xi},\ \eta_i=e_{vi},\ i\in\overline{\mathcal{V}}_F,\tag{21}$$

and $\boldsymbol{\xi}=(\xi_1,\xi_2,\cdots,\xi_N)^{\mathrm{T}}$, $\boldsymbol{\eta}=(\eta_1,\eta_2,\cdots,\eta_N)^{\mathrm{T}}$. Taking the derivatives of $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ along the MASs (9) and (10), then the error systems are

$$\begin{cases}\dot{\boldsymbol{\xi}}=\Phi(t)\boldsymbol{\xi}+\varphi(t)\boldsymbol{\eta},\\ \dot{\boldsymbol{\eta}}=-a\varphi(t)\boldsymbol{H}\boldsymbol{\xi}-a\varphi(t)\boldsymbol{H}\boldsymbol{\eta}+\boldsymbol{e}_f,\end{cases}\tag{22}$$

where

$$\Phi(t)=\begin{cases}\varphi(t)/\theta, & t\in[0,T),\\ 0, & t\in[T,+\infty).\end{cases}\tag{23}$$

Theorem 1 Suppose that Assumptions 1–3 hold; for any given time $T>0$, the prescribed-time group-bipartite consensus for the MASs (9) and (10) can be achieved within time T by the control protocol (17). If for all $i\in\overline{\mathcal{V}}$, there exist $c_i>0$, $i=1,2,\cdots,m$, $a>1$, and $\theta>2$, satisfy the following conditions:

$$c_i\lambda_{\min}(\boldsymbol{\Xi}_i)+\lambda_{\min}(\boldsymbol{\Pi})>2\lambda_{\max}(\boldsymbol{P}),\tag{24}$$

$$\gamma_1<0,\tag{25}$$

$$\gamma_2<0,\tag{26}$$

where

$$\gamma_1=(2a-a\theta)\lambda_{\min}(\boldsymbol{\Gamma})+\lambda_{\max}(\boldsymbol{P}),$$

$$\gamma_2 = -a\theta\lambda_{\min}(\boldsymbol{\Gamma}) + (1+2\theta)\lambda_{\max}(\boldsymbol{P}),$$
$$\boldsymbol{\Xi}_i = \boldsymbol{P}_i(\boldsymbol{L}_{ii} + \boldsymbol{B}_i) + (\boldsymbol{L}_{ii} + \boldsymbol{B}_i)^{\mathrm{T}}\boldsymbol{P}_i,$$
$$\boldsymbol{\Pi} = \boldsymbol{P}\boldsymbol{L}_d + \boldsymbol{L}_d^{\mathrm{T}}\boldsymbol{P},$$
$$\boldsymbol{L}_d = \boldsymbol{H} - \mathrm{blkdiag}(c_1(\boldsymbol{L}_{11} + \boldsymbol{B}_1), c_2(\boldsymbol{L}_{22} + \boldsymbol{B}_2), \cdots, c_m(\boldsymbol{L}_{mm} + \boldsymbol{B}_m)),$$
$$\boldsymbol{\Gamma} = \boldsymbol{P}\boldsymbol{H} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{P} = \boldsymbol{\Pi} + \mathrm{blkdiag}(c_1\boldsymbol{\Xi}_1, c_2\boldsymbol{\Xi}_2, \cdots, c_m\boldsymbol{\Xi}_m),$$
$$\boldsymbol{\Omega} = \begin{bmatrix} a\boldsymbol{\Gamma} & \boldsymbol{P} \\ \boldsymbol{P} & \boldsymbol{P} \end{bmatrix}, \tag{27}$$

$\boldsymbol{P} = \mathrm{blkdiag}(\boldsymbol{P}_1, \boldsymbol{P}_2, \cdots, \boldsymbol{P}_m)$, and $\boldsymbol{P}_i$ is defined in Lemma 5. Furthermore, for each follower i, the input u_i is bounded.

Proof Based on Lemma 4, we get

$$\lambda_{\min}(\boldsymbol{\Gamma}) > \lambda_{\min}(\boldsymbol{\Pi}) + \lambda_{\min}(\mathrm{blkdiag}(c_1\boldsymbol{\Xi}_1, c_2\boldsymbol{\Xi}_2, \cdots, c_m\boldsymbol{\Xi}_m)). \tag{28}$$

If we choose appropriate parameters $c_i > 0$, $i = 1, 2, \cdots, m$ such that condition (24) is satisfied, then the following equality holds

$$\lambda_{\min}(\boldsymbol{\Gamma}) > \lambda_{\min}(\boldsymbol{\Pi}) + \lambda_{\min}(\mathrm{blkdiag}(c_1\boldsymbol{\Xi}_1, c_2\boldsymbol{\Xi}_2, \cdots, c_m\boldsymbol{\Xi}_m)) > 2\lambda_{\max}(\boldsymbol{P}) > 0, \tag{29}$$

which means $\boldsymbol{\Gamma} > 0$. Since $a > 1$, we can get $a\boldsymbol{\Gamma} - \boldsymbol{P} > 0$, then $\boldsymbol{\Omega} > 0$ is guaranteed. Construct the Lyapunov function as

$$V = \frac{1}{2}[\boldsymbol{\xi}^{\mathrm{T}}, \boldsymbol{\eta}^{\mathrm{T}}]\boldsymbol{\Omega}\begin{bmatrix}\boldsymbol{\xi} \\ \boldsymbol{\eta}\end{bmatrix}, \tag{30}$$

then $V \geqslant 0$ and $V = 0$ if and only if $\boldsymbol{\xi} = 0$ and $\boldsymbol{\eta} = 0$.

Consider the time interval $[0, T)$, taking the time derivative of V along the solution of system (22) yields

$$\dot{V} = [\boldsymbol{\xi}^{\mathrm{T}}, \boldsymbol{\eta}^{\mathrm{T}}]\begin{bmatrix} a\boldsymbol{\Gamma} & \boldsymbol{P} \\ \boldsymbol{P} & \boldsymbol{P} \end{bmatrix}\begin{bmatrix} \frac{\varphi(t)}{\theta}\boldsymbol{\xi} + \varphi(t)\boldsymbol{\eta} \\ \boldsymbol{e}_f - \varphi(t)\boldsymbol{H}\boldsymbol{\xi} - \varphi(t)\boldsymbol{H}\boldsymbol{\eta} \end{bmatrix} = \frac{a\varphi(t)}{\theta}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{\xi} + \frac{\varphi(t)}{\theta}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta} + \varphi(t)\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta} - \frac{a\varphi(t)}{2}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{\xi} - \frac{a\varphi(t)}{2}\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{\eta} + (\boldsymbol{\xi}^{\mathrm{T}} + \boldsymbol{\eta}^{\mathrm{T}})\boldsymbol{P}\boldsymbol{e}_f. \tag{31}$$

Noting that $1/\varphi(t) \leqslant T/\theta$, and based on Young's inequality, it can be concluded that

$$(\boldsymbol{\xi}^{\mathrm{T}} + \boldsymbol{\eta}^{\mathrm{T}})\boldsymbol{P}\boldsymbol{e}_f = \sum_{i=1}^{N} p_i(\xi_i + \eta_i)e_{f_i} \leqslant p_i\sum_{i=1}^{N}(|\xi_i| + |\eta_i|)(r_1|e_{xi}| + r_2|e_{vi}|) \leqslant p_i\sum_{i=1}^{N}\left[\frac{r_1}{\varphi(t)}|\xi_i|^2 + r_2|\eta_i|^2 + \left(r_2 + \frac{r_1}{\varphi(t)}\right)|\xi_i|\cdot|\eta_i|\right] = \frac{3Tr_1 + \theta r_2}{2\theta}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\xi} + \frac{3\theta r_2 + Tr_1}{2\theta}\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta}, \tag{32}$$

and

$$\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta} = \sum_{j=1}^{N} p_i\xi_i\eta_i \leqslant \sum_{j=1}^{N} p_i|\xi_i||\eta_i| \leqslant \sum_{j=1}^{N}\frac{p_i}{2}(|\xi_i|^2 + |\eta_i|^2) = \frac{1}{2}(\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\xi} + \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta}). \tag{33}$$

Substituting the inequalities (32) and (33) into (31) yields

$$\dot{V} \leqslant \varphi(t)\boldsymbol{\xi}^{\mathrm{T}}\left[\left(\frac{2a - a\theta}{2\theta}\right)\boldsymbol{\Gamma} + \frac{1}{2\theta}\boldsymbol{P}\right]\boldsymbol{\xi} + \varphi(t)\boldsymbol{\eta}^{\mathrm{T}}\left[-\frac{a}{2}\boldsymbol{\Gamma} + \left(\frac{1+2\theta}{2\theta}\right)\boldsymbol{P}\right]\boldsymbol{\eta} + \left(\frac{3Tr_1 + \theta r_2}{2\theta}\right)\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\xi} + \left(\frac{3\theta r_2 + Tr_1}{2\theta}\right)\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta} \leqslant \varphi(t)\boldsymbol{\xi}^{\mathrm{T}}\left[\frac{(2a - a\theta)\lambda_{\min}(\boldsymbol{\Gamma}) + \lambda_{\max}(\boldsymbol{P})}{2\theta}\right]\boldsymbol{\xi} + \left(\frac{3Tr_1 + \theta r_2}{2\theta}\right)\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\xi} + \varphi(t)\boldsymbol{\eta}^{\mathrm{T}}\left[\frac{-a\theta\lambda_{\min}(\boldsymbol{\Gamma}) + (1+2\theta)\lambda_{\max}(\boldsymbol{P})}{2\theta}\right]\boldsymbol{\eta} + \left(\frac{3\theta r_2 + Tr_1}{2\theta}\right)\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\eta}. \tag{34}$$

According to Eq. (30), it can be obtained that

$$\frac{\lambda_{\min}(\boldsymbol{\Omega})}{2}(\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\xi} + \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta}) \leqslant V \leqslant \frac{\lambda_{\max}(\boldsymbol{\Omega})}{2}(\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\xi} + \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta}). \tag{35}$$

Combined with inequalities (34) and (35), if conditions (25) and (26) hold, one gets

$$\dot{V} \leqslant -k\varphi(t)V + bV, \tag{36}$$

where

$$\begin{cases} k = 2\cdot\min\{-\gamma_1/2\theta, -\gamma_2/2\theta\}/\lambda_{\max}(\boldsymbol{\Omega}), \\ b = 2\cdot\max\left\{\dfrac{3Tr_1 + \theta r_2}{2\theta}, \dfrac{3T\theta r_2 + Tr_1}{2\theta}\right\}/\lambda_{\min}(\boldsymbol{\Omega}). \end{cases} \tag{37}$$

Thus, from Lemma 2, $V \leqslant \exp(-k\bar{\mu}(t, T) + k\bar{\mu}(0, T) + bt)$.

Noting that $\varphi(t) \geqslant \theta/T$ and $\lim_{t\to T}\exp(-k\bar{\mu}(t,T) + k\bar{\mu}(0,T) + bt) = 0$, the following inequality holds

$$e_x^{\mathrm{T}}e_x + e_v^{\mathrm{T}}e_v \leqslant \gamma(\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\xi} + \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta}) \leqslant \frac{2\gamma V}{\lambda_{\min}(\boldsymbol{\Omega})} \leqslant \frac{2\gamma\exp(-k\bar{\mu}(t,T) + k\bar{\mu}(0,T) + bt)}{\lambda_{\min}(\boldsymbol{\Omega})}, \tag{38}$$

where $\gamma = \max\{T^2/\theta^2, 1\}$. Thus, when $t \to T$, we have that $\boldsymbol{e}_x \to 0$ and $\boldsymbol{e}_v \to 0$. For $t \in [T, +\infty)$, it can be obtained that $V \equiv 0$, which implies that $\boldsymbol{e}_x \equiv 0$, $\boldsymbol{e}_v \equiv 0$. That is, the prescribed-time group-bipartite consensus is achieved within time T according to Definition 2.

Next, we will prove that the control input is bounded. For $t \in [0, T)$,

$$\|\boldsymbol{U}\|^2 = a\varphi^2(t)\|\boldsymbol{H}\boldsymbol{\xi} + \boldsymbol{H}\boldsymbol{\eta}\|^2 \leqslant a\varphi^2(t)\|\boldsymbol{H}\|^2\|\boldsymbol{\xi} + \boldsymbol{\eta}\|^2 \leqslant a\varphi^2(t)\|\boldsymbol{H}\|^2(\|\boldsymbol{\xi}\|^2 + \|\boldsymbol{\eta}\|^2 + 2\|\boldsymbol{\xi}\|\|\boldsymbol{\eta}\|) \leqslant 2a\varphi^2(t)\|\boldsymbol{H}\|^2(\|\boldsymbol{\xi}\|^2 + \|\boldsymbol{\eta}\|^2) \leqslant \frac{4a\varphi^2(t)\|\boldsymbol{H}\|^2}{\lambda_{\min}(\Omega)}V \leqslant \frac{4a\|\boldsymbol{H}\|^2\varphi^2(t)\exp(-k\bar{\mu}(t, T) + k\bar{\mu}(0, T) + bt)V(0)}{\lambda_{\min}(\boldsymbol{\Omega})}. \tag{39}$$

Since $\lim\limits_{t\to T}\mu^2(t,T)\exp(-k\bar{\mu}(t,T)) < +\infty$, it is obvious that $\|\boldsymbol{U}\|^2$ is bounded. For $t\in[T,+\infty)$, we have proved that $\boldsymbol{e}_x=\boldsymbol{e}_v\equiv 0$, and thus $\boldsymbol{U}\equiv 0$ can be derived from Eq. (19). The proof is completed.

2.2 Case 2

In Case 1, each sub-topology $\mathcal{G}_i$, $i=1,2,\cdots,m$ is structurally balanced. To satisfy Assumption 3, the whole topology cannot be structurally balanced. To address this limitation, we have incorporated the leader's dynamic model into the controller design, thereby relaxing the constraints on the communication topology requirements.

Assumption 4 The digraph $\bar{\mathcal{G}}$ is structurally balanced and strongly connected.

Assumption 5 The sets satisfy $\bar{\mathcal{V}}_{ij}\in\bar{\mathcal{V}}_j$, $i=1,2,\cdots,m$, and $j=1,2$.

The new Laplacian matrix is defined as $\boldsymbol{L}=[l_{ij}]_{N\times N}$, where

$$l_{ij}=\begin{cases}\sum\limits_{j\in\bar{\mathcal{V}}}|a_{ij}|, & i=j,\\ -a_{ij}, & i\neq j.\end{cases}\tag{40}$$

Define $\bar{\boldsymbol{H}}=\boldsymbol{L}+\boldsymbol{B}$, then we have Lemma 6.

Lemma 6[13] $\boldsymbol{D}\bar{\boldsymbol{H}}\boldsymbol{D}$ is a nonsingular M-matrix, and similar to the proof of Lemma 5, there exists a positive diagonal matrix $\boldsymbol{Q}>0$, such that $\boldsymbol{\Delta}=\boldsymbol{Q}\bar{\boldsymbol{H}}+\bar{\boldsymbol{H}}^T\boldsymbol{Q}>0$.

The prescribed-time consensus is described by

$$\begin{aligned}u_i=&-\varphi(t)^2\bar{a}\Big[\sum_{j\in\bar{\mathcal{V}}}a_{ij}(\operatorname{sgn}(a_{ij})x_i-x_j)-\sum_{j\in\bar{\mathcal{V}}}d_jl_{ij}x_j+b_i(x_i-d_ix_i)\Big]-\\&\varphi(t)\bar{a}\Big[\sum_{j\in\bar{\mathcal{V}}}a_{ij}(\operatorname{sgn}(a_{ij})v_i-v_j)-\sum_{j\in\bar{\mathcal{V}}}d_jl_{ij}v_j+b_i(v_i-d_iv_i)\Big],\end{aligned}\tag{41}$$

where $\bar{a}>0$ is a parameter to be designed later.

Similar to Case 1, the following error system can be obtained

$$\begin{cases}\dot{\boldsymbol{\xi}}=\boldsymbol{\Phi}(t)\boldsymbol{\xi}+\boldsymbol{\Phi}\varphi(t)\boldsymbol{\eta},\\ \dot{\boldsymbol{\eta}}=-\bar{a}\varphi(t)\bar{\boldsymbol{H}}\boldsymbol{\xi}-\bar{a}\varphi(t)\bar{\boldsymbol{H}}\boldsymbol{\eta}+\boldsymbol{e}_f.\end{cases}\tag{42}$$

Theorem 2 Under Assumptions 1–2 and 4–5, for a given time T, the prescribed-time group-bipartite consensus for MASs (9) and (10) can be achieved within time T by the control protocol (41), if for all $i\in\bar{\mathcal{V}}$, there exists $\bar{a}>0$ which satisfies the following conditions:

$$\bar{a}\lambda_{\min}(\boldsymbol{\Delta})>\lambda_{\max}(\boldsymbol{Q}),\tag{43}$$

$$\bar{\gamma}_1<0,\tag{44}$$

$$\bar{\gamma}_2<0,\tag{45}$$

where

$$\begin{cases}\bar{\gamma}_1=(2\bar{a}-\bar{a}\theta)\lambda_{\min}(\boldsymbol{\Delta})+\lambda_{\max}(\boldsymbol{Q}),\\ \bar{\gamma}_2=-\bar{a}\lambda_{\min}(\boldsymbol{\Delta})+(1+2\theta)\lambda_{\max}(\boldsymbol{Q}),\\ \boldsymbol{\Lambda}=\begin{bmatrix}\bar{a}\boldsymbol{\Delta} & \boldsymbol{Q}\\ \boldsymbol{Q} & \boldsymbol{Q}\end{bmatrix}.\end{cases}\tag{46}$$

Furthermore, for each follower i, the input u_i is bounded.

Proof Construct the Lyapunov function as

$$V=\frac{1}{2}[\boldsymbol{\xi}^{\mathrm{T}},\boldsymbol{\eta}^{\mathrm{T}}]\boldsymbol{\Lambda}\begin{bmatrix}\boldsymbol{\xi}\\ \boldsymbol{\eta}\end{bmatrix}.\tag{47}$$

According to condition (43), $V\geqslant 0$ and $V=0$ if and only if $\boldsymbol{\xi}=\boldsymbol{\eta}=0$. The proof that follows is similar to that of Theorem 1, and hence we omit it.

3 Simulation Examples

In this section, two simulation examples are provided to verify the effectiveness of control protocols. We take the same nonlinear function as $f(t,x_i,v_i)=\sin(x_i(t)+v_i(t))$, and then we can get $r_1=r_2=1.5$.

3.1 Example 1

Consider an MAS with 12 agents, and the communication topology between agents is shown in Fig. 1. The followers are divided into three groups as $\mathcal{V}_1=\{1,2\}$, $\mathcal{V}_2=\{3,4,5,6\}$, and $\mathcal{V}_3=\{7,8,9\}$, where agents 10, 11, and 12 represent the leader of $\mathcal{V}_1$, $\mathcal{V}_2$, and $\mathcal{V}_3$, respectively. Set $T=5$ s, and we can verify that conditions (24)–(26) in Theorem 1 are satisfied when choosing $\theta=5$, $c_1=5$, $c_2=16$, $c_3=8$, and $a=24$. The initial values are taken as

$$\begin{cases}\boldsymbol{x}(0)=[5.5,-4.0,3.8,1.1,-2.0,-8.7,5.6,\\ \qquad -3.2,2.1,-8.5,-8.2,5.9]^{\mathrm{T}},\\ \boldsymbol{v}(0)=[4.8,-7.9,-7.4,1.0,-0.3,7.8,\\ \qquad 5.9,4.6,-8.9,8.8,3.6,-7.3]^{\mathrm{T}}.\end{cases}$$

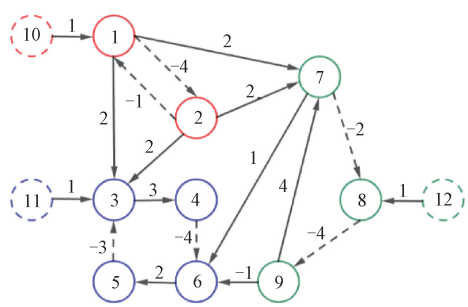


Fig. 1 Communication topology in Example 1

The trajectories of the position and velocity of all

agents are shown in Fig. 2. The tracking errors of followers are presented in Fig. 3. As shown in Fig. 3, the tracking errors of the position and velocity of all followers converge to zero within $T=5$ s, which means that all agents can achieve group-bipartite consensus within the prescribed time $T=5$ s.

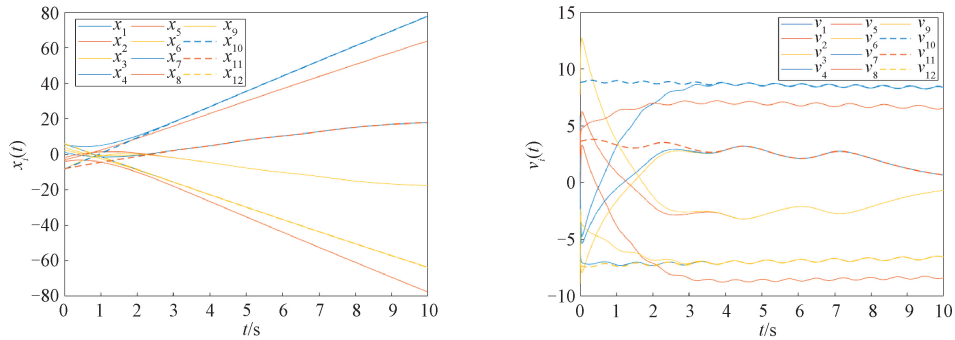


Fig. 2 System trajectories in Example 1 with $T=5$ s: (a) trajectories of position; (b) trajectories of velocity

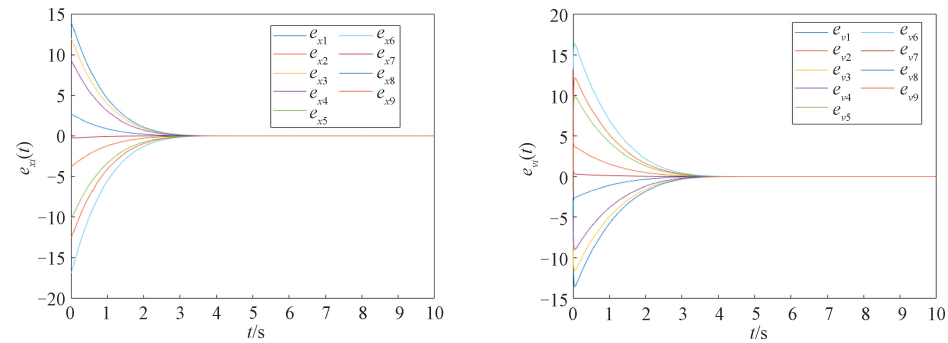


Fig. 3 Tracking errors of followers in Example 1: (a) position errors; (b) velocity errors

3.2 Example 2

Consider an MAS with nine agents, and the communication topology between the agents is shown in Fig. 4. The agents are divided into two groups as $\mathcal{V}_1=\{1, 2, 3\}$ and $\mathcal{V}_2=\{4, 5, 6, 7\}$. Agents 8 and 9 represent the leaders of $\mathcal{V}_1$ and $\mathcal{V}_2$, respectively. Choose $\theta=4$, $\bar{a}=4$, then conditions (43) – (45) in Theorem 2 are satisfied. The initial values are taken as

$$\begin{cases}\boldsymbol{x}(0)=[3.6, -4.4, -9.4, 8.8, 0.6, 3.4, 2.1, 3.6, 4.4]^{\mathrm{T}},\\ \boldsymbol{v}(0)=[4.1, 9.2, 4.5, 7.1, 9.8, 5.1, -7.2, 5.1, 9.6]^{\mathrm{T}}.\end{cases}$$

The trajectories of the position and velocity of all agents are shown in Fig. 5, and the tracking errors of followers are presented in Fig. 6. As shown in Fig. 6, the tracking errors of the position and velocity of all followers converge to zero within $T=5$ s, which means that all agents can achieve prescribed-time group-bipartite consensus within $T=5$ s.

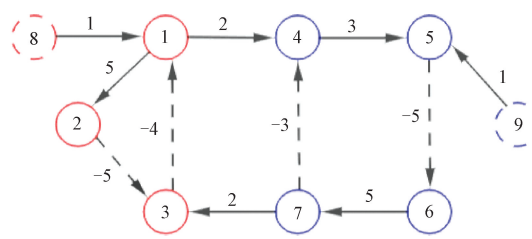


Fig. 4 Communication topology in Example 2

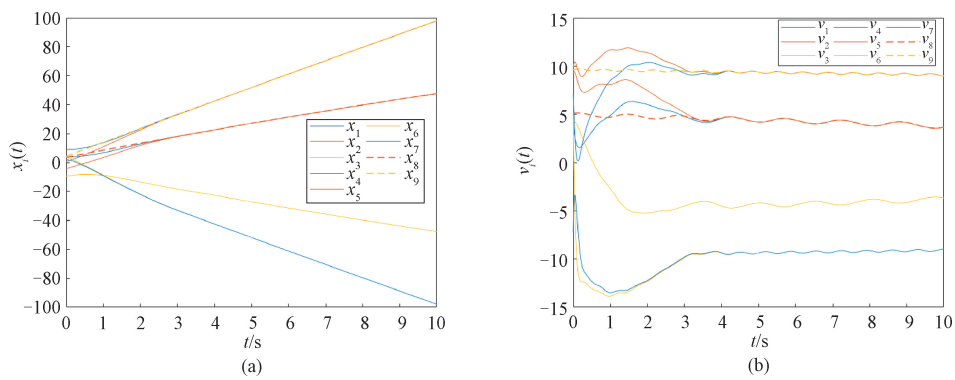


Fig. 5 System trajectories in Example 2 with $T=5$ s: (a) trajectories of position; (b) trajectories of velocity

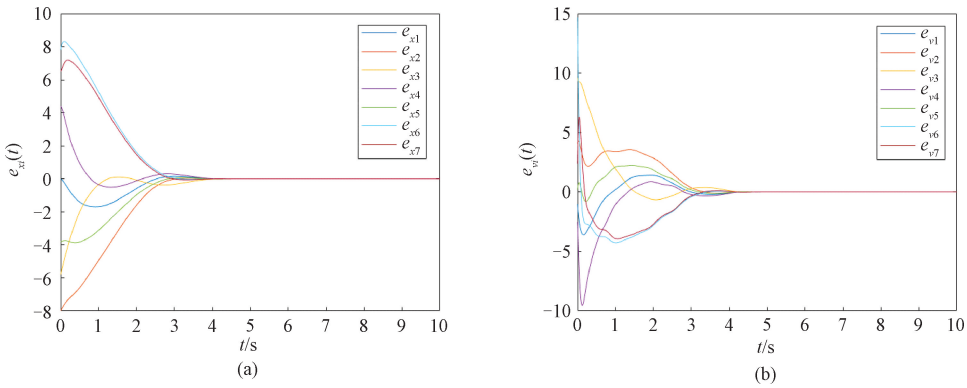


Fig. 6 Tracking errors of followers in Example 2: (a) position errors; (b) velocity errors

4 Conclusions

The problem of prescribed-time group-bipartite consensus of second-order nonlinear MASs is studied. In Case 1, each sub-topology is structurally balanced. However, due to the constraints of the inter-group balance condition, the overall topology cannot be structurally balanced unless there is no communication between the different groups. To overcome this problem, leader dynamics are introduced into the controller, so that only the overall topology needs to satisfy the structural balance condition. Finally, the effectiveness of the results is verified through two numerical simulation examples. In future research, we will consider the prescribed-time group-bipartite consensus under switching topologies.

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二阶非线性多智能体系统的预定时间群二分一致性

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摘 要：本文研究符号通信拓扑下一类二阶非线性多智能体系统的预定时间领导跟随群二分一致性。首先，针对每个子拓扑结构平衡的情况，选取适当的时变函数作为控制增益，提出了一种新颖的预定时间控制协议；通过在控制器中引入额外的增益参数，简化了闭环系统稳定的充分条件，同时使得参数的计算过程更为简便。其次，通过在控制器中引入领导者的动态，放宽了对拓扑的约束，只需要多智能体系统的全局拓扑满足结构平衡条件，并结合智能体邻居信息与领导者动态设计了全新的一致性协议。基于矩阵理论和Lyapunov 稳定性理论，构造多智能体系统实现一致性的充分条件。最后，通过两组数值仿真算例，验证了两种控制协议的有效性。

关键词：群二分一致性；多智能体系统；预定时间；符号图