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Equivalence principles exotica

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Abstract This is a short review of the different principles of equivalence stated and used in the context of the gravitational interaction. We emphasize the need for precision in stating and differentiating these different equivalence principles, especially in the context of prevalent confusion regarding the applicability of the weak equivalence principle in quantum mechanics. We discuss several empirical results pertaining to the validity of the equivalence principle in exotic physical situations not directly amenable to experimental tests. We conclude with a section on the physical basis of the universal validity of the equivalence principle, as manifest in the universality of free fall, and discuss its link to cosmic gravity.

Keywords equivalence principle, universality of free fall, general relativity, experimental gravity, conservation laws

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1 The equivalence principles

The universality of free fall, first discussed in detail by Galileo is the basis of all forms of equivalence principle (EP). In the most discussed version, the weak equivalence principle asserts that the acceleration of test masses in a uniform gravitational field is independent of the mass and composition of the test object. This is equivalent to the statement that the ratio of the gravitational mass to the inertial mass of any object is a universal value inde-

pendent of elementary composition or other properties. The statement generalizes to include all forms of energy, since energy is a source of gravity. However, a precise statement of the weak equivalence principle, in the form useful for the theory of gravitation, is that the dynamical behaviour of a physical system under non-gravitational interactions in a uniform gravitational field $\mathbf{g}$ is identical to its behaviour in a uniformly accelerating frame with its acceleration $\mathbf{a} = -\mathbf{g}$.

Theoretical possibilities in constructing a theory of gravitation allow making a distinction between the source charge of gravity—the active mass—and the mass that undergoes dynamics under the gravitational influence—the passive mass. An equivalence of the active mass and the passive mass is also postulated, and well supported by experiments. In a theory where there is a violation of this active-passive mass equivalence principle, there is an asymmetry in action and reaction and this can lead to self acceleration.

While the experimental tests of the universality of free fall (UFF), and hence that of the weak equivalence principle (WEP), are impressively precise, theoretical suggestions exist where the equivalence might be broken in a regime that is barely two or three orders of magnitude lower than what is experimentally established. This makes conceiving and performing such tests an important current research activity in the field of experimental gravitation. In fact, most modern experiments that search for a modification of standard gravity at short ranges, inspired by string theory and higher dimensional theories, draw their lineage and inspiration from experiments on the UFF and WEP. Moreover, there are several situations of great importance where the UFF cannot be tested with good precision directly, and one has to find indirect yet decisive methods of constraining possible violations if the standard theory of gravity—the general theory of relativity—has to be preserved intact. Some examples where direct tests are difficult concerns

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the UFF for antimatter, zero-point energy, Fermi energy, neutrinos, etc.

Equally important is the question regarding the validity of the weak equivalence principle in quantum mechanics. Imprecise statements of the EP has lead to several statements and claims of the violation of the EP in the quantum regime, all of which require additional scrutiny. From our viewpoint, quantum mechanics is in complete harmony with the equivalence principle at the same level at which it is tested in the classical macroscopic regime.

Finally one may ask whether there is a deeper reason for the UFF and WEP to be true. If there is a physical reason that can be traced, then instead of remaining as a hypothesis with good empirical support, UFF and WEP can be treated as fundamental principles, perhaps at the level of conservation laws. We will argue that this indeed is the case.

tational fields by noting that physics in uniformly accelerated frames is equivalent to physics in uniform gravitational fields. This allows for a geometric formulation. The equivalence of physics in accelerated frames and in a uniform gravitational field is postulated on the basis of the universality of free fall (UFF), or the universality of the ratio of the gravitational to inertial mass [1]. This weak equivalence principle is then the fundamental principle of equivalence. From this one generalizes to a situation to include gravitational phenomena, and insists on the UFF of gravitational binding energy as well, leading to a stronger form of the equivalence principle (SEP). The Einstein equivalence principle (EEP) is the grand generalization that asserts the equivalence of all laws of physics in a uniform gravitational field and in a uniformly accelerated frame. Thus one may schematically depict the hierarchy as $\text{WEP} \rightarrow \text{SEP} \rightarrow \text{EEP}$ (Fig. 1).

While the geometric formulation of gravity more or less disregards the role of the test mass, since its gravitational dynamics is independent of its characteristics or internal structure, it makes formal sense to distinguish between the role of gravitational mass in its active and

2 The equivalence tree

The standard theory of gravity, the general theory of relativity, is constructed by applying special relativistic physics in inertial frames to situations involving gravi-

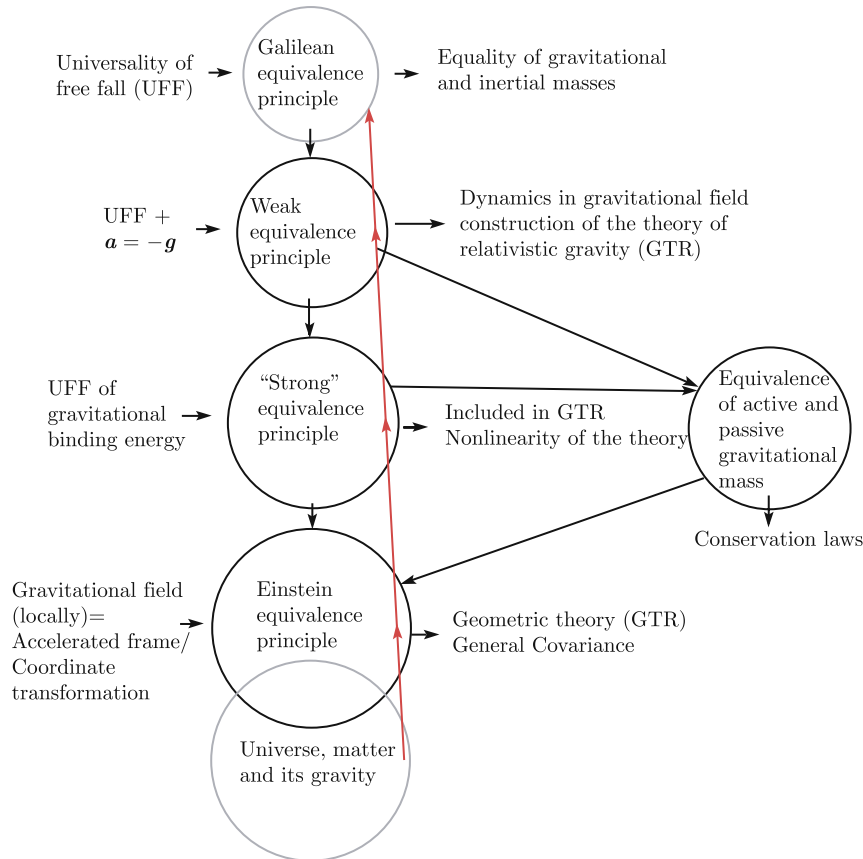


Fig. 1 The tree of equivalence principles. The new aspect is the emergence of the WEP and UFF from the gravitational interaction of the test bodies with cosmic matter.

the passive form. The active gravitational mass is the source of gravity, and the passive gravitational mass is then the “test charge” on which a gravitational field acts. Tests of the equivalence principle are statements of the universality of the ratio of the passive gravitational mass to the inertial mass. This by itself does not imply, from empirical evidence alone, that all forms of matter and energy generate gravitational fields. Therefore, the desired equivalence of the active and passive gravitational masses, or the universality of the ratio of the passive to active gravitational masses, is explicitly stated as a separate equivalence principle [1, 2].

There is another level of hierarchy that needs to be mentioned. All significant tests of the WEP are done with macroscopic bodies, by comparing their trajectory in a uniform gravitational field. This then allows the construction of the classical general relativity. In the present understanding classicality is supposed to be the apparent subset of a more general quantum mechanical physical world, even though this is by no means a proven fact. How is it possible to conclude that the principle that is verified in the classical regime is applicable to the microscopic quantum world? There is a need for stating precisely a quantum equivalence principle—WEP that is applicable to quantum dynamics. Fortunately this turns out to be the exactly the same statement as in the classical case. The result of a quantum mechanical experiment in a uniform gravitational field is identical to that in a uniformly accelerating frame [3].

3 Tests of violations of the WEP

Since this paper is not a review of the standard tests of the WEP, we mention these tests only briefly here, just to indicate the established constraints that will be useful in the discussion ahead [1, 4, 5]. Typically there are two types of tests—those in which the free fall of two test bodies with different nuclear composition is directly compared in the field of earth, as in the original Galilean discussion, and those in which the differential acceleration, Δa , is monitored as the deflection of a torsion balance in the field of the sun. The quantity inferred is the Galileo-Eötvös parameter $\eta = \Delta a/a$. While the intrinsic sensitivity of such tests approach $\eta \approx 10^{-14}$, systematic noise of various kinds limits the final sensitivity to about $\eta > 10^{-13}$ at present. The best constraint from such a terrestrial experiment is from the University of Washington, Seattle group [6, 7], and $\eta \leq 2 \times 10^{-13}$. A currently running torsion balance experiment at TIFR, Mumbai aims to reach a similar level of precision soon [8]. Atom interferometry based experiments hope to reach a level of

$\eta \leq 10^{-15}$, but this is proposed with two isotopes of rubidium [9], differing in their nuclear properties by a small amount, and will not effectively reach the usefulness of testing the WEP with materials differing widely in their nuclear properties at the level of $\eta \approx 10^{-14}$. The point is that the crucial quantity is not just the differential acceleration sensitivity, η , but the product of η and the difference in the amount of a relevant charge for coupling to a new force or to modified gravity, like the specific difference in baryon or lepton number, $\delta Q/Q$. Experiments that could use a light element like Lithium and heavy Rubidium or Cesium, all of which can be laser cooled to a few microkelvin temperature might be a more relevant pair, but such experiments needing multiple lasers become complicated, with larger systematic noise effects, and the final sensitivity might turn out to be less competitive.

Since the Earth and the Moon are in free fall towards the Sun, these can be considered as test bodies for a test of the WEP, and the precision with which the changes in the distance between the earth and the moon can be monitored has improved significantly in the past two decades, due to the advances in Laser Ranging technology. Today, with millimeter level precision for the long duration measurement of this distance, the constraint on any possible violation of the WEP is at the level of $\eta \approx 10^{-14}$ [10]. Since earth’s gravitational binding energy is of the order of 10^{-9} of its rest mass, and since the moon has a relatively smaller amount of gravitational binding energy, LLR provides the only test of the SEP that asserts the UFF for gravitational binding energy. This interpretation has been confirmed with additional input from the Seattle group on the WEP for appropriate laboratory test bodies with negligible gravitational energy [11].

There are several space-based experiments that are proposed, and in advanced stages of development and testing in the terrestrial laboratory [12, 13]. “MICROSCOPE” aims for $\eta \leq 10^{-14}$, and “GG (Galileo Galilei)”, a non-cryogenic differential experiment with rotational modulation of the signal, aims for $\eta \leq 10^{-17}$.

Before we discuss the UFF for specific forms of matter and energy, it is important to discuss the interpretation of a positive signal in the experimental test. An apparent violation of UFF could mean that the long range gravitational interaction is not universal and that it depends on some internal properties of matter. This will of course discredit the general theory of relativity, or any geometric theory for that matter. A positive signal could also mean that there is a new long range interaction that couples to some internal charge in matter, like a quantum number associated with the nucleons (quarks) or leptons,

while gravity itself respects UFF. Indeed, this is the reason why many experiments that look for a violation of the WEP are also successful in constraining hypothetical “fifth forces” in a wide range of their parameter space of coupling constant and range. These two interpretations have widely different implications for the theoretical descriptions, and therefore any positive signal in such experiments will have to be pursued further for arriving at the correct interpretation. However, we will argue, in the concluding section, that a true understanding of the UFF involves cosmic gravity and that a violation of UFF in experiments would point to new intermediate or short range forces, and not to a violation of WEP within long range gravity.

3.1 Specific constraints

The typical way of determining whether a particular form of energy (which is also the source of gravity) obeys the UFF is to estimate the energy content in the particular form relative to the rest mass energy of the test body, and then to evaluate the difference in the relative energy content for the two test bodies employed in the experiment. This allows for separating the contribution to UFF from each form of energy in the test body, assuming linearity. If any of these energies violate UFF that will then show up as a differential acceleration and this can be compared with the observed constraint on the differential acceleration to arrive at a constraint on the extent to which the particular form of energy obeys the UFF [1]. For example, the nuclear binding energy in matter is typically less than 10 MeV/nucleon, and therefore the ratio of nuclear binding energy to rest mass (ε_n) is about 10^{-2} . However, the difference in this specific nuclear energy between different materials ($\delta\varepsilon_n$) is smaller, and in typical test bodies used in experiments this could be a few times 10^{-3} . If the constraint on the relative differential acceleration in an WEP experiment is η , then one can conclude that the nuclear binding energy obeys UFF at a level that is good to $\eta/\delta\varepsilon_n \approx 10^3\eta$. With η in the range of 10^{-12} , one may then conclude that there is very good evidence that nuclear binding energy obeys the UFF and WEP. When it comes to electrostatic binding, $\delta\varepsilon$ is much smaller, and the constraints weaken relatively, but remain very stringent in an absolute sense due to the smallness of η . Since the gravitational binding energy for laboratory bodies is only about 10^{-25} of the rest mass, laboratory experiments do not give any useful constraint on the UFF for gravitational energy, and this requires massive celestial objects in free fall, like the earth, and an exceptional experiment like the LLR.

Now we are in a position to discuss specific constraints

on the UFF and WEP for a variety of exotic yet physically real and relevant forms of energy.

3.2 WEP for antimatter

Questions on the gravitational properties of antimatter have been discussed from the very beginning of the introduction of the concept of antimatter, and speculations continue to be raised even today. Since antimatter is rare, and difficult to produce and store in stable form, questions on its gravitational properties are not easily answered. Direct experiments on the WEP for electrons and positrons were attempted with very interesting and important consequences, though without a result on this particular goal [14]. It is only recently that antimatter in stable neutral atomic form (anti-hydrogen) has been produced and stored in relatively large quantities to perform direct experimental tests of various kinds. Significant gravitational experiments are still far from realization. Therefore, the early idea by Schiff on obtaining a result on the WEP of antimatter by combining reliable theoretical arguments with experiment using matter was very important [15]. The argument invokes the matter-antimatter virtual pairs surrounding matter, and ascribes a physical reality to them. From quantum electrodynamics, one can estimate the average density of such electron-positron pairs surrounding charged nucleus or electrons, this can be estimated to be the fraction $\frac{m_e}{m_p}(Z/\alpha_{em})^2 \approx 5 \times 10^{-4} \times 1/100 \approx 5 \times 10^{-6}$ for typical test bodies in experiments. The difference in the virtual antimatter content between different test bodies could be a factor of 10 lower, and we can conclude that antimatter obeys the WEP with η better than 10^{-5} . Since this argument assumes a theoretical construction that is not directly verified, it is desirable that direct experiments are performed.

While a terrestrial experiment with K mesons and B mesons might be expected to be sensitive to the issue of the differential gravitational behaviour of particles and antiparticles, the gravitational phases introduced due to the gravitational potential of the earth during a very short time of propagation are too small to arrive at a significant model-independent constraint.

It is possible to come up with a more direct argument, with relatively less dependence on theory, to establish the validity of WEP for antimatter. The argument has a validity that goes beyond the constraining of violation of WEP for antimatter. Since it is experimentally established with good precision that matter and antimatter annihilates to form photons, and since the photons are known to obey the WEP with significant precision (at

least at the level of 10^{-5} from standard GR tests), it can be immediately concluded that antimatter has to obey the WEP at this level. Otherwise, it will become possible to violate the conservation of energy, and create perpetual motion. Consider the “fall” of a photon in a uniform gravitational field g through some distance d . The gain in energy (blueshift) is $gd(h\nu/c^2)$. If the total energy is sufficient for pair creation, a particle and antiparticle can be created, with equal inertial mass. The two particles “lifted” up through height d will need an energy $2m_ggd$ only if both the particles obey the WEP, and hence have the same gravitational mass. If the gravitational masses are different, then the total energy will not add up to what we started with and hence perpetual motion becomes possible.

3.3 WEP for neutrinos

Since neutrinos are weakly interacting near-zero mass particles, and hence fast, it is not easy to test directly the validity of the equivalence principle for them. Indeed, it is difficult to defend empirically the claim that neutrinos are gravitationally active, and that they fall as photons do in a gravitational field. Several authors have considered the gravitationally induced phase shifts of neutrinos in a gravitational field that is of relevance in the astrophysical context, and these phases are too small to be of significant importance in standard neutrino oscillation phenomenology. Of course, if new flavour-dependent long range interactions are postulated, these may have a significant effect on neutrino oscillations. The near equal arrival times of photons and neutrinos from supernova 1987 indicate that the propagation of neutrinos respects standard gravity since the Shapiro time delay that encodes the integral of the gravitational potential over the propagation trajectory is found to be the same for both light and neutrinos within 10^{-3} [16–18]. Essentially the gravitational refractive index for the quantum mechanical particles affects their phase and velocity. The supernova data confirms that the integrated gravitational refractive index effect for neutrinos and light are identical to within 10^{-3} . The analysis by Pakvasa *et al.* [18] compared the effect of gravity on neutrinos and antineutrinos, and observed that their gravitational time delays are equal to within 10^{-6} .

One may invoke the indirect argument mentioned in the last section, since the neutrino is produced in a nuclear process under various conservation constraints that involve particles that are known to obey the equivalence principle like the proton, neutron and the electron. Therefore, a neutrino that does not obey the WEP will also imply a violation of the conservation of energy [3].

3.4 WEP for zero-point energy

The situation regarding the validity of the WEP becomes vague and difficult to address when one deals with more exotic and practically non-tangible forms of matter-energy. The most important of these is the zero-point energy (ZPE). Quantization of fields in the standard way, in quantum field theories, invariably has an infinite zero-point energy background. Even with cut-offs of various kinds, a large energy density in zero-point modes is unavoidable, and this of course is the source of the well-known cosmological constant problem. The fundamental question in the physics of zero-point energy is whether it is gravitationally active and acts as a source of gravity, and whether it has a universal coupling to gravity generated by other massive sources. The difficulty in performing a direct test of the WEP for zero-point energy is obvious; the ZPE content relative to the rest mass energy in ordinary matter cannot be more than about 10^{-25} or so, even in specially engineered layered materials. For the same reason, there is no direct experiment possible to test whether ZPE is a source for gravity. Given that ZPE is one of the most important concepts in the foundations of physics, and that we believe in its reality in the sense of a reality for energy associated with real particles and photons, it becomes extremely important to get some hint as to its gravitational properties.

The universal argument based on the conservation of energy comes to our rescue here admirably. In the standard theory, the Casimir force between a pair of conducting plates is due to the difference in the zero-point energy density inside and outside a pair of conducting plates that act as a boundary for zero-point modes. Since there is direct experimental evidence that the Casimir force causes two conducting plates to move towards each other and gain kinetic energy, it is clear that if ZPE does not obey WEP we can set up a perpetually energy violating cycle. Kinetic energy content itself is known to obey the WEP from direct experiments. For example, one may consider the energy motion associated with earth’s rotation, which is about 10^{-8} of its rest mass energy. The LLR results show that the earth and the moon have the same rate of free fall towards the sun, good to about 10^{-14} , and this means that rotational kinetic energy obeys the WEP accurately to 1 part in 10^6 since the moon has relatively negligible kinetic energy. A confirmation of the Casimir force in a measurement accurate to 5 %, along with empirical knowledge that kinetic energy respects WEP with high precision is then sufficient to conclude that the ZPE obeys the WEP and UFF accurate to at least 5 %.

Nevertheless, one can ask the question whether it is

possible to do a direct experiment. Apart from the reliability and relative independence from theoretical assumptions, there is also the possibility of a better constraint in a direct experiment.

A proposal for a space-based test of the WEP for ZPE by Moffat and Gillies [19] was based on an earlier discussion by Ross on the exclusion of ZPE by a conducting material [20]. The argument is the following. If space is filled with ZPE in vacuum modes, a conducting metal sphere should exclude the ZPE, up to some cut off wavelength that depends on the properties of the metal. Therefore, there is a void in the all pervading ZPE in the region where the metal is. The missing ZPE has been calculated by Ross as

$$\int 4\pi\hbar ck^3 dk = \int_0^{E_c} \frac{4\pi}{(\hbar c)^3} E^3 dE \quad (1)$$

where E_c is the cut-off energy, taken as about 15.5 eV for Aluminium. The estimated missing ZPE is then 2.4×10^{19} eV/cm³ which is about 10^{-14} of the rest mass energy. This is just outside the range of terrestrial experiments on the WEP, but well within the sensitivity of the planned space-based experiments [12, 13, 21]. This is the basis on which Moffat and Gillies thence suggested that the presently planned space-based experiments, in particular Project SEE [21], study the case in which the free fall of an Aluminium ball is compared to that of a similar ball made of another material, or of a hollow ball, to obtain a 1 % test of the WEP for ZPE.

It is perhaps important to note a counter-point to this possibility that arose during discussions of this proposal. The issue is whether the absence, rather than the presence, of ZPE in a region can be used for inferring its gravitational properties. This issue requires further discussion, and if feasible, the importance of such an experiment cannot be overstated. However, we need to record a conflict between the feasibility of this experiment and cosmology, much the same as the conflict between the concept of large zero-point energy density and cosmology. Since the rate of expansion of the universe, or the Hubble parameter, determines the energy density of the universe to be only about 10^{-29} g/cm³, or about 10^4 eV/cm³, the (missing) ZPE in all the fields together could be constrained to be less than 10^{-15} of what Ross had estimated for Aluminium. If this is the case, then it would mean that the excluded ZPE in metals is only 10^{-29} of their rest mass, and therefore not accessible in the standard WEP experiments. This argument could be turned around to get a significant conclusion about cosmology itself, if the experiment indeed shows a violation of the UFF due to excluded ZPE at the level of

$\eta \approx 10^{-15}$. Then our present ideas about the cosmological expansion being driven by the total energy (density) content in the universe according to the Robertson-Walker solution of the Einstein's equation will need a drastic modification.

We would like to point out here that the picture of zero-point modes of radiation contributing to the Casimir force is perhaps incorrect, as indicated by another conflict with cosmology. The experiments that measure Casimir force seem to measure the theoretically predicted force, which is the gradient of the Casimir energy contained within the conducting boundary walls. This gradient dE/dr is of the order of 10^{-3} N at a separation of 100 nm, for plates of unit area. Consider a small change of 1 nm around 100 nm, where the force remains constant to a few percent. The ZPE in the volume $1 \text{ nm} \times 1 \text{ cm}^2$, as dictated by agreement between theory and experiment, is $10^{-3} \text{ N} \times 1 \text{ nm} = 10^{-12}$ J; a definite result required to be true because experiments have made measurements in this range of separation [22]. This means that the energy density in zero-point modes, if they have a reality, is at least 10^{-5} J/cm³. But the maximum average density of the universe is only 10^{-14} J/cm³. We see that even the renormalized calculation of Casimir energy assuming the reality of zero-point modes cannot be sustained in consistency with cosmological observations and theory. This implies that the true reason for the Casimir force is not a differential pressure created by imbalanced zero-point modes inside and outside a cavity.

The question of the validity of WEP for the ZPE of matter, as opposed to that of fields, is comparatively straightforward. Molecular oscillators and similar microscopic quantum matter oscillators that constitute bulk matter should contain zero-point energy corresponding to their typical vibrational frequencies. Crystal lattice vibrations also have zero-point modes of high frequency. A rough estimate for crystalline bulk material with a Debye frequency of about a THz indicates that this could be at least be at the level of milli-electron volts, and consequently 10^{-12} of the proton mass. Clearly a test of the WEP at the level of $10^{-13} - 10^{-14}$ will start to give definite answers about the UFF for ZPE of matter oscillators. There is no conflict with cosmology in this case since this ZPE density is always much smaller than the matter energy density itself.

3.5 WEP for Fermi energy

A related issue is whether quantum mechanical energy associated with the confinement of fundamental particles, like that of the nucleon in a heavy nucleus, obeys

the UFF. At zero temperature, the fermionic nucleons are distributed subject to the quantum uncertainty requirement and the Pauli exclusion principle. The typical momentum of the nucleon due to the confinement is given by

$$p_r \approx \frac{\hbar}{\Delta r} \quad (2)$$

where Δr is the size of the confining volume, about a femtometer (10^{-15} m). The equivalent energy associated with this zero point momentum, $E_z \approx p_r^2/2m_n$, about 10^7 eV, is a non-negligible fraction of the rest mass. Clearly, even a crude test of the equivalence principle at the level of 10^{-3} is sufficient to confirm that the Fermi energy of nucleons, constituting a few percent of the rest mass, obeys the UFF and WEP.

The question whether the associated pressure, the Fermi pressure, is a source of gravity is more subtle and we will discuss this in a later section.

3.6 WEP for spin

Whether or not there are anomalous couplings between a quantum mechanical or classical spin (angular momentum) and a gravitating mass have been discussed extensively. Several experiments have searched for spin dependent anomalous couplings to gravity, and none that is beyond the standard general relativistic coupling has been found [23]. The standard coupling arises very similar to the current-current coupling in electromagnetism, and hence these can be described in the language of gravito-magnetic couplings. An anomalous coupling can arise, for example, in the form $\alpha_{sg} \mathbf{s} \cdot \mathbf{g}/c$. The dimensionless coupling constant α_{sg} is expected to be of order unity. This is usually motivated in the context of a future quantum theory of gravity, or in the context of Einstein-Cartan theory with torsion, and clearly such a coupling is anomalous in various ways. It violates parity and time reversal symmetries, and it violates the equivalence principle in gravity. Therefore, it is considered very important to obtain constraints on the coupling constant α_{sg} at a level below unity. Indeed, there are dedicated experiments [24] that specifically look for effects of such couplings, hoping to reach a constraint of $\alpha_{sg} \approx 1$. To get an idea of the coupling energy or forces that are involved in such experiments, we note that the energy of coupling in the earth's gravitational field is $\alpha_{sg} s (3 \times 10^{-8})$ joule. Therefore, the quantum mechanical spin precession from the anomalous coupling is only at the level of 10^{-8} Hz in the earth's field. The best constraint on α_{sg} till now is from a spectroscopy experiment using Mercury, and $\alpha_{sg} < 60$ [25] For a macroscopic spinning object in the

laboratory, the energy of interaction could be a fraction of a joule, a miniscule 10^{-18} of the rest mass energy.

Recently, we have noticed that the data from the ever resourceful Lunar Laser Ranging mission can be effective in constraining the value of α_{sg} well below unity [26]. For a macroscopic spinning object, the anomalous force is proportional to a component of the gravitational curvature or the gradient in the gravitational field. We get

$$F_{sg} = \alpha_{sg} \nabla(\mathbf{s} \cdot \mathbf{g})/c = \alpha_{sg} \mathbf{s} \cdot \hat{\mathbf{g}} \frac{dg}{dr}/c \quad (3)$$

where we consider the variation of the magnitude of gravitational field as a function of distance from the centre of a gravitating massive object. In the field of the earth, for example, we get for the magnitude of the force

$$F_{sg} \approx \alpha_{sg} s (2 \times 10^{-14}) \text{ N} \quad (4)$$

Since this can be written as $F_{sg} = g(\delta m)$, this will lead to apparent weight changes of a spinning body at the level of $\alpha_{sg} s (2 \times 10^{-15})$ kg. For practically attainable spin rates, this apparent weight change is typically smaller than 10^{-15} kg, and not of much help in obtaining a constraint on α_{sg} in the laboratory on earth.

If we look at the magnitude of acceleration of a spinning macroscopic body due to the anomalous coupling, we get

$$a_{sg} = F_{sg}/m = \alpha_{sg} s \frac{dg}{dr}/(mc) \approx \alpha_{sg} R^2 \omega \frac{dg}{dr}/c \quad (5)$$

R is the size of the spinning object. Though this is independent of the mass, it violates the universality of free fall and hence the equivalence principle. In the earth's field, dg/dr is about $3 \times 10^{-6} \text{ m} \cdot \text{s}^{-2}/\text{m}$, and therefore the anomalous acceleration of a spinning sphere with 3 cm radius and spin rate of 100 Hz is only about $3 \times 10^{-15} \text{ m/s}^2$. This indicates that a terrestrial experiment on the universality of free fall can barely surpass the present best constraint on α_{sg} since the projected sensitivity of such experiments is limited to about 10^{-15} m/s^2 . However, the proposed space based experiments on the test of the equivalence principle can probe α_{sg} less than unity, in principle.

The gradient of the sun's gravitational field near the earth is orders of magnitude smaller than of the earth's field near its surface. dg/dr of the sun near earth is approximately $4 \times 10^{-14} \text{ m} \cdot \text{s}^{-2}/\text{m}$. Therefore, the anomalous acceleration of a spinning body is

$$a_{sg} \approx \alpha_{sg} R^2 \omega \frac{dg}{dr}/c \approx \alpha_{sg} R^2 \omega (1 \times 10^{-22}) \text{ m/s}^2 \quad (6)$$

This expression and the observation that the quantity $R^2 \omega$ is much larger for the earth compared to the moon

is the basis of our new best constraint on α_{sg} . $R^2\omega$ for the earth is approximately 10^9 . The spin of the earth is not perpendicular to the orbital plane, and therefore $\mathbf{s} \cdot \mathbf{g}$ is nonzero. Putting in the angle factor, we see that the anomalous acceleration is about $a_{sg} \approx 4 \times 10^{-14} \text{ m/s}^2$. To be conservative in our estimate, and to justify the approximations, we set the expected acceleration as $a_{sg} \approx 10^{-14} \text{ m/s}^2$. The Lunar Laser Ranging observes that the differential acceleration between the earth and the moon towards the sun, normalized to the average acceleration ($\Delta a/a_\odot$) is less than about 10^{-14} . This means that a spin dependent anomalous acceleration of the earth is less than $a_{sg} < 10^{-14} \times a_\odot \approx 10^{-14} \times 0.006 \text{ m/s}^2 = 6 \times 10^{-17} \text{ m/s}^2$. Comparing this with the expected anomalous acceleration for $\alpha_{sg} = 1$, gives us the constraint $\alpha_{sg} < 6 \times 10^{-3}$. This is a remarkable constraint, given the history of attempts to obtain stringent constraints on α_{sg} .

We note that it is still worth pursuing experiments with quantum spins, aiming to reach $\alpha_{sg} < 1$, since one might imagine a fundamental difference in the angular momentum associated with a quantum spin, and that associated with a classical spin, even though the angular momentum from a collection of quantum spins can be converted to a rotation of a classical macroscopic object.

4 WEP and quantum mechanics

Much has been written about the question of whether quantum dynamics obeys the WEP. Unfortunately, there is also much confusion regarding this issue and there are several papers that claim the apparent violation of the equivalence principle in a quantum mechanical context. All these incorrect conclusions arise from imprecise statement of the equivalence principle itself [3]. Since the classical tests of WEP depend on tests of UFF, somehow the impression has been created that trajectories and also other dynamical characteristics of propagation of a particle in space should not depend on its mass and other internal properties for the WEP to be valid. The confusion in the quantum mechanical situation arises mostly from the observation that the dynamical phase depends on the energy and mass, and that there is energy-dependent phase change in a gravitational field. However, the correct statement of the WEP is not that the trajectories and dynamics are independent of the mass. The correct and precise statement is that the dynamics in a uniform gravitational field is the same as in a frame under uniform acceleration with $\mathbf{a} = -\mathbf{g}$.

It is possible to prove that quantum dynamics obeys the WEP at the same level as classical dynamics does [3].

Before making a general claim, we consider a specific example to bring out the essential point. Consider a matter interferometer consisting of coherently split paths x_1 and x_2 , recombined at a detector. The relative phase accumulated by the quantum wavefunction representing the particle propagating in a gravitational potential $\phi(x)$ is

$$\delta\varphi = \frac{m_g}{\hbar} \left(\int_{x_1} \phi_1(x) dt_1 - \int_{x_2} \phi_2(x) dt_2 \right) \quad (7)$$

The integrals are over the two relevant paths. This phase shift is the same as the gravitational phase shift measured in neutron interferometry experiments. The phase shift in a gravitational field for this interferometer is well known and it is given by

$$\delta\varphi = \alpha \frac{m_g m_i g A \lambda}{\hbar^2} \quad (8)$$

where g is the gravitational acceleration, A is the area of the interferometer, and λ is the de Broglie wavelength. α is a geometrical factor that depends on the orientation of the interferometer. Since the phase shift depends explicitly on the gravitational mass, many authors write that this term, as well as similar terms in analogous situations, demands a separate quantum version of the equivalence principle. The presence of the Planck's constant is sometimes interpreted as "some quantum signature in gravity" [27]. However, this term is fully consistent with the WEP. To see this, we now calculate the expression for the phase shift, without gravity, in an accelerating frame. This can be easily done, for an accelerating frame and for the case in which there is a gravitational field, using the perturbation technique in path integral formalism [3]. The Lagrangian in both cases is quadratic and the gravitational field or the acceleration is treated as a perturbation. The phase shift is given by the time integral of the perturbation of the Lagrangian over the unperturbed classical path. For the gravitational case, the perturbation to the Lagrangian is $m_g g z$, for a uniform gravitational field in the vertical direction, z . Then the phase shift is

$$\delta\varphi = \frac{1}{\hbar} \oint m_g g z(t) dt = \frac{m_g g A}{\hbar v_0} = \frac{m_g m_i g A \lambda}{\hbar^2} \quad (9)$$

Now we calculate the phase shift in the same interferometer in an accelerating frame, with uniform acceleration a . The perturbation to the free Lagrangian, to first order, is $\delta L = \frac{1}{2} m_i v^2(t) - \frac{1}{2} m_i v_0^2 = m_i v_0 a t$. The phase shift then is

$$\delta\varphi = \frac{v_0}{\hbar} \oint m_i a t dt = \frac{v_0 m_i a A}{\hbar v_0^2} = \frac{m_i a A}{\hbar v_0} = \frac{m_i m_i a A \lambda}{\hbar^2} \quad (10)$$

The quantum phase shifts in Eqs. (9) and (10) are identical, if the classical equivalence principle, ($m_i = m_g$), is valid for the particles in the interferometer. The fact that the phase shift depends on the gravitational mass is no indication of any violation of the equivalence principle. We can conclude that if the classical equivalence principle is verified for atoms or particles using a torsion balance or by any other technique, then quantum interferometry using these particles cannot violate the equivalence principle. The issue of whether the atoms are bound in the form of a macroscopic object or not is not relevant, since the equivalence principle is also verified to great accuracy for various forms of binding energies.

Now we give a general proof, discussed earlier in Ref. [3]. We consider the evolution from time t_a to t_b of a quantum state consisting of an arbitrary superposition of mass eigenstates. The Lagrangian of the quantum system in a gravitational field as well as in an accelerating frame is at most quadratic in velocities and coordinates. The quantum propagator is

$$K(t_b, t_a) \equiv \langle \psi_b | U(t_b, t_a) | \psi_a \rangle = F(t_b, t_a) \exp(iS_{cl}/\hbar) \quad (11)$$

The quantum path integral with phase contributed entirely from the classical path gives the exact result for such a situation. The classical equivalence principle dictates that the classical path in a gravitational field g is exactly same as the classical path in an accelerated frame with acceleration $-a$. Then the classical actions are identical for the two situations. Therefore, the path integrals, and the phase shifts are identical in the two situations, without any approximations. Therefore, $F(t_b, t_a)$ is identical for the cases of an external gravitational field and that of an accelerating frame, for evolution between fixed times t_a and t_b .

We get

$$K(t_b, t_a)|_g = K(t_b, t_a)|_{a=-g} \quad (12)$$

where the subscripts identify the gravitational field and the accelerating frame. Since the quantum propagators are identical, the quantum evolutions in the two situations are identical for arbitrary quantum states.

Several results on the WEP and UFF in tests involving quantum mechanical systems are discussed in a review article by Gillies and Unnikrishnan [5]. The equivalence of classical and quantum evolution under the assumption of the validity of the WEP is discussed also by them.

5 Equivalence principle and exotic sources

Experiments that look for a violation of the universality

of free fall compare the acceleration of two test bodies with different internal structure, especially their nuclear properties, towards a massive source body. The source bodies could be large laboratory masses, the earth or the sun, with their mean gravity fields of the order of 10^{-7} m/s², 10 m/s² or 6×10^{-3} m/s². While the sun is the most convenient of sources when a standard torsion balance is used, the earth's centrifugal acceleration field of about 10^{-2} m/s² could also be used, as in the classic Eötvös experiment. Space experiments benefit from the relatively much larger field from the earth, a gain of about 1000 compared to a torsion balance experiment in the sun's field. The standard experiments can also be analyzed for anomalous differential acceleration towards exotic hypothetical sources of gravity, like the dark matter in our galaxy. The gravitational field of the dark matter halo is less than 10^{-10} m/s², and therefore the Galileo-Eötvös parameter is limited to about $\eta \leq 5 \times 10^{-5}$ for the state of the art differential acceleration sensitivity of 3×10^{-15} m/s² [6, 7]. Since the existence and properties of the dark matter halo are inferred to fit the observations within the standard Newtonian gravity paradigm, tests at this level of sensitivity are not expected to help in inferring new aspects of the gravitational properties of dark matter. Nevertheless, such analyses have their value in diversifying our way of thinking about the various ways in which the UFF might be violated. Since the hypothetical dark energy itself is supposed to be smoothly distributed, its local gravitational field is essentially zero, and experiments on UFF do not address the gravitational properties of dark energy, nor of the other smooth components in the universe like the massive neutrinos. However, interferometry with light and matter waves is sensitive to any anomalous coupling since the phase shift depends on the interaction energy rather than on the force.

6 The equivalence of the active and passive mass

There has been a progressive refinement and consequent detailed structure of the concept of mass from Newtonian gravity to modern general relativity and its variants. In the context of a generalized theory of gravitation there are three distinct types of "masses" that are relevant. The "passive gravitational mass" is the same mass that is acted upon by gravity. The response of a body to the gravitational interaction (or to any force) is decided by the "inertial mass". The mass that is the source of gravity, the gravitational charge, is the "active gravitational mass" [1, 2].

The standard tests of WEP and UFF probe the universality of the ratio of gravitational mass to the iner-

tial mass. To establish the assumed universality of the ratio of active and passive gravitational mass (EAPM) requires a conceptually different type of test, as first performed by Kreuzer [28]. The EAPM of the active gravitational mass and the passive gravitational mass is not that well established in the laboratory, when the numbers are compared to the tests of WEP. This is partially because the theoretical issues involved are not considered as fundamental as in the case of a violation of the WEP. Also, it is possible to constrain a hypothetical violation by invoking conservation laws since it can be shown that a violation of EAPM will lead to self acceleration of a body. In spite of this, this is considered an important measurement in the context of Post-Newtonian gravity [1].

The best constraint on any violation of the EAPM is from the absence of any observable self acceleration of the moon in its orbit, derived from the Lunar Laser Ranging (LLR) data [29]. To see the important argument, consider two spherical bodies A and B connected rigidly, with their geometrical centres separated by distance R . Let M_{Ap} and M_{Bp} be their passive gravitational masses. Further, assume that the body A has an active gravitational mass of $M_{Aa} = M_{Ap}$. This means that its gravitational force on the body B is

$$F_{A-B} = -\frac{GM_{Aa}M_{Bp}}{R^2} \quad (13)$$

Now, if the body B has an active gravitational mass M_{Ba} that is different from M_{Bp} , there will be self acceleration. If $M_{Ba} = M_{Bp}(1 + \delta)$, then the gravitational force of B on A is

$$F_{B-A} = -\frac{GM_{Ba}M_{Ap}}{R^2} = -\frac{GM_{Bp}M_{Aa}(1 + \delta)}{R^2} \quad (14)$$

Therefore there is an imbalance of forces,

$$\Delta F = F_{B-A} - F_{A-B} = -\frac{\delta GM_{Bp}M_{Aa}}{R^2} \quad (15)$$

resulting in self-acceleration

$$a_s = -\frac{\delta GM_{Bp}M_{Aa}}{(M_{Ap} + M_{Bp})R^2} = -\frac{\delta GM_{Aa}}{(M_{Ap}/M_{Bp} + 1)R^2} \quad (16)$$

Here we have assumed that the passive gravitational mass and the inertial mass are the same, using strong evidence from the experiments on the UFF. If the two passive masses are approximately of the same magnitude, then the self acceleration is $a_s \approx -\delta GM_{Aa}/(2R^2)$. Since the moon's internal structure consists of an asymmetric nonspherical distribution of iron and silicon, violation of EAPM will lead to self-acceleration and secular change in the period of the orbit, and one can test for the hypothesis that the ratio of the active and passive grav-

itational masses for the two materials are different. A detailed analysis of the LLR data by Bartlett and van Buren provided the constraint, $\delta < 4 \times 10^{-12}$. Since the difference in the lepton (electron) fractions in iron and Aluminium is 5 %, this result confirms the equality of the active and passive gravitational masses of electrons with high confidence.

A direct result from laboratory experiments turns out to be very difficult. In fact, Unnikrishnan and Gillies [30, 31] pointed out that only the most recent high precision measurements of the gravitational constant at 100 ppm or better are able to provide direct evidence that leptons like electrons have a nonzero active gravitational mass. They also discussed ideas for new experiments that could improve the existing constraints by two orders of magnitude.

In this context, an issue that had been discussed is whether the Fermi pressure of confined nucleons contribute to gravity, as the Einstein's equation demands, as it equates the geometric distortions due to gravity to the energy-momentum tensor that contains the pressure terms [32, 33]. This again turns out to be a difficult question to answer directly from a laboratory experiment in spite of its importance as a new test of general relativity. Theoretically one expects that the source term of gravity is not just the density ρ , but the combination $(\rho + 3p)$, where p is the pressure. For relativistic matter $p \approx \rho/3$, and can contribute significantly to the external gravitational field. For non-relativistic matter, this is less, but still measurably significant. But the issue of measurement is made subtle, as observed by Ehlers *et al.* in that the surface tension that keeps the nucleons confined in the nucleus contributes a cancelling "negative gravity" effect, rendering the " $3p$ " term unobservable [32]. The new generation precision " G " experiments are sensitive enough to address the issue, and as discussed by Unnikrishnan and Gillies [34, 35], the absence of any anomalous dependence of G on the material of the source masses used (water and mercury in this case) in Zürich G experiment does give the definite indication that the active and passive gravitational masses corresponding to the Fermi pressure in nuclei are equal.

7 The equivalence principle and higher dimensional gravity

Most discussions of the gravitational interaction in the context of higher dimensional physics speculate deviations from the standard inverse square behaviour of gravity, even in the non-relativistic limit. Two popular models under continued test are the ADD model [36] of large

extra dimensions with the unification (quantum gravity) scale close to the electro-weak scale of 1 TeV, and the Randall-Sundrum model [37] with a 4D-brane with matter and weak gravity and another hidden 4D-brane with Planck scale gravity, both embedded in the higher dimensional bulk, with a certain folded symmetry imposed in the configuration. In both cases, there is a predicted departure from the inverse square law at short distances, and in the ADD model this deviation was expected to start at about a few hundred microns, accessible in experiments with torsion balances and cantilevers [38, 39]. This departure by itself does not give rise to a violation of the universality of free fall. But the additional couplings possible in such theories could in principle give rise to a violation of UFF and WEP. However, testing a violation of UFF at short distance is much more difficult than testing for a deviation from the inverse square law, and at present experiments have not started addressing the validity of the WEP in higher dimensional theories with large extra dimensions.

8 Why equivalence principle?

Is there an underlying “cause” for the near perfect universality of free fall, allowing a “principle” of equivalence to be formulated, serving as the basis for a fundamental theory of gravity? In the present scenario of standard gravity and general relativity, the UFF is treated as empirically valid, and its idealization is the principle of equivalence, assumed to be perfectly true. The only attempt at “deriving” the equivalence principle as a consequence of something more fundamental and universal was Mach’s speculation about the origin of inertial forces [40, 41]. If inertial forces have something to do with the surrounding matter, then the inertial mass itself can be considered as a secondary quantity and therefore one might be able to get to a causal basis for the equivalence between the gravitational and inertial masses.

This possibility has been recently explored rigorously by Unnikrishnan in the context of his theory of Cosmic Relativity [42]. The essential point is that a freely falling body does so in a local gravitational field in the presence of all the matter in the universe, and hence in their nearly isotropic and homogeneous gravitational potential. In a frame comoving with the freely falling object, the large matter-current of all the galaxies and other matter in the universe generates a gravito-magnetic vector potential $\mathbf{A}_U = (\mathbf{v}/c)\Phi_U$, where Φ_U is the local gravitational potential due to all the matter in the Universe. Therefore, there is a nonzero time derivative of this potential in an accelerating frame. A time dependent vector po-

tential signals a reactive gravitational force that opposes the motion,

$$\frac{m_g}{c} \frac{d\mathbf{A}_U}{dt} = -\mathbf{E}_g = m_g \frac{\partial \mathbf{v}}{\partial t} \frac{\Phi_U}{c^2} \quad (17)$$

If we identify $-\mathbf{E}_g$ as the force required to accelerate, and $m_g \Phi_U/c^2$ as the inertial mass m_i , we get Newton’s law of motion, from the gravity of the Cosmos! (The quantity $\Phi_U/c^2 \approx 1$ for our universe.) We see that Newton’s law is both relativistic and gravitomagnetic in nature [43]. This a true surprise since physicists are struggling to measure gravitomagnetic effects with high precision whereas our most familiar of laws is a direct manifestation of the large gravitomagnetism of the matter in the Universe, as felt in any frame moving relative to this matter. The velocity appearing is the “absolute velocity” to be identified with the velocity relative to the CMBR. Since $m_i = m_g \Phi_U/c^2$, the ratio m_i/m_g is a property of the Universe and hence universal. One can see here the true foundation of the weak equivalence principle.

This can also be derived in the same form starting from the fact that the actual local metric with all the matter around us, in a freely falling frame, is the Robertson-Walker metric (transformed by the velocity) and not the Minkowski metric as usually assumed. Both are flat, but the former encodes a gravito-magnetic potential proportional to the velocity through the cosmic frame. So, we have now a rigorous demonstration of the secondary nature of the inertial mass and of the equality of the gravitational and the inertial mass [42, 43].

This result has a very interesting implication. In the standard frame work, the UFF and WEP are violated if the long range gravitational coupling (G) is composition dependent for some reason. Since the acceleration of a test body in the gravitational field of the source mass is $a = m_g g(G)/m_i$, the acceleration for different test bodies made of different material could be different. In Cosmic Relativity, however, the UFF is preserved because the masses are freely falling in the presence of all the matter in the universe (Fig. 2). Therefore, the inertial mass in the denominator is $m_i(G) = m_g \Phi_U(G)/c^2$, and the dependence on the material dependent G vanishes from the relation $a = m_g g(G)/m_i(G)$. This result goes beyond the premise of general relativity, and indicates that long range gravity will not violate the UFF even if the gravitational coupling G is composition dependent! Experiments that look for a violation of the UFF and the equivalence principle will therefore see a signal only in those cases where there is a short range component to gravity (then the matter in the universe is irrelevant), or if there is a new composition dependent force, not related to gravity.

Thus we have established the causal foundation of the equivalence principle and the universality of free fall.

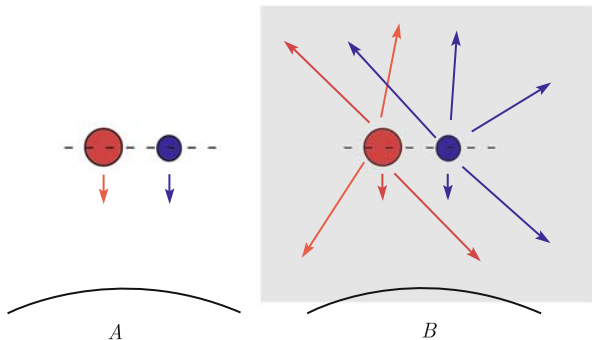


Fig. 2 The left panel shows the conventional view of a “free-fall” experiment. The right panel shows the factual situation, and the shaded area represents the matter distribution in the universe. Since the test masses are falling in the presence of all the matter to which they are gravitationally coupled, the UFF remains intact even with a composition dependent gravitational coupling—a surprising result.

9 Summary

We have reviewed some exotic aspects of the Equivalence Principle and the Universality of Free Fall, commenting on their validity in the context of the “free-fall” of nonclassical matter such as quantum matter, neutrinos, zero-point energy, Fermi energy etc. We have shown why quantum matter in general would obey the UFF, at the level it is tested with classical matter. Indirect arguments based on energy conservation and cosmology that help to yield constraints on the violation of UFF in situations where a laboratory experiment is not feasible were mentioned. The possibility of a test of WEP and UFF for zero-point energy and Fermi energy was discussed. Finally we have discussed a profound physical basis of the validity of the UFF, proving its general validity even in the case when the gravitational coupling is composition dependent, based on the long-range gravitational link to all the matter in the universe.

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