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Full-wavefield modeling using a decoupled viscoacoustic wave equation with vector-reflectivity

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Abstract

Conventional Q-compensated least-squares reverse time migration generally employs a linearized viscoacoustic modeling operator based on the first-order Born approximation, which simulates only primary reflections in synthetic data. As a result, it cannot properly match seismic data containing multiples, leading to prominent crosstalk artifacts and reduced image quality. To overcome this limitation, we started with the relaxation function of the generalized standard linearized solid model and derived a viscoacoustic wave equation with vector-reflectivity. This formulation enabled the simulation of full viscoacoustic wavefields that included both primary and multiple reflections, and it could be numerically solved using finite-difference algorithms. Numerical experiments demonstrated that the wavefields generated using the proposed vector-reflectivity equation were equivalent to those produced using the original variable-density viscoacoustic wave equation. To further investigate the attenuation characterizations, we decoupled the dissipation and dispersion effects in the viscoacoustic wave equation with vector-reflectivity and derived a corresponding decoupled formulation. Numerical experiments demonstrated that the decoupled viscoacoustic wave equation with vector-reflectivity accurately simulated both dissipation and phase-dispersion wavefields. Based on decoupled characteristics and full-wavefield simulation capabilities, the proposed formulation provides an effective linearized forward-modeling engine for Q-compensated least-squares reverse time migration.

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1. Introduction

The anelastic properties of subsurface media lead to the conversion of seismic wave energy into heat as waves propagate through attenuating media.¹ This intrinsic attenuation effect, quantified by the quality factor Q , is especially significant in hydrocarbon reservoirs and gas-bearing sandstones, causing both amplitude loss and phase distortion in the seismic wavefield, thereby degrading the quality of the seismic data.² When

numerical simulation methods for seismic wavefields neglect attenuation effects, significant discrepancies arise between synthetic and field data in both amplitude and phase, ultimately leading to imaging results characterized by weak amplitudes, mispositioned events, and reduced resolution. Therefore, it is essential to develop an accurate and efficient viscoacoustic wave equation incorporating intrinsic attenuation that simulates the attenuation effects on seismic wavefields.

In seismic exploration, scholars employ attenuation models to characterize constant-Q behavior within the seismic frequency band, thereby constructing various viscoacoustic wave equations with distinct characteristics. The constant-Q model,³ capable of accurately characterizing attenuation behavior across the seismic exploration frequency band, has garnered increasing attention in recent years. Based on the constant-Q model, decoupled fractional Laplacians viscoacoustic wave equation with amplitude-loss and phase-dispersion terms have been developed and widely used in attenuating wavefield forward modeling,⁴⁻⁹ Q-compensated reverse time migration (Q-RTM),^{10,11} and Q-compensated least-squares reverse time migration (Q-LSRTM).¹²⁻¹⁵ However, numerically solving the fractional Laplacian operators requires multiple Fourier transforms using the pseudo-spectral method at each time step, leading to prohibitive computational costs in three-dimensional applications and industrial-scale implementations.¹⁶⁻¹⁸ Moreover, as a global algorithm, the pseudo-spectral method is difficult to scale to graphics processing unit-based parallel computing. Another numerical challenge is that the fractional Laplacian operator is a space-wavenumber mixed-domain operator. Direct implementation using the pseudo-spectral method introduces strong Gibbs phenomena, thereby impairing the accuracy of wavefield simulation.

Another commonly used attenuation model is the generalized standard linearized solid (GSLs) model.^{19,20} Different from the constant-Q model, which is constructed through mathematical formulations, the GSLs model is built using physical components—springs and dashpots—as its fundamental elements. Viscoacoustic wave equations derived from this model can be efficiently solved using finite-difference methods, making them broadly applicable in seismic forward modeling and migration.²¹⁻²⁹ The GSLs model estimates constant-Q behavior by configuring multiple standard linear solid (SLS) relaxation parameters,^{19,30} yielding time-domain wave equations with numerous, intricately related relaxation terms. Consequently, attenuation effects are implicitly represented through parameter relationships rather than explicitly appearing in the equations.^{31,32} Theoretically, as

the number of SLS models increases, the GSLs attenuation model provides a more accurate estimation of constant-Q behavior, enabling the derived viscoacoustic wave equations to better describe seismic wave propagation in attenuating media. However, this indiscriminate increase in the number of SLS models inevitably increases model complexity and substantially raises computational costs. Clearly, a judicious balance must be struck among parameter complexity, physical model accuracy, and computational efficiency.

Recently, a relaxation function containing an explicit Q factor and the viscoacoustic wave equation derived from it has attracted considerable attention in the research community.³¹ The explicit appearance of the seismic quality factor Q in the viscoacoustic wave equation not only avoids the complexity associated with relaxation parameters but also facilitates the derivation of gradient formulas with respect to Q in both Q-LSRTM and Q Full Waveform Inversion. Beyond parameter complexity, conventional GSLs-based viscoacoustic wave equations face a fundamental limitation in their implementation for migration and inversion: the mathematical coupling between amplitude dissipation and phase dispersion. Bai *et al.*²² eliminated the memory variable from the single-SLS viscoacoustic wave equation and isolated an amplitude loss term characterized by a pseudo-differential operator. However, the phase dispersion term was omitted in their formulation, resulting in a decoupled viscoacoustic wave equation that accounted only for amplitude loss and was incapable of simulating phase dispersion effects in attenuated wavefields. Li *et al.*¹¹ and Fathalian *et al.*²⁴ derived a decoupled viscoacoustic wave equation with amplitude-loss and phase-dispersion terms based on the single-SLS model. However, in these single-SLS decoupled viscoacoustic wave equations, the amplitude loss term took the form of a pseudo-differential operator, whose numerical implementation relies on the computationally expensive pseudo-spectral method. This not only hinders their extension to viscoelastic media but also limits their applicability in industrial production. Moreover, a single-SLS relaxation mechanism can only approximate constant-Q behavior over a relatively narrow seismic exploration frequency band. Consequently, the single-SLS decoupled viscoacoustic wave equations fail to achieve satisfactory numerical simulation performance in strongly attenuating regions. The migration process requires reversing the sign of the amplitude attenuation term in the decoupled viscoacoustic wave equation while maintaining the sign of the phase dispersion term, thereby constructing an attenuation-compensated viscoacoustic wave equation.^{3,4} This inherent coupling in the GSLs viscoacoustic wave equation substantially limits their

applicability in both Q-compensated migration and inversion.

Q-compensated least-squares reverse time migration effectively mitigates attenuation effects, significantly enhancing deep-target illumination, improving resolution, and preserving amplitude fidelity.^{5-7,33-36} However, conventional Q-LSRTM typically relies on linearized modeling operators constructed under the first-order Born approximation, which simulate only primary reflections. This creates a mismatch between synthetic records and actual seismic records that contain full-wavefield information, including multiples and converted waves. This data mismatch leads to slow iterative convergence and introduces crosstalk artifacts in the imaging results, primarily due to multiples. To obtain high-quality Q-LSRTM images, seismic data are usually preprocessed to suppress multiples,^{37,38} thereby avoiding crosstalk artifacts in the final images. Nevertheless, accurate identification and effective separation of multiples remain technically challenging, particularly in complex geological structures. Consequently, there is a clear need to develop a novel linearized viscoacoustic wave equation that is both suitable for least-squares inversion and capable of accurately describing attenuation while simulating full-wavefield responses—including primaries and multiples.^{39,40} Establishing such an equation would enable high-precision attenuation-compensated imaging directly from full-wavefield data, without relying on multiple suppression. This advancement would enhance the practicality, robustness, and ultimate imaging quality of Q-LSRTM.

Traditional seismic forward modeling typically considers only velocity parameters while neglecting the influence of density variations on wavefield amplitudes. This simplification introduces amplitude discrepancies between synthetic and observed data in least-squares inversion, with amplitude distortions becoming particularly pronounced in zones of strong lateral density contrast. In recent years, an acoustic wave equation with vector-reflectivity has been proposed that avoids explicit density parameters.⁴¹⁻⁴³ This approach enables full-wavefield forward modeling using only a velocity model and iteratively updated reflectivity images of least-squares inversion. Studies have shown that the wavefields simulated by this method achieve numerical accuracy comparable to those from variable density acoustic equations and effectively reproduce complex wavefield responses, including primaries and multiples.^{41,44} Compared to linearized forward operators based on the first-order Born approximation, the vector-reflectivity equation can directly utilize the iteratively updated velocity

and reflectivity models to accurately simulate full-wavefield data that includes multiple reflections. This results in improved kinematic and dynamic consistency between synthetic and observed records. Therefore, developing a viscoacoustic wave equation with vector-reflectivity promises several enhancements for Q-LSRTM: (i) more accurate amplitude information without requiring density parameters; (ii) effective suppression of imaging artifacts caused by multiple-related crosstalk; and (iii) accelerated convergence and enhanced inversion stability due to better data misfit reduction. This approach offers a promising technical pathway for high precision Q-LSRTM.

In this paper, we started from the relaxation function of the GSLS model and derived a variable-density viscoacoustic wave equation with explicitly expressed Q. Based on this formulation, we then introduced a vector-reflectivity representation and derived the viscoacoustic wave equation with vector-reflectivity that enables the simulation of viscoacoustic wavefields containing both primary reflections and multiples without explicit knowledge of density. Subsequently, we decoupled the amplitude dissipation and phase dispersion effects of the viscoacoustic wave equation with vector-reflectivity, thereby making it suitable for Q-LSRTM. Finally, the numerical experiments demonstrated the feasibility and effectiveness of the proposed approach.

2. Methods

2.1. Derivation of the viscoacoustic wave equation with vector-reflectivity

In attenuating media, stress is represented as the convolution of a relaxation function with the strain history of time. According to the GSLS model, the relaxation function $M(t)$ in attenuating media can be given as **Equation 1**.^{20,32,45,46}

$$M(t) = M_0 \left[1 + \frac{1}{Q} \sum_{l=1}^L D_l \exp(-t/\tau_l) \right] H(t) \quad (1)$$

where $M_0 = \rho v_{p0}^2$ denotes the relaxed modulus, t is the time variable, ρ is density, v_{p0} is the relaxed P-wave velocity, Q is the P-wave quality factor, $l \in [1, L]$ represents the number of relaxation mechanisms, and $H(t)$ is the step function. D_l and τ_l are the weights and decay times of l th relaxation mechanism, which are calculated by optimizing algorithm to approximate constant-Q effects over a preset seismic exploration frequency band. In this paper, we use $L = 3$, and the resulting parameters are defined in [Table 1](#).^{31,32}

Table 1. The coefficient of D_l and τ_l

Coefficients	Values
D_1	1.74278563
D_2	1.41221145
D_3	1.72183357
τ_1	0.00159180
τ_2	0.01450505
τ_3	0.14020500

Based on the relaxation function defined in **Equation 1**, combining the stress-strain relationship and the second Newton's law, the second-order variable-density viscoacoustic wave equation with the explicitly expressed Q is derived as **Equations 2 and 3**.⁴⁵

$$\frac{\partial^2 p}{\partial t^2} = \rho v^2 \left[1 + \frac{1}{Q} \sum_{l=1}^L D_l \right] \nabla \cdot \frac{1}{\rho} \nabla p - \frac{1}{Q} \sum_{l=1}^L r_l \quad (2)$$

$$\frac{\partial r_l}{\partial t} = -\frac{1}{\tau_l} \left\{ r_l - D_l \rho v^2 \nabla \cdot \frac{1}{\rho} \nabla p \right\} \quad (3)$$

where r_l is the memory variable defined in l th SLS relaxation mechanism, and p is the pressure wavefield, v is the particle velocity. ∇ is the gradient operator and $\nabla \cdot$ is the divergence operator. The above variable-density viscoacoustic wave equation accurately captures the characteristics of the full seismic wavefields in attenuating media. Forward modeling using **Equation 1** requires three parameters as input: velocity, density, and Q . However, obtaining an accurate density field remains particularly challenging. By incorporating the impedance definition, we rewrite **Equations 2 and 3** into **Equations 4 and 5**:

$$\frac{\partial^2 p}{\partial t^2} = vz \left[1 + \frac{1}{Q} \sum_{l=1}^L D_l \right] \nabla \cdot \frac{v}{z} \nabla p - \frac{1}{Q} \sum_{l=1}^L r_l \quad (4)$$

$$\frac{\partial r_l}{\partial t} = -\frac{1}{\tau_l} \left\{ r_l - D_l zv \nabla \cdot \frac{v}{z} \nabla p \right\} \quad (5)$$

where $z = \rho v$ denotes the acoustic impedance. Using a simple mathematical divergence algorithm, the above velocity-impedance viscoacoustic wave equation can be further rewritten as **Equations 6 and 7**:

$$\frac{\partial^2 p}{\partial t^2} = \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l \right) \left(v^2 \nabla^2 p + v \nabla v \cdot \nabla p + v^2 z \nabla \frac{1}{z} \cdot \nabla p \right) - \frac{1}{Q} \sum_{l=1}^L r_l \quad (6)$$

$$\frac{\partial r_l}{\partial t} = -\frac{1}{\tau_l} \left\{ r_l - D_l v^2 \nabla^2 p - D_l v \nabla v \cdot \nabla p - D_l v^2 z \nabla \frac{1}{z} \cdot \nabla p \right\} \quad (7)$$

Here, we define the vector-reflectivity $\mathbf{R}$ as the normalized rate of impedance change in each vector direction (**Equation 8**):⁴¹

$$\mathbf{R} = -\frac{1}{2} z \nabla \frac{1}{z} \quad (8)$$

Substituting **Equation 8** into **Equations 6 and 7**, we can obtain **Equations 9 and 10**:

$$\frac{\partial^2 p}{\partial t^2} = \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l \right) \left(v^2 \nabla^2 p + v \nabla v \cdot \nabla p - 2v^2 \mathbf{R} \cdot \nabla p \right) - \frac{1}{Q} \sum_{l=1}^L r_l \quad (9)$$

$$\frac{\partial r_l}{\partial t} = -\frac{1}{\tau_l} \left\{ r_l - D_l v^2 \nabla^2 p - D_l v \nabla v \cdot \nabla p + D_l 2v^2 \mathbf{R} \cdot \nabla p \right\} \quad (10)$$

As a result, **Equations 9 and 10** are the derived viscoacoustic wave equations with vector-reflectivity. In this way, we can conduct forward modeling using **Equations 9 and 10** with velocity, Q , and impedance as input parameters. The impedance is usually estimated from the migrated images. In seismic exploration, imaging results are more readily obtainable than estimates of density parameters. Moreover, imaging results inherently contain information related to density. Compared to using constant or inaccurate density as input for forward modeling, using estimated imaging results as input is more reasonable, as it accounts for the influence of density on wavefield simulation. To be noted, **Equations 2, 4, and 9**, as well as **Equations 3, 5, and 10**, are mathematically equivalent, and no approximations are introduced in the derivation. Ignoring the influence of density, **Equations 9 and 10** reduce to the viscoacoustic wave equation with constant density. In Section 3, we demonstrate that the wavefield simulated by **Equations 9 and 10** is equivalent to that simulated in **Equations 2 and 3**.

2.2. Derivation of the decoupled viscoacoustic wave equation with vector-reflectivity

Although the viscoacoustic wave equation derived in Section 2.1 with vector-reflectivity accurately captures the feature of seismic waves propagating in attenuating media,

it couples dissipation and dispersion effects in memory variables, which are not readily used to implement Q-compensated migrations. To decouple the dissipation and dispersion effects, we now proceed to derive the decoupled form of **Equations 9** and **10**. Here, we first expand the gradient operator and rewrite the derived equation as **Equations 11** and **12**:

$$\frac{\partial^2 p}{\partial t^2} = \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l\right) [v^2 \nabla^2 p + v(\partial_x v \cdot \partial_x p + \partial_z v \cdot \partial_z p) + v^2 z \left(\partial_x \frac{1}{z} \cdot \partial_x p + \partial_z \frac{1}{z} \cdot \partial_z p\right)] - \frac{1}{Q} \sum_{l=1}^L \tilde{r}_l \quad (11)$$

$$\frac{\partial \tilde{r}_l}{\partial t} = -\frac{1}{\tau_l} \{r_l - D_l v^2 \nabla^2 p - D_l v(\partial_x v \cdot \partial_x p + \partial_z v \cdot \partial_z p) - D_l v^2 z (\partial_x \frac{1}{z} \cdot \partial_x p + \partial_z \frac{1}{z} \cdot \partial_z p)\} \quad (12)$$

The symbol ∂_i represents the directional derivative of the variable in i direction. We then apply the Fourier transform to **Equations 11** and **12** in the time domain to obtain the frequency-domain viscoacoustic wave equation with vector-reflectivity (**Equations 13** and **14**):

$$\omega^2 P = \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l\right) [v^2 k^2 P + v(k_x v k_x P + k_z v k_z P) + v^2 z \left(k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P\right)] + \frac{1}{Q} \sum_{l=1}^L \tilde{r}_l \quad (13)$$

$$i\omega \tilde{r}_l \tau_l = -\{\tilde{r}_l + D_l v^2 k^2 P + D_l v(k_x v k_x P + k_z v k_z P) + D_l v^2 z \left(k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P\right)\} \quad (14)$$

where ω denotes the angular frequency, P and $\tilde{r}_l$ are the frequency domain form of the time domain wavefield p and memory variables r_l , respectively. The memory variables in **Equation 14** can be expressed as

$$\tilde{r}_l = -\frac{1}{1 + i\omega \tau_l} \{D_l v^2 k^2 P + D_l v(k_x v k_x P + k_z v k_z P) + D_l v^2 z (k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P)\} \quad (15)$$

Substituting **Equation 15** into **Equation 13** and eliminating the memory variables, we can obtain the dispersion relation of the viscoacoustic wave equation

(**Equation 16**):

$$\begin{aligned} \omega^2 P = & \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l\right) [v^2 k^2 P + v(k_x v k_x P + k_z v k_z P) \\ & + v^2 z (k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P)] \\ & - \frac{1}{Q} \left\{ \sum_{l=1}^L \frac{D_l}{1 + \omega^2 \tau_l^2} (v^2 k^2 P + v(k_x v k_x P + k_z v k_z P) \right. \\ & + v^2 z (k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P)) \} \\ & + \frac{i\omega \tau_l}{Q} \left\{ \sum_{l=1}^L \frac{D_l}{1 + \omega^2 \tau_l^2} (v^2 k^2 P + \right. \\ & \left. v(k_x v k_x P + k_z v k_z P) + v^2 z (k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P)) \} \end{aligned} \quad (16)$$

Equation 16 is decomposed into two components: one containing only the real part controls phase dispersion, and the other containing only the imaginary part controls amplitude dissipation.^{2,28} If we ignore the dissipation-dominated effects, namely, the imaginary part, the dispersion-dominated dispersion relation can be expressed as **Equation 17**:

$$\begin{aligned} \omega^2 P = & \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l\right) [v^2 k^2 P + v(k_x v k_x P + k_z v k_z P) \\ & + v^2 z (k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P)] \\ & - \frac{1}{Q} \left\{ \sum_{l=1}^L \frac{D_l}{1 + \omega^2 \tau_l^2} (v^2 k^2 P + v(k_x v k_x P + k_z v k_z P) \right. \\ & \left. + v^2 z (k_x \frac{1}{z} k_x P + k_z \frac{1}{z} k_z P)) \} \end{aligned} \quad (17)$$

The frequency-related term $D_l / (1 + \omega^2 \tau_l^2)$ in **Equation 17**, transformed into the time-space domain, it becomes a space-wavenumber mixed domain operator, which cannot be solved using a finite-difference method. To effectively estimate this frequency-related term, we perform a polynomial fitting method. Moreover, to preserve the original intent of explicitly incorporating Q into the viscoacoustic wave equation, we select a polynomial $\sum_{l=1}^L D_l / [1 + \sum_{n=0}^N (a_n Q^{-n})]$ that includes Q to fit the term $\sum_{l=1}^L D_l / (1 + \omega^2 \tau_l^2)$, $n \in [0, N]$ is the degree of the polynomial. A higher polynomial order n yields a more accurate approximation, but at the cost of increased equation complexity and computational resources. In this study, we choose $n = 1$ for equation simplicity and computational cost. Thus, we make the following

approximation for $Q > 10$ and seismic frequency band [1, 125] Hz (**Equation 18**):⁴⁶

$$\sum_{l=1}^L \frac{D_l}{1 + \omega^2 \tau_l^2} \approx \sum_{l=1}^L \frac{D_l}{1 + 1/Q} \quad (18)$$

By inserting the approximation **Equation 18** into **Equation 17** and transforming **Equation 17** into the time-space domain, **Equation 19** can be derived:

$$\frac{\partial^2 p}{\partial t^2} = \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l - \frac{1}{Q} \sum_{l=1}^L D'_l \right) \left[v^2 \nabla^2 p + v(\partial_x v \cdot \partial_x p + \partial_z v \cdot \partial_z p) + v^2 z \left(\partial_x \frac{1}{z} \cdot \partial_x p + \partial_z \frac{1}{z} \cdot \partial_z p \right) \right] \quad (19)$$

Applying **Equation 8** in **Equation 19**, we have the dispersion-dominated viscoacoustic wave equation with vector-reflectivity (**Equation 20**):

$$\frac{\partial^2 p}{\partial t^2} = \left(1 + \frac{1}{Q} \sum_{l=1}^L D_l - \frac{1}{Q} \sum_{l=1}^L D'_l \right) (v^2 \nabla^2 p + v \nabla v \cdot \nabla p - 2v^2 \mathbf{R} \cdot \nabla p) \quad (20)$$

where $D'_l = D_l / (2 + 1/Q)$. In **Equation 20**, the unit 1 in parentheses represents the acoustic wave effect, while the term $\frac{1}{Q} \sum_{l=1}^L D_l - \frac{1}{Q} \sum_{l=1}^L D'_l$ corresponds to dispersion components. According to Mu and Huang,²⁶ the dissipation-dominant term equals the full viscoacoustic term minus the dispersion-dominant term. Thus, the dissipation-dominated viscoacoustic wave equation with vector-reflectivity can be derived as **Equations 21** and **22**:

$$\frac{\partial^2 p}{\partial t^2} = \left(1 + \frac{1}{Q} \sum_{l=1}^L D'_l \right) (v^2 \nabla^2 p + v \nabla v \cdot \nabla p - 2v^2 \mathbf{R} \cdot \nabla p) - \frac{1}{Q} \sum_{l=1}^L r_l \quad (21)$$

$$\frac{\partial r_l}{\partial t} = -\frac{1}{\tau_l} \{ r_l - D_l v^2 \nabla^2 p - D_l v \nabla v \cdot \nabla p + D_l 2v^2 \mathbf{R} \cdot \nabla p \} \quad (22)$$

Finally, the decoupled viscoacoustic wave equation with vector-reflectivity is derived. To unify the representation of viscoacoustic wave equations with vector-reflectivity with different characteristics using a single formulation, we adopt the following form (**Equations 23** and **24**):

$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} = & \left[1 + \kappa_1 \left(\frac{1}{Q} \sum_{l=1}^L D_l - \frac{1}{Q} \sum_{l=1}^L D'_l \right) \right] (v^2 \nabla^2 p + v \nabla v \cdot \nabla p - 2v^2 \mathbf{R} \cdot \nabla p) \\ & + \kappa_2 \left[\frac{1}{Q} \sum_{l=1}^L D'_l \right] (v^2 \nabla^2 p + v \nabla v \cdot \nabla p - 2v^2 \mathbf{R} \cdot \nabla p) \\ & - \kappa_2 \frac{1}{Q} \sum_{l=1}^L r_l \end{aligned} \quad (23)$$

$$\frac{\partial r_l}{\partial t} = -\frac{1}{\tau_l} \{ r_l - D_l v^2 \nabla^2 p - D_l v \nabla v \cdot \nabla p + D_l 2v^2 \mathbf{R} \cdot \nabla p \} \quad (24)$$

where κ_1 and κ_2 are two dimensionless control parameters independent of the equation itself, and their values are restricted to the following four combinations: When $\kappa_1 = 1$ and $\kappa_2 = 1$, **Equation 23** degenerates into viscoacoustic wave **Equation 9** that describes the viscoacoustic wavefield; when $\kappa_1 = 1$ and $\kappa_2 = 0$, **Equation 23** becomes the dispersion-dominated viscoacoustic wave **Equation 20** that characterizes only phase dispersion effects of viscoacoustic wavefield; when $\kappa_1 = 0$ and $\kappa_2 = 1$, **Equation 23** becomes the dissipation-dominated viscoacoustic wave **Equation 21** that describes only amplitude loss effects of viscoacoustic wavefield; when $\kappa_1 = 1$ and $\kappa_2 = -1$, the sign of dissipation term is reversed while the sign of phase dispersion term is unchanged, **Equation 23** becomes the attenuation-compensated viscoacoustic wave equation that characterizes the energy-compensated effects of viscoacoustic wavefield. Notably, all formulations derived from **Equation 23** remain amenable to numerical solution using the finite-difference method. Moreover, after incorporating an appropriate regularization operator,⁴⁶ the attenuation-compensated equation can also achieve stable propagation in the time domain and be solved using the finite-difference method.

Compared to previous decoupled viscoacoustic wave equations, the newly proposed formulation incorporates an explicit Q factor, facilitating the derivation of gradient formulas with respect to Q in subsequent inversion imaging. Moreover, the new equation includes vector-reflectivity, enabling accurate simulation of the full wavefield using velocity and estimated imaging results, thereby circumventing inaccuracies arising from the absence of density parameters in wavefield modeling.

3. Numerical example

In this section, we demonstrate that the newly derived decoupled viscoacoustic wave equation with vector-reflectivity effectively simulates dissipation and dispersion effects independently. We then present synthetic examples to validate that this equation reproduces wavefields equivalent to those generated by the variable-density viscoacoustic wave equation. Numerically, we employ a staggered-grid finite-difference scheme with tenth-order spatial accuracy and second-order temporal accuracy⁴⁷ to solve the viscoacoustic wave equation.

3.1. Decoupled dissipation- and dispersion-characteristics

To validate the decoupled characteristics of the dispersion-

dominated Equation 20 and the dissipation-dominated Equation 21 in the newly derived decoupled viscoacoustic wave equation with vector-reflectivity, we conducted forward modeling for a homogeneous model. The model had 401×401 grid points with 10 m spacing in both the x - and z -directions. We excited a Ricker wavelet with a dominant frequency of 25 Hz at the model center. Figure 1 presents wavefield snapshots simulated at 0.5 s using four distinct wave equations: the acoustic wave equation with vector-reflectivity (upper left), the dissipation-dominated equation (upper right), the dispersion-dominant equation (lower left), and the viscoacoustic wave equation with vector-reflectivity (lower right). When compared to the acoustic reference (upper left), the viscoacoustic wavefield (lower right) exhibited remarkable amplitude attenuation and a clear phase advance in the wavefront. Notably, the dissipation-dominated wavefield (upper right) maintained wavefront alignment with the acoustic reference while reproducing amplitude loss consistent with the full viscoacoustic wavefield. Conversely, the dispersion-dominated wavefield (lower left) preserved the phase characteristics and wavefront geometry of the viscoacoustic wavefield without showing noticeable amplitude decay.

In addition, we conducted forward modeling for the same homogeneous model with $Q = 40$ and 15 , respectively, using the four wavefield propagators above. A receiver was deployed 1 km from the source to record waveforms from different wavefields. Figure 2 shows the waveforms and the corresponding amplitude spectra. As the Q value decreased, the horizontal distance between the red and black curves increased, while their peak amplitudes remained essentially consistent. The red and blue curves maintained the same horizontal position. This indicates that the phase-dispersion-dominant wave equation primarily governs the velocity-dispersion behavior of viscoacoustic waves. The green curve consistently aligned horizontally with the black curve, yet its amplitude closely matched that of the blue curve. This indicates that the dissipation-dominant wave equation primarily governs the amplitude behavior of viscoacoustic waves. In the amplitude spectrum (Figure 2C and 2D), the dissipation-dominant term largely coincided with the full viscoacoustic term, and similarly, the phase dispersion-dominant term closely aligned with the acoustic term.

These results provide clear, direct evidence of the decoupling achieved in the proposed equations. The

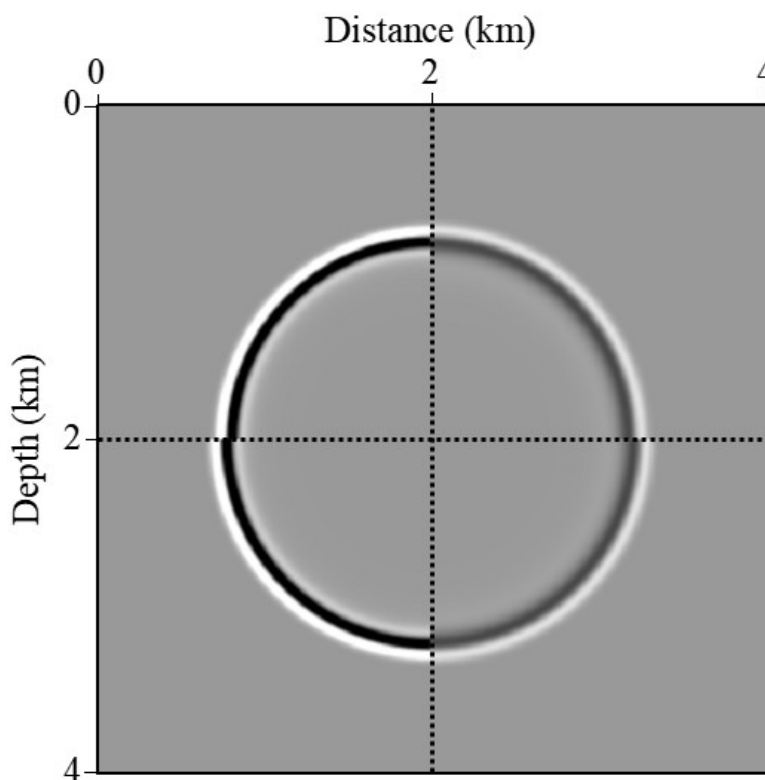


Figure 1. Wavefield snapshots simulated at 0.3 s using four distinct wave equations with vector-reflectivity: The acoustic (top left), the dissipation-dominated (top right), the dispersion-dominated lower left), and the viscoacoustic wave equation (lower right). The parameters of this homogeneous viscoacoustic model are: $v_p = 2000$ m/s, $Q = 15$, $\rho = 2$ g/cm³.

dissipation-dominated equation independently captures amplitude effects, whereas the dispersion-dominated equation faithfully models velocity-dispersion effects. This explicit separation of physical effects not only confirms the theoretical validity of the decoupled formulation but also establishes a practical foundation for implementing amplitude compensation and phase correction in seismic imaging and inversion workflows.

3.2. Comparison with the variable-density viscoacoustic wave equation

The theoretical analysis in Section 3.1 demonstrated that the variable-density viscoacoustic wave equation and the viscoacoustic wave equation with vector-reflectivity are mathematically and physically equivalent. To evaluate the numerical simulation accuracy of the proposed viscoacoustic wave equation with vector-reflectivity, we conducted forward modeling using the complex modified Marmousi model, as shown in Figure 3. The modified Marmousi model was discretized into 801×561 grids with a uniform spacing of 12.5 m in both the x - and z -directions. The horizontal and vertical components of the vector-reflectivity were derived from Equation 8. A Ricker wavelet with a dominant frequency of 25 Hz was excited at the surface. Two receivers, positioned 2.0 km and 3.0 km from the source, respectively, were used to record the waveforms. The data were sampled at 0.8 ms intervals over a total recording time of 5.0 s. We adopted the variable-density viscoacoustic wave Equation 2 from Guo *et al.*⁴⁵ as the reference solution, given its accurate representation of attenuation behavior.

Figure 4 displays the wavefield snapshots simulated at 2.8 s using the variable-density viscoacoustic wave equation and the viscoacoustic wave equation with vector-reflectivity. The reference result (Figure 4A) and the wavefield simulated by the proposed method (Figure 4B) exhibited nearly identical kinematic and dynamic characteristics. The residual between the two (Figure 4C) remained negligible, confirming the high fidelity of the proposed formulation. Both equations can accurately and stably simulate the wavefield in subsurface attenuating media. For a more detailed comparison, Figure 5 presents the recorded waveforms at the two receivers. We observed that the green curve and the red dashed curves were essentially coincident, regardless of the waveform recording location. The residual between the two curves was negligible relative to the waveform amplitude. The high consistency in both kinematic and dynamic features between the reference and proposed solutions demonstrates that the viscoacoustic wave equation with vector-reflectivity achieves numerical simulation accuracy essentially equivalent to that of the variable-density

viscoacoustic wave equation. Furthermore, the single-shot records are shown in Figure 6. We observed that the deep reflection events in the shot records exhibit low energy and poor coherence, attributable to attenuation. The shot records simulated by the two methods were highly similar, with negligible residuals between them. This indicates that the wavefields generated throughout the numerical simulation process exhibit essentially identical kinematic and dynamic characteristics. However, due to grid discretization, large impedance contrasts lead to inaccuracies in reflectivity estimates, which in turn impair the accuracy of wavefield simulation at those locations.

Additionally, we employed a scaled version of the Society of Exploration Geophysicists Advanced Modeling model to further evaluate the simulation accuracy of the proposed viscoacoustic wave equation with vector-reflectivity. The model, shown in Figure 7, consisted of 876×751 grid points with uniform 10 m spacing in horizontal and vertical directions. A Ricker wavelet source with a dominant frequency of 25 Hz was excited at the surface at a horizontal position of 4.4 km. Data were recorded with a time sampling of 0.8 ms over 6.0 s.

Figure 8A and 8B displays wavefield snapshots simulated at 2.0 s computed using the reference variable-density viscoacoustic equation and the proposed vector-reflectivity viscoacoustic wave equation, respectively. The two snapshots exhibited excellent agreement, with only minor residuals, as shown in Figure 8C. Figure 9 presents a common-shot gather recorded by 876 receivers spaced at 10 m intervals. Figure 9A and 9B is essentially identical, yet their residual—shown in Figure 9C—contains a relatively strong reflection. This reflection originates from the first interface of the velocity and density model. We hypothesize that it results from excessively large density contrasts at the interface, leading to pronounced gradient discontinuities. A vertical trace extracted at 4.4 km from Figure 9 is plotted in Figure 10. Both the shot gather and the extracted trace demonstrated that results from the vector-reflectivity formulation closely matched those of the variable-density reference, confirming that the proposed wave equation accurately captures both kinematic behavior and attenuation effects in heterogeneous viscoacoustic media.

However, in practical seismic exploration, the reflectivity is typically estimated from stacked images obtained with small-offset data using the deconvolution imaging condition. This reflectivity is input only as the vertical reflectivity estimate, while the horizontal reflectivity estimate remains absent.

4. Discussion

In the previous sections, numerical forward-modeling

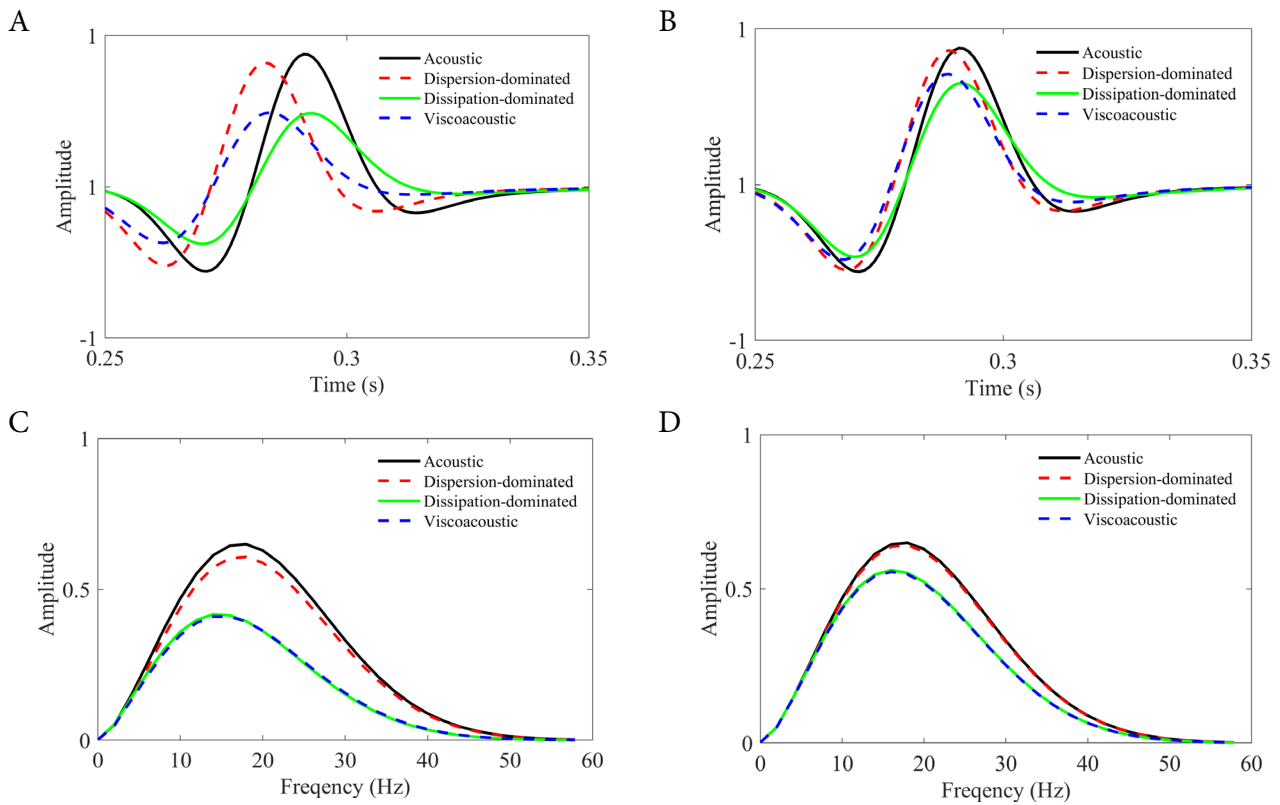


Figure 2. The waveforms recorded by the receiver are numerically simulated by the acoustic wave equation (black curve), dispersion-dominated equation (red dashed curve), dissipation-dominated equation (green curve), and viscoacoustic wave equation (blue dashed curve) with: (A) $Q = 15$, (B) $Q = 40$ in homogeneous media. (C) and (D) are the amplitude spectra of the waveforms in (A) and (B).

experiments confirmed that the viscoacoustic wave equation with vector-reflectivity and the variable-density viscoacoustic wave equation achieved comparable accuracy in simulating heterogeneous attenuating media. This demonstrates that the viscoacoustic wave equation with vector-reflectivity (**Equation 9**) can serve as a substitute for the variable-density viscoacoustic wave equation (**Equation 2**) as the wavefield propagation operator in both Q-RTM and Q-LSRTM. Given their identical simulation accuracy, the computational efficiency of the numerical solutions of the two equations must also be considered. Comparing **Equation 2** and **Equation 9**, **Equation 2** contains one gradient operator ∇p and one divergence operator $\nabla \cdot \left(\frac{1}{\rho} \nabla p \right)$. Whereas **Equation 9** includes two gradient operators ∇p and ∇v , and one divergence operator $\nabla \cdot (\nabla p)$. Theoretically, this implies that solving **Equation 9** requires additional computational resources to handle the extra gradient operator ∇v . Nevertheless, since **Equation 9** does not rely on accurate density parameters as input and can instead utilize estimated reflectivity images, it retains distinct advantages.

Since we decoupled the dissipation and phase dispersion effects of the viscoacoustic wave equation with vector-reflectivity (**Equation 9**), we can derive the corresponding attenuation-compensated viscoacoustic wave equation with vector-reflectivity ($\kappa_1 = 1$ and $\kappa_2 = -1$ in **Equation 24**) to implement Q-RTM. Yet, this appears to fall into a circular dilemma: numerically solving the equation requires reflectivity images as input, while the purpose of numerically solving the equation in Q-RTM is to obtain attenuation-compensated reflectivity images. However, this circular dependency can be resolved within the Q-LSRTM framework because least-squares inversion utilizes the updated reflectivity images as the input to the linearized viscoacoustic wave equation to compute the predicted data. Having derived the forward and migration operators within Q-LSRTM, we now turn to the linearized forward operator.

The linearized viscoacoustic wave equation derived under the first-order Born approximation is widely used in Q-LSRTM. However, this linearized operator only generates primary reflections and inherently excludes full-wavefield information such as multiples. To highlight the advantages

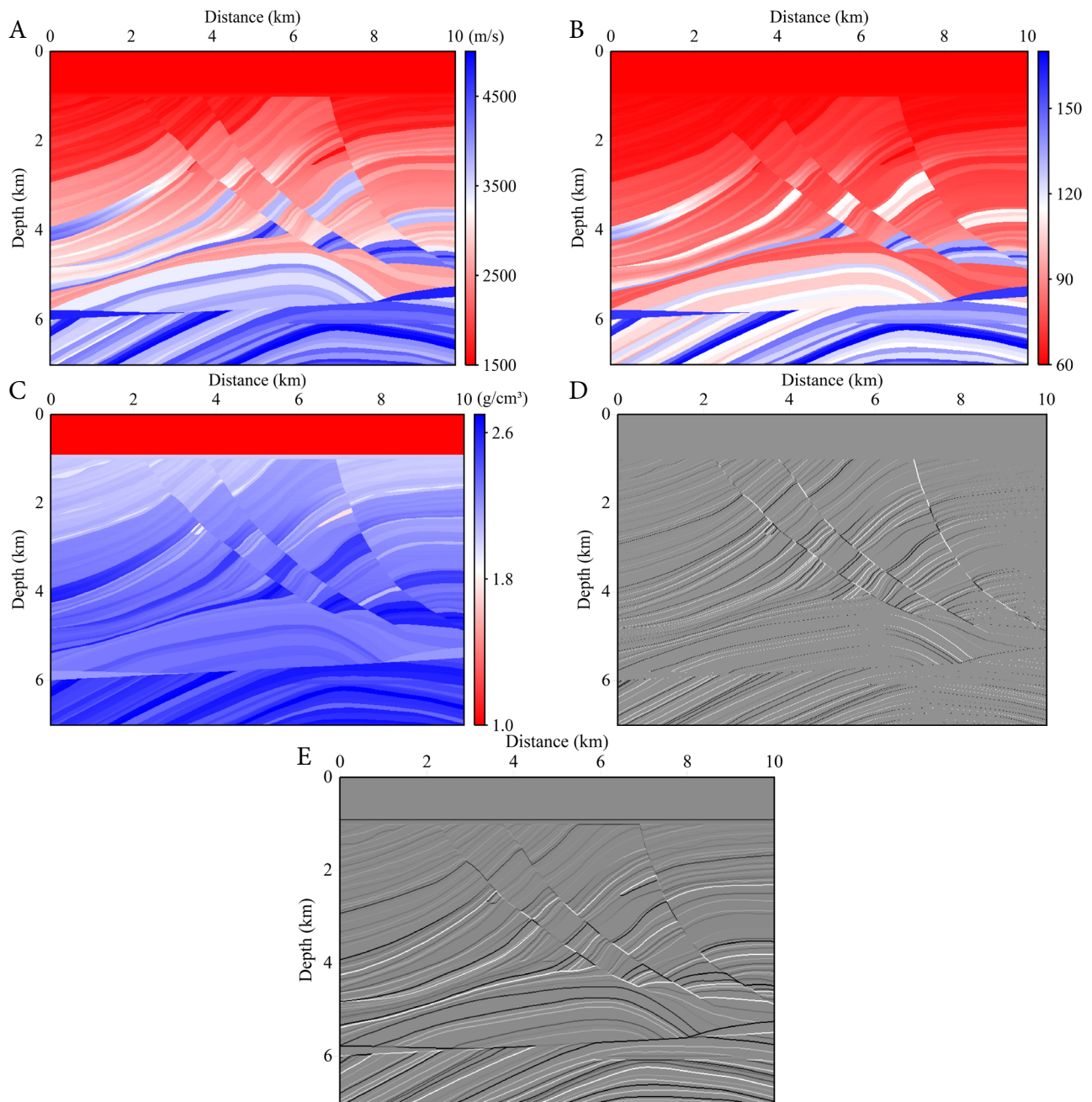


Figure 3. The modified Marmousi model. (A) P-wave velocity v_p . (B) P-wave quality factor Q . (C) Density ρ . (D) and (E) Horizontal and vertical components of the vector-reflectivity.

of the proposed vector-reflectivity viscoacoustic wave equation over the first-order Born linearized operator, we conducted forward modeling using a constant-velocity, variable-density model. Figure 11 illustrates the model parameters: constant velocity, constant Q , variable density, and the vector-reflectivity components in the horizontal and vertical directions. Reflectivity and the associated wavefields originate solely from density variations. A

Ricker wavelet with a dominant frequency of 25 Hz was excited at the surface at a horizontal position of 1.5 km. Modeling experiments were performed using three distinct equations: the variable-density viscoacoustic wave equation (full-wavefield reference), the proposed vector-reflectivity viscoacoustic wave equation, and the variable-density linearized viscoacoustic wave equation based on the first-order Born approximation.

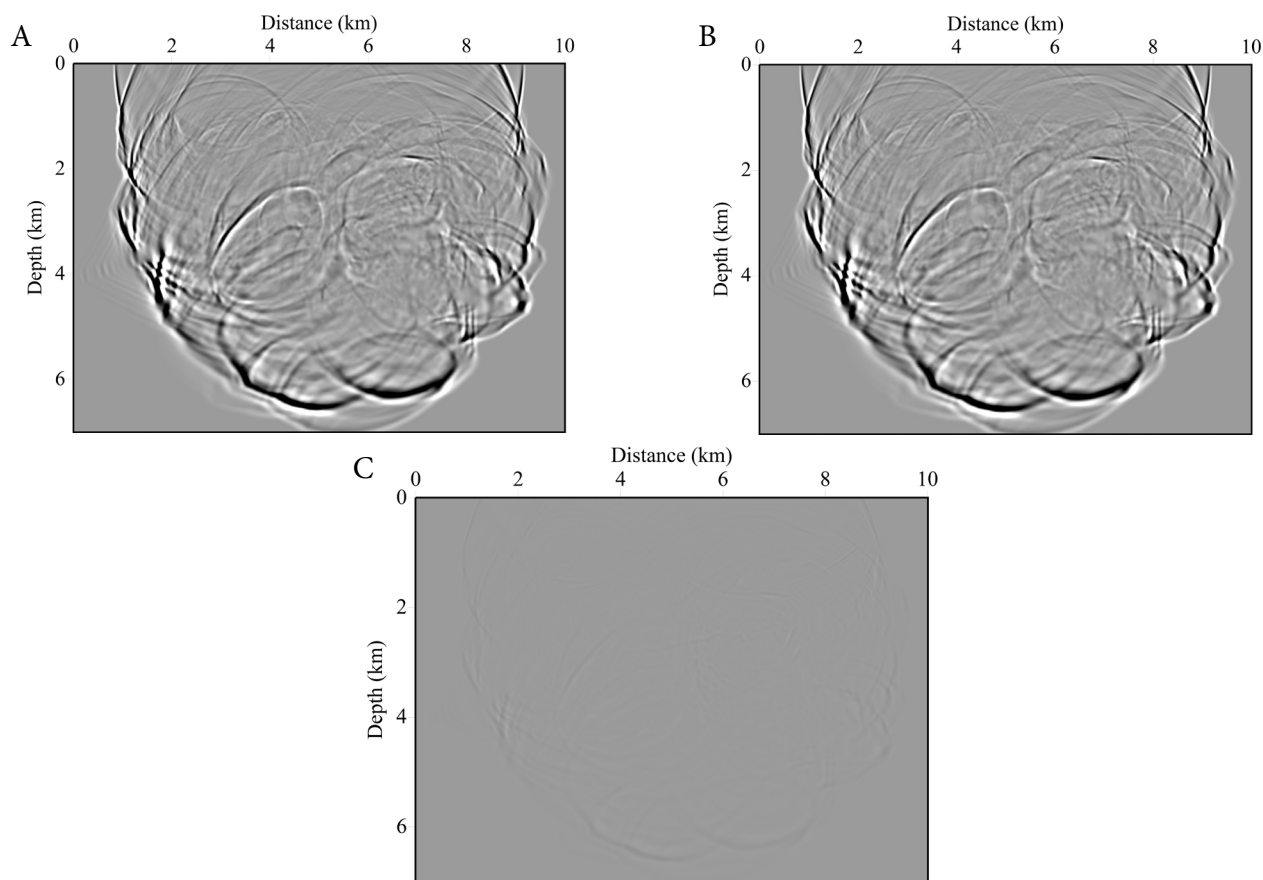


Figure 4. Wavefield snapshots of the modified Marmousi model simulated at 2.8 s using (A) variable-density viscoacoustic wave equation and (B) viscoacoustic wave equation with vector-reflectivity. (C) The residual of (A) and (B). The same color scale was used for all subplots.

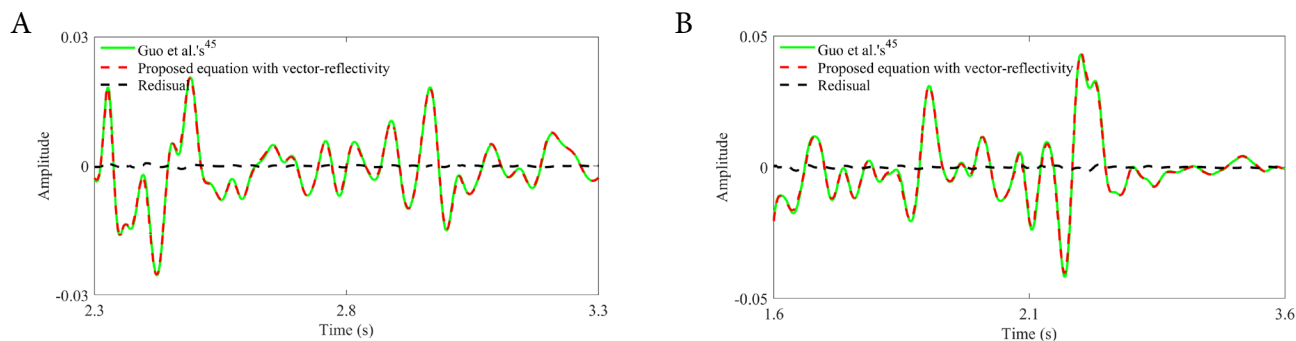


Figure 5. The recorded waveforms located (A) 2.0 km and (B) 3.0 km from the source, simulated using the viscoacoustic wave equation of Guo *et al.*⁴⁵ (green curves) and the viscoacoustic wave equation with vector-reflectivity (red dashed curves). The black dashed curves are the residual between the green curves and the red dashed curves.

The corresponding synthetic shot records are shown in Figure 12. The first four reflection events in the seismograms arose from density contrasts, while the two later events marked by black arrows corresponded to internal multiples. The record generated using the variable-density viscoacoustic wave equation (Figure

12A) served as the full-wavefield reference. Comparison between Figure 12A and 12B revealed that the vector-reflectivity equation correctly simulated internal multiples, whereas the linearized equation (Figure 11C) failed to model these events. This demonstrates that the proposed vector-reflectivity viscoacoustic wave equation can

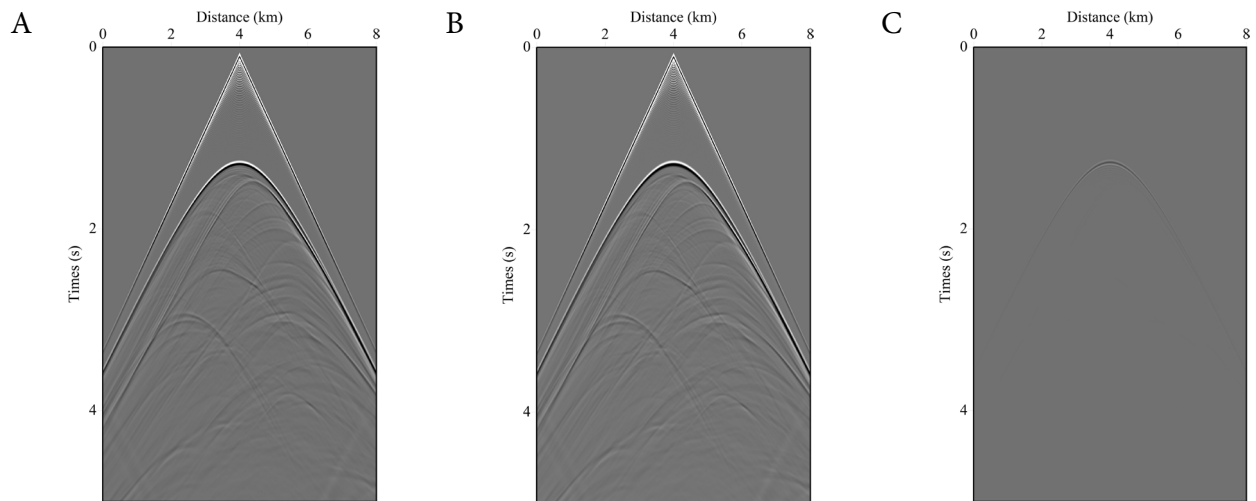


Figure 6. Shot gathers of the modified Marmousi model simulated using (A) the viscoacoustic wave equation with vector-reflectivity and (B) the variable-density viscoacoustic wave equation. (C) The residual of (A) and (B). The same color scale was used for all subplots.

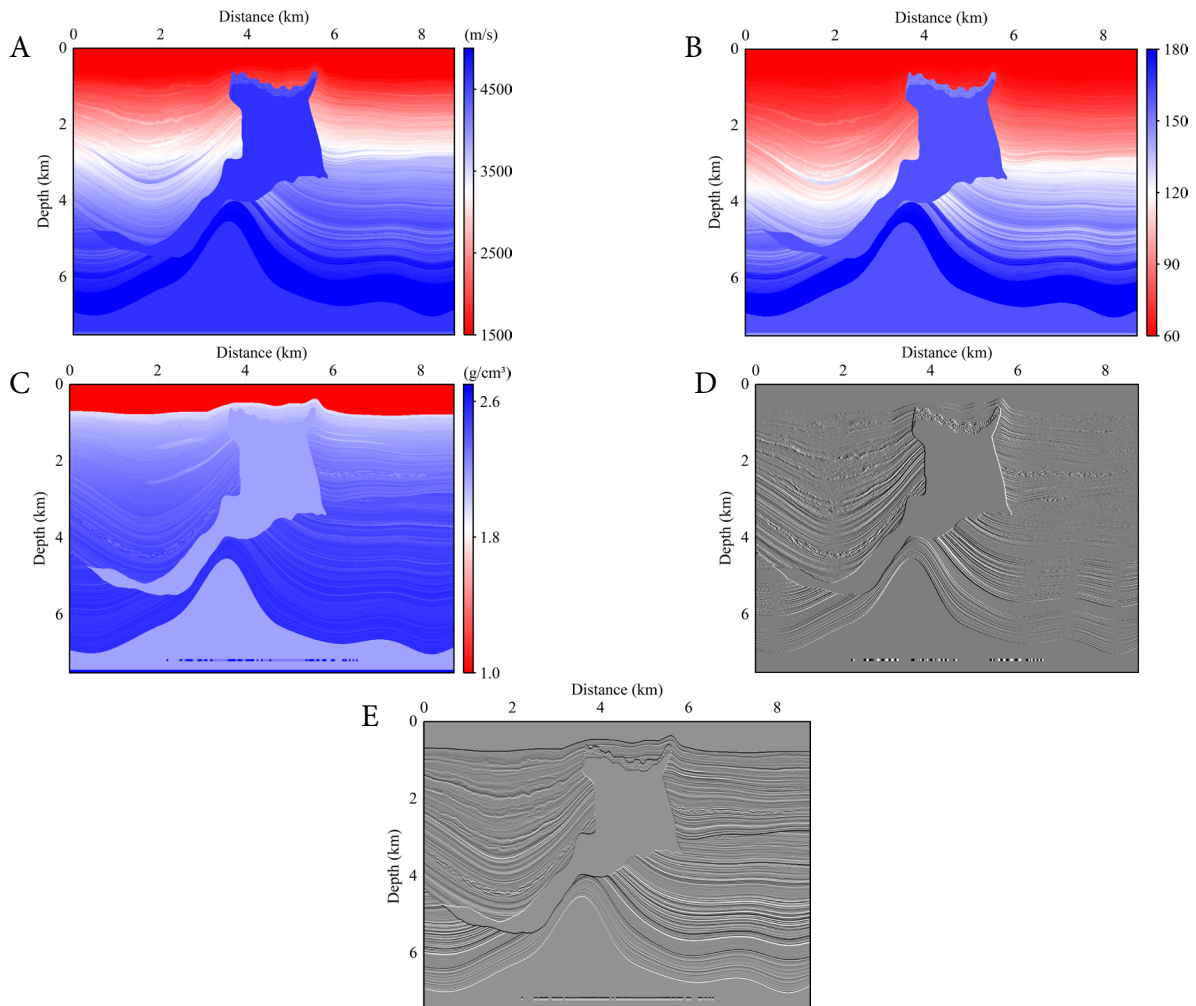


Figure 7. The modified Society of Exploration Geophysicists Advanced Modeling model with a grid point of 876×751 and a grid spacing of 10 m. (A) P-wave velocity v_p . (B) P-wave quality factor Q . (C) Density ρ . (D) and (E) Horizontal and vertical components of the vector-reflectivity.

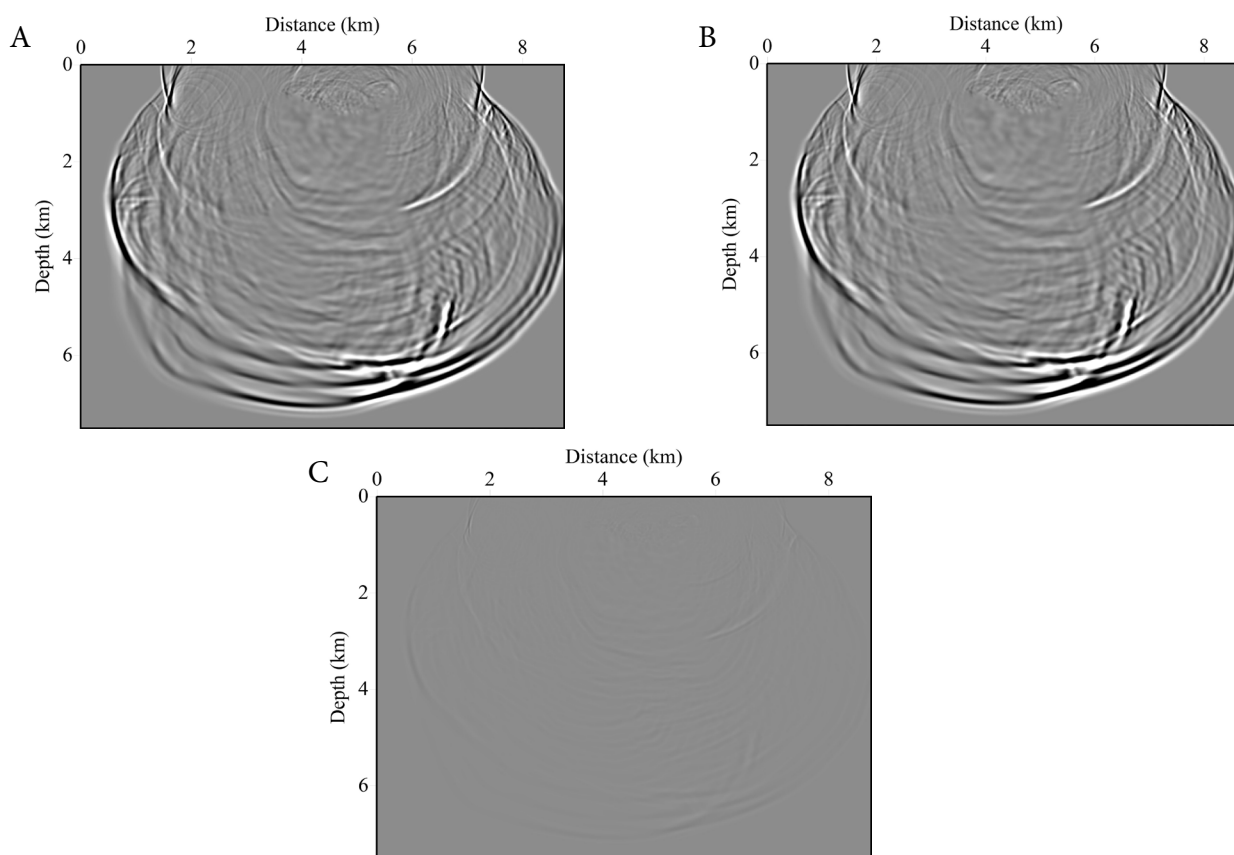


Figure 8. Wavefield snapshots of the modified Society of Exploration Geophysicists Advanced Modeling model simulated at 2.0 s using (A) variable-density viscoacoustic wave equation and (B) viscoacoustic wave equation with vector-reflectivity. (C) The residual of (A) and (B). The same color scale was used for all subplots.

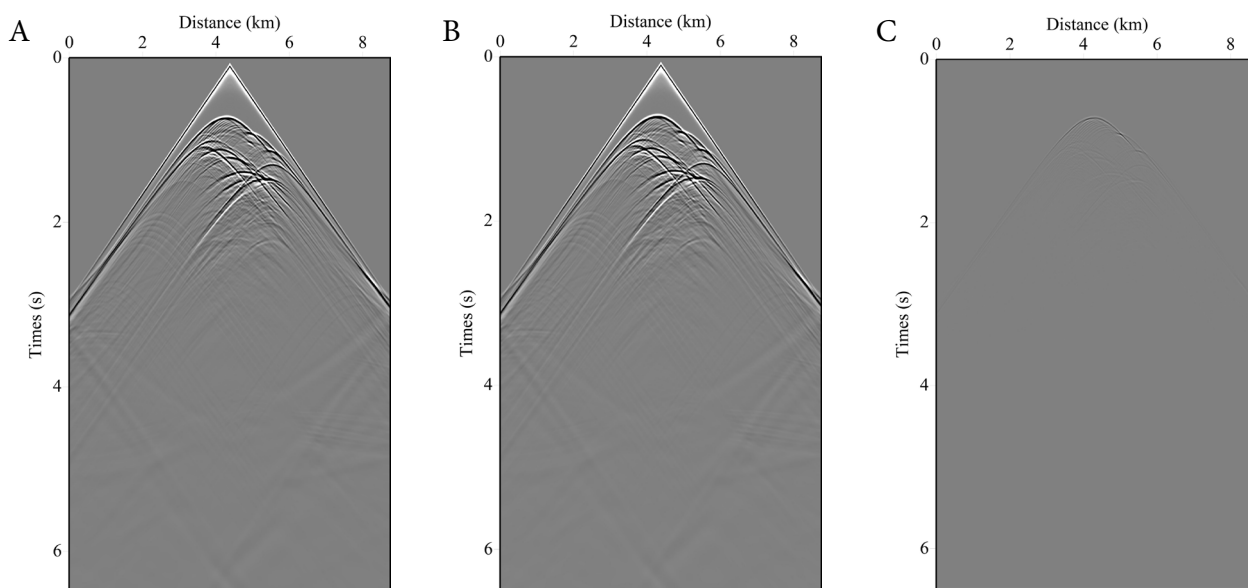


Figure 9. Shot gathers of the modified Society of Exploration Geophysicists Advanced Modeling model simulated using (A) the viscoacoustic wave equation with vector-reflectivity and (B) the variable-density viscoacoustic wave equation. (C) The residual of (A) and (B). The same color scale was used for all subplots.

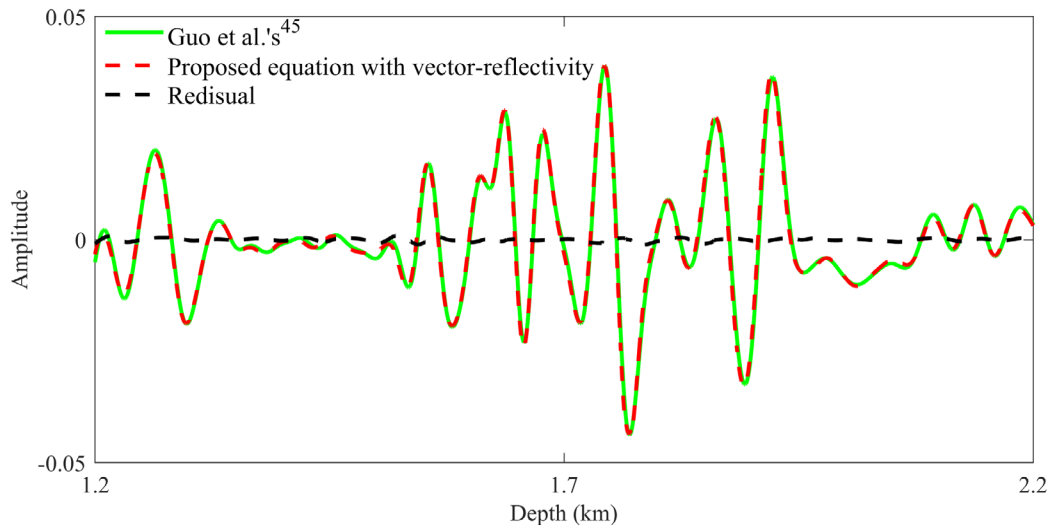


Figure 10. Vertical trace extracted at 4.4 km in horizontal distance from **Figure 9**. Notes: Green curves: viscoacoustic wave equation of Guo *et al.*,⁴⁵ Red dashed curves: viscoacoustic wave equation with vector-reflectivity; Black dashed curves: residual between green curves and red dashed curves.

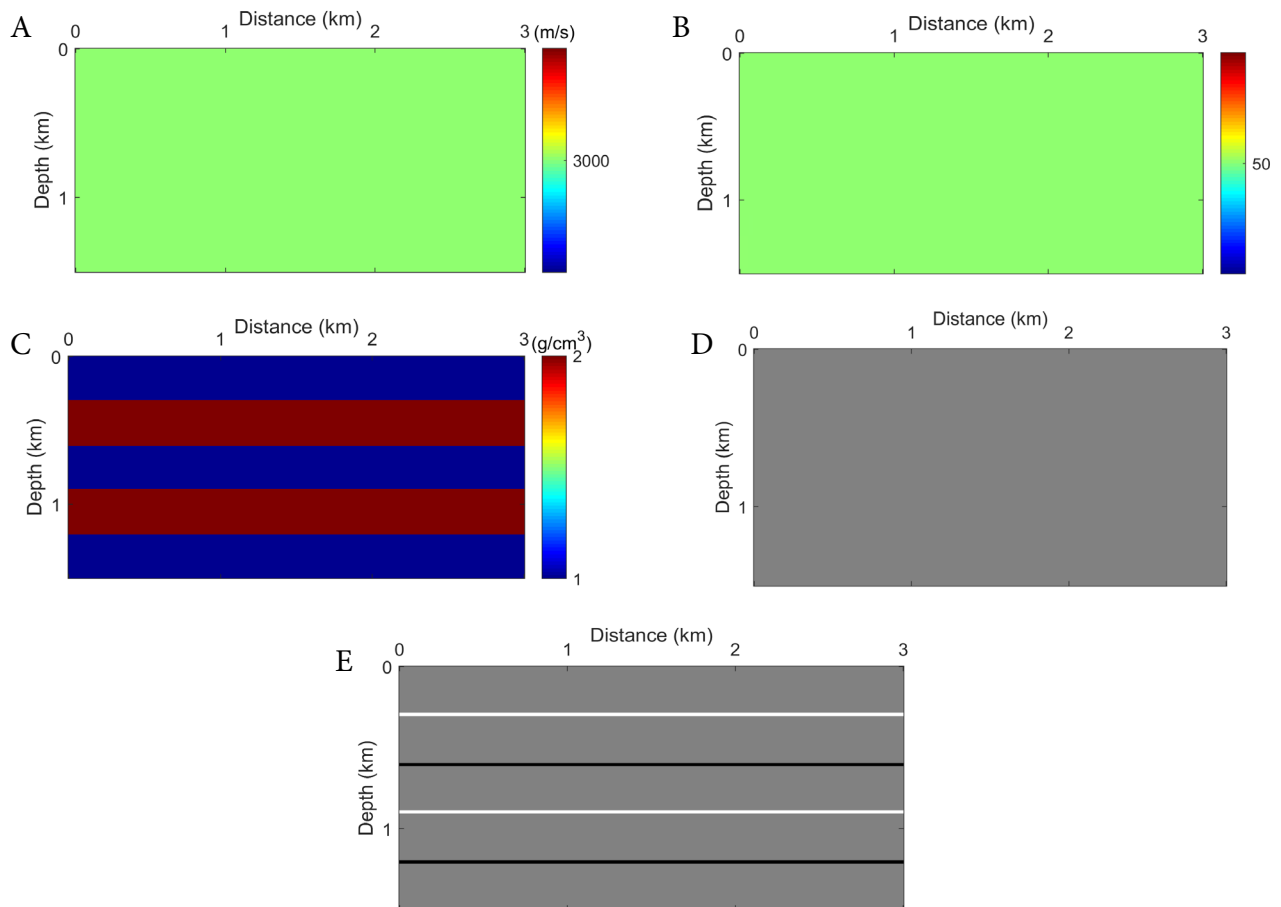


Figure 11. The constant-velocity, variable-density model with a grid point of 301×151 and a grid spacing of 10 m. (A) P-wave velocity v_p . (B) P-wave quality factor Q . (C) Density ρ . (D) and (E) Horizontal and vertical components of the vector-reflectivity.

effectively replace the first-order Born linearized operator in Q-LSRTM, as the simulated multiples can be matched with those in the field data and subsequently eliminated through subtraction.⁴⁴

5. Conclusion

This study presents a viscoacoustic wave equation with a vector-reflectivity forward-modeling framework to overcome the limitations of conventional Q-LSRTM. Compared to the forward operators in conventional Q-LSRTM, the viscoacoustic wave equation with vector-

reflectivity accurately simulated the full-wavefield without requiring explicit density parameters. Moreover, the proposed equation explicitly incorporates the Q factor, thereby avoiding the need for complex relaxation-time parameters. We further derived a decoupled formulation in which dissipation and dispersion effects were treated separately, serving as the migration operator to accelerate convergence in Q-LSRTM. Compared to the linearized viscoacoustic operator derived from the first-order Born approximation, the proposed viscoacoustic wave equation with vector-reflectivity accurately described attenuation effects and simulated full-wavefield responses—including

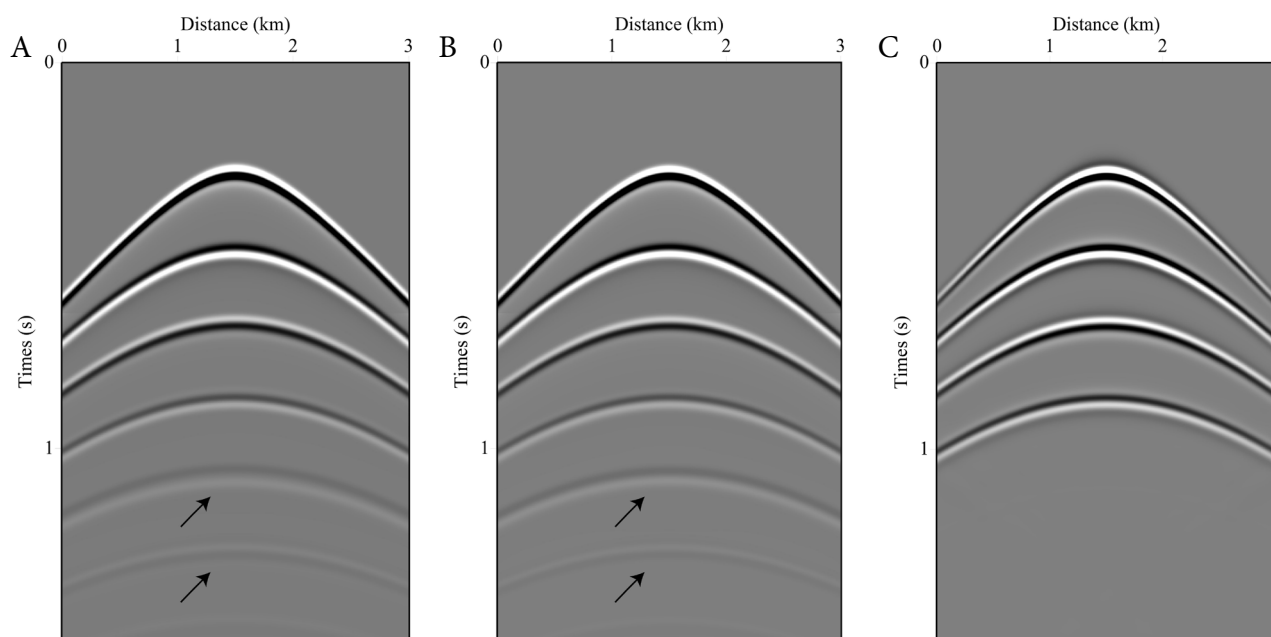


Figure 12. Shot gathers of the constant-velocity, variable-density simulated using (A) the variable-density viscoacoustic wave equation, (B) the viscoacoustic wave equation with vector-reflectivity, and (C) the linearized variable-density viscoacoustic wave equation. The same color scale was used for all subplots.

both primaries and multiples. Numerical experiments demonstrated that the proposed formulation is mathematically equivalent to the original variable-density viscoacoustic wave equation. Future work should focus on applying this decoupled vector-reflectivity viscoacoustic wave equation to Q-LSRTM to effectively suppress crosstalk artifacts and produce high-resolution seismic images in attenuating media.

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Conflict of interest

The authors declare they have no competing interests.

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Writing—review & editing: Fei Li, Zilong Ye, Jianping Huang

Availability of data

All data are available upon reasonable request from the corresponding authors.

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