Front. Phys. 12(5), 124203 (2017)
DOI 10.1007/s11467-016-0613-2

RESEARCH ARTICLE

Propagation dynamics of finite-energy Airy beams in nonlocal nonlinear media

Zhen-Kun Wu, Peng Li, Yu-Zong Gu†

Institute of Microsystem Physics, School of Physics and Electronics, Henan University, Kaifeng 475004, China
Corresponding author. E-mail: †yzgu@henu.edu.cn
Received April 15, 2016; accepted July 26, 2016

We investigate periodic inversion and phase transition of normal and displaced finite-energy Airy beams propagating in nonlocal nonlinear media with the split-step Fourier method. Numerical simulation results show that parameters such as the degree of nonlocality and amplitude have profound effects on the intensity distribution of the period of an Airy beam. Nonlocal nonlinear media will reduce into a harmonic potential if the nonlocality is strong enough, which results in the beam fluctuating in an approximately cosine mode. The beam profile changes from an Airy profile to a Gaussian one at a critical point, and during propagation the process repeats to form an unusual oscillation. We also briefly discus the two-dimensional case, being equivalent to a product of two one-dimensional cases.

Keywords Airy beam, nonlocal nonlinear, phase transition, intensity distribution

PACS numbers 03.65.Ge, 3.65.Sq, 42.25.Gy

1 Introduction

Airy beams were first theoretically demonstrated in 1979 by Berry and Balazs [1], and experimentally observed in 2007 by Siviloglou *et al.* [2, 3]. Theoretical and experimental studies of Airy beams have been performed worldwide owing to their peculiar properties during propagation such as being self-healing, diffraction free, and demonstrating transverse acceleration over many Rayleigh lengths [2–6]. Such optical beams have potential applications including dielectric microparticles [7], self-bending plasma channels [8], material modifications [9, 10], and Airy light bullets accelerating both in transverse dimensions and in time [11]. In the nonlinear regime, the propagation dynamics of Airy beams have been studied exhaustively. For example, the interactions of two Airy beams have been investigated in Kerr [12] and saturable nonlinear media [13]. However, all the works stated above were studied in local nonlinear media, and many studies have shown that nonlocality has strong effects on the propagation of the solitons [14, 15]. In addition, it has been shown that the boundary conditions of nonlocal media strongly affect the propagation dynamics of self-accelerating beams [16]. The propagation of an Airy beam in a nonlocal nonlinear media (NNM) was theoretically studied by Zhou *et al.* [17, 18]. Shen *et al.* have studied the anomalous interaction of Airy beams in nonlocal nonlinear media [19]. Here, nonlocal nonlinearity means that the high induced refractive index change of a material at a particular location is governed by the light intensity in a certain neighborhood of this location. Such a nonlocal optical nonlinearity exists in nematic liquid crystals [20] and thermal media [21].

In this paper, we numerically investigate the nonlinear dynamics of normal and transversely displaced finite-energy Airy beams in NNM. We find that the beam exhibits some unusual propagation properties, such that for half of the inversion period the Airy beam accelerates in one transverse direction, with the main Airy beam lobe leading the train of pulses, whereas in the other half of the period it accelerates in the opposite direction, with the main lobe still leading, but now the whole beam is inverted. The inversion happens at a critical point, at which the beam profile changes from an Airy profile to a Gaussian one, and during propagation the process repeats to form an unusual oscillation. The structure of this paper is organized as follows: in Section 2 we introduce the theoretical model that will be used briefly and construct the initial Airy beam; in Section 3 we successively investigate the propagation properties of finite energy Airy beam in local and nonlocal nonlinear media

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in detail, and also compare the results with those in the previous studies; the two-dimensional (2D) case is briefly discussed in Section 4; and we present our conclusions in Section 5.

2 Theoretical model

We begin our analysis by considering Airy beam propagation in nonlocal nonlinear media, which is described by the following dimensionless nonlocal nonlinear Schrödinger equation:

$$\mathrm{i}\frac{\partial\psi}{\partial z}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+N(I)\psi=0, \tag{1}$$

where ψ is the Airy beam envelope. The variables x and z are the normalized transverse coordinates and the propagation distances, respectively, scaled by a characteristic transverse width x_0 and the corresponding Rayleigh range kx_0^2. Here, $k=2\pi n/\lambda_0$ is the wavenumber, n is the ambient index of refraction, and λ_0 is the wavelength in free space. The nonlinear expression is $N(I)=\int R(x-x')|\psi(x',z)|^2\mathrm{d}x'$, with $R(x)$ corresponding to the normalized symmetrical real nonlocal response function of the medium. In general, the realistic forms of the nonlocal response functions depend on the underlying physical processes of the materials. Without loss of generality, we consider the case of so-called hyperbolic secant nonlocal response functions: $R(\boldsymbol{x})=\omega\sec h(x/\sigma^2)$, with the characteristic width σ representing the degree of nonlocality. Although the hyperbolic secant nonlocal response function is phenomenological, it can describe the general properties of other actual nonlocal media. Especially, without the nonlinearity $N(I)=0$, Eq. (1) is just the linear Schrödinger equation for which many particular solutions exist. One of the exact solutions is expressed in term of the Airy function, and to make it finite energy the solution is generalized into [1, 2]

$$\psi(x,z)=A_0\mathrm{Ai}\left(x-\frac{z^2}{4}+\mathrm{i}az\right)\times\exp\left[\frac{\mathrm{i}}{12}(6a^2z-12\mathrm{i}ax+6\mathrm{i}az^2+6xz-z^3)\right], \tag{2}$$

where A_0 is the amplitude of the Airy beam and the Airy function is defined as $\mathrm{Ai}(x)=\frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty}\exp(xt-t^3/3)\mathrm{d}t$. The term $z^2/4$ in Eq. (2) represents this ballistic trajectory, and $a\geq 0$ is an arbitrary real decay constant to ensure containment of the infinite Airy tail and the finite power of the Airy beam. This solution is generated from an initial condition $\psi(x)=A_0\mathrm{Ai}(x)\exp(ax)$, and represents a finite-energy Airy wave. In accordance with previous literature [12, 13], we have set the parameter $a=0.1$ throughout this paper.

3 Numerical simulations

The numerical simulations in this paper were obtained by utilizing the fourth-order split-step fast Fourier transform method [22] in double precision. Starting from the propagation Eq. (1), we first consider the case of local cubic media with $\sigma=0$, and use Eq. (2) at $z=0$ as an input profile. The evolution results of this profile are displayed in Figs. 1(a), (b), and (c), respectively, corresponding to different values of amplitude A_0. Specifically, for small amplitude ($A_0=1$), the Airy beam first evolves by performing acceleration in space, and then it

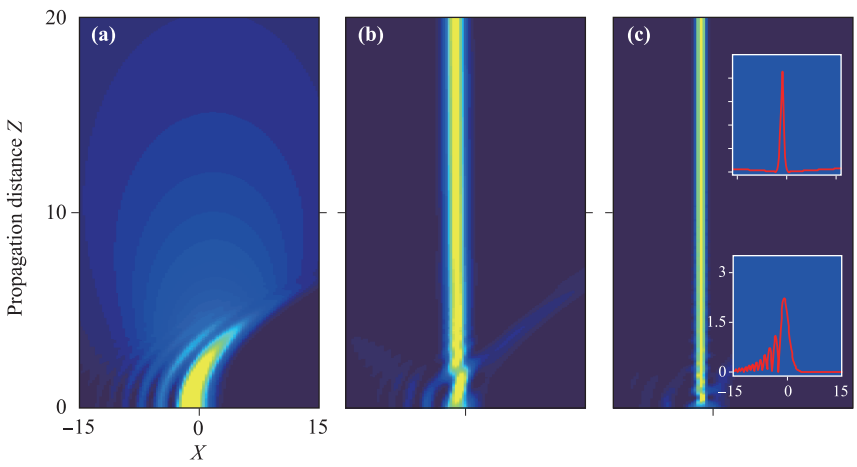

Fig. 1 Evolution of Airy beam with $\sigma=0$ and the amplitudes $A_0=1$ **(a)**; 3 **(b)**; 5 **(c)** in local nonlinear media for several propagation distances. Inset: The intensity distribution at propagation distance $z=0$ (bottom) and $z=20$ (top). The other parameters $\omega=5$.

exhibits diffraction properties as displayed in Fig. 1(a). When one sets $A_0 = 3$ and $A_0 = 5$, and redo the evolution, the corresponding results are shown in Figs. 1(b) and (c), respectively. The distribution of the main lobe of the Airy beam tends to be a Gaussian distribution, and the intensity of the side lobe becomes weaker and weaker. However, when A_0 is sufficiently large, a stationary soliton will be formed out of the centered energy about the Airy main lobe as shown in Fig. 1(c), where the bottom and top insets present the intensity distribution of the finite energy Airy beam at $z = 0$ and $z = 20$, respectively. For the reason that the tail power of the Airy beam is almost trapped into the main lobe to shed a soliton. It should be noted that the soliton peak intensity is larger than that of the Airy beam main lobe. Moreover, the solitons, once formed, do not accelerate, but travel along straight trajectories, and exhibit periodic oscillations of the soliton amplitude and width as a function of propagation distance.

Figure 2 shows the periodic inversion of an Airy beam during propagation with different degree of nonlocality. One can clearly see that the nonlinear media cannot support stationary solitons, but instead a periodic intensity distribution of the Airy beam with opposite bending directions is formed in NNM. Specifically, for $\sigma = 10$ and $A_0 = 20$, the propagation dynamic exhibits oscillation properties such that the centroid oscillates back and forth with certain periods across the optical axis because of the nonlocality, as depicted in Fig. 2(a). The beam intensity maintains an Airy-like profile with a main lobe and the decaying tails. If we fix $\sigma = 10$, but the amplitude parameter is increased to $A_0 = 30$, as displayed in Fig. 2(b), the period will become larger and the beam width increases in comparison with Fig. 2(a). This is possibly because the nonlocal nonlinearity cannot cause the Airy beam to bend in the opposite direction in a short propagation distance for the increasing beam intensity. When the degree of nonlocality σ increases to 15, corresponding to the strongest nonlocal media, the beam exhibits perfect oscillation in an approximately cosine mode during propagation, while the propagation is diffraction-free, with the beam staying localized at all times, as shown in Fig. 2(c). One can also see that at a critical point the Airy beam loses its multi-peak profile and propagates as an asymmetric single-hump beam. These behaviors are in good agreement with the case shown in Fig. 1(a) displayed in Ref. [23], in which the finite energy Airy beam propagates in a linear medium with an external parabolic potential. The reason is that the hyperbolic secant function $R(\boldsymbol{x}) = \omega \sec h(x/\sigma^2)$ will reduce into a parabolic potential if the degree of nonlocality σ is large enough. Therefore, the similar periodic oscillation phenomenon of Airy beam also can be obtained in the regime of strong nonlocality. Here, we would like to point out that as parameter ω is increased further to $\omega = 10$, the parabolic potential well becomes deeper, and the period becomes smaller, as depicted in Fig. 2(d). It is worth noting that similar Airy beam propagation properties also can be exhibited in a photonic lattice [24].

However, although the whole beam exhibits perfect harmonic oscillation as displayed in Figs. 2(c) and (d), that of the maximum intensity point is anharmonic. The dynamics of the maximum of the Airy beam intensity can be captured by direct numerical simulation during propagation as shown in Fig. 3, where the solid curve is

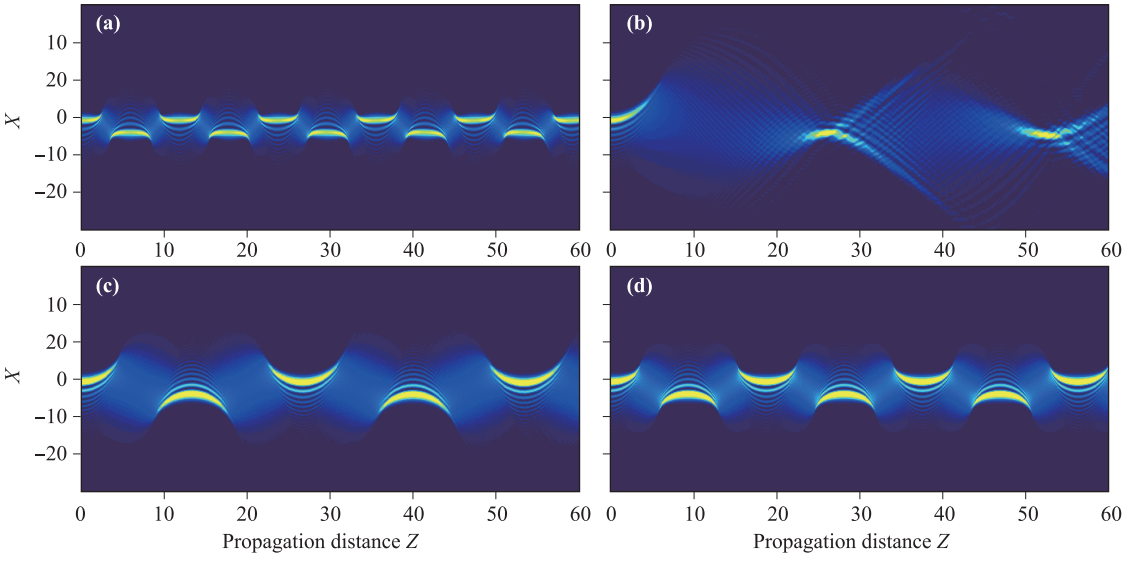

Fig. 2 Propagation dynamics of Airy beam in NNM. The degrees of nonlocality are: **(a, b)** $\sigma = 10$, and **(c)** $\sigma = 15$. The amplitudes are: (a) $A_0 = 20$, (b, c) $A_0 = 30$, and (a, b, c) $\omega = 5$. **(d)** The figure setup is as in (c), but the parameter $\omega = 10$.

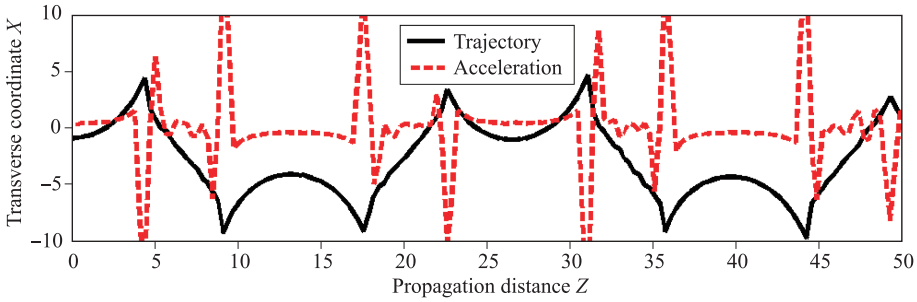

Fig. 3 Propagation trajectory (*solid curves*) and acceleration (*dashed curves*) of a finite Airy beam in NNM. The parameters is the same as in Fig. 2(c).

the trajectory of the maximum beam intensity and the dashed curve is the corresponding acceleration. The figure can explain why the motion is not harmonic. As is well known, harmonic motion indicates that the curve for trajectory would be the same as that for acceleration, but of the opposite sign and scaled by a factor [23]. In addition, Figs. 2 and 3 show that the Airy beam loses its multi-peak profile but possesses a symmetric Gaussian profile at the critical point during propagation. Especially, the beam decelerates first, then accumulates at the edge (at a certain point), loses its multi-peak structure and becomes a single-peak, and finally accelerates in the opposite direction. As described in Refs. [23, 25], we refer to this phenomenon as the "phase transition" of the finite-energy Airy beam.

4 Two-dimensional case

At this stage, we extend the analysis to the 2D case. The 2D finite-energy Airy beam is also characterized by periodic inversion and phase transition during propagation dynamics in 2D NNM. This is a consequence of the linearity of the problem, such that the 2D case can be considered as a product of two 1D cases. Hence, the extension from 1D to 2D is straightforward; the propagation of an Airy beam in the nonlocal nonlinear media is governed by the nonlocal nonlinear Schrödinger equation

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+N[I(x,y)]\psi=0, \tag{3}$$

where

$$N[I(x,y)]=\iint R(r-r')|\psi(x',y',z)|^2\mathrm{d}x'\mathrm{d}y'. \tag{4}$$

In Eq. (4), the hyperbolic secant nonlocal response functions is given by $R(x,y)=\omega\sec h(x/\sigma^2)\sec h(y/\sigma^2)$. The solutions of Eq. (3) are easily found by using the inverse scattering transform [22]. The reported solutions are all related to the Hermite, Bessel, or Airy functions. In this section, we take only the finite-energy Airy beam as an example. The Airy beam in the source plane $z=0$ takes the form of

$$\psi(x,y,0)=A_0\mathrm{Ai}(x)\mathrm{Ai}(y)\exp[a(x+y)] \tag{5}$$

with $A_0=90$. In Fig. 4, we depict the propagation dynamics of a 2D Airy beam in nonlocal nonlinear media by direct numerical integration of Eq. (3). As shown in Fig. 5, the 3D surface plot in the top panel depicts that the beam undergoes two-phase oscillation and periodic inversion with a period T.

To explore the phase transition more clearly, we show the intensity distribution of the beam at certain distances in Figs. 4(a)–(d), where the intensity snapshots are taken from the same simulation at propagation distances $z=0$, $z=T/4$, $z=T/2$, and $z=3T/4$, respectively. From the panels, one can clearly see that intensity profile oscillates between an Airy beam and a Gaussian beam. Figure 4(a) is the initial 2D finite-energy Airy beam distributed in the third quadrant. In Fig. 4(b), at $z=T/4$ one can see that the intensity is exhibiting a Gaussian profile owing to the strongly nonlocal nonlinearity. When the propagation distance is increased to $z=T/2$, as presented in Fig. 4(c), the intensity distribution oscillates to be an Airy beam, and is located in the first quadrant. When the beam propagates further and reaches $z=3T/4$, the beam intensity is characterized by a Gaussian profile, as shown in Fig. 4(d).

5 Conclusion

In summary, we have numerically investigated the dynamics of 1D and 2D finite-energy Airy beams in an NNM. We find that nonlinearity has strong effects on the Airy beam during propagation. It is worth noting that it will reduce into a harmonic potential if the nonlocality is strong enough, which results in the beam fluctuating in an approximately cosine mode. The whole beam exhibits a phase transition at an odd integer multiple of quarters of the oscillation period, and undergoes

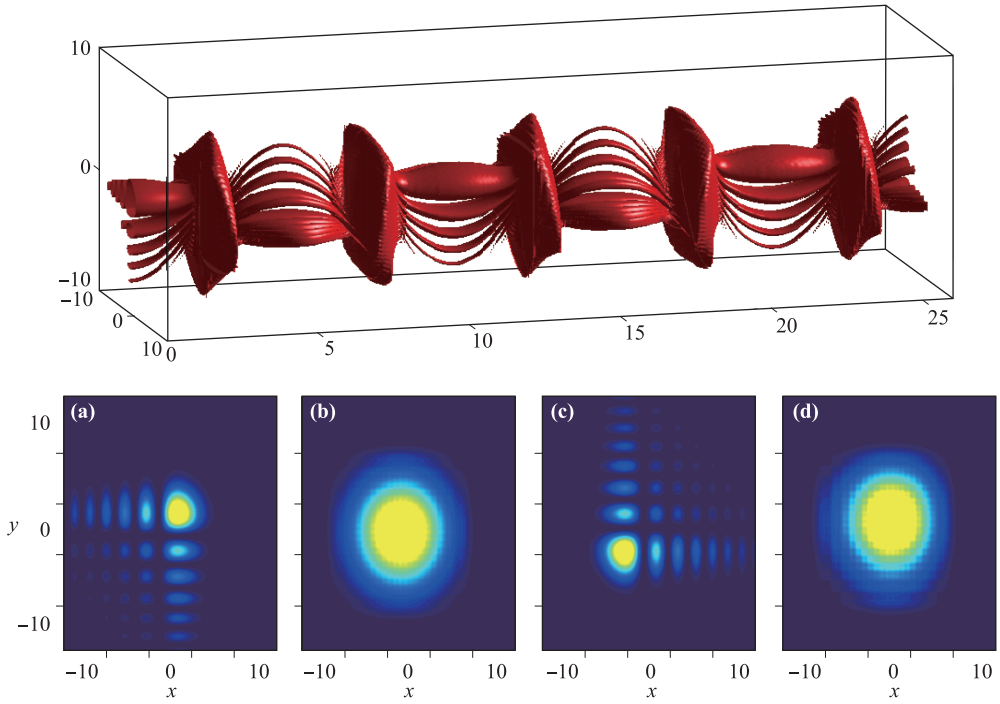

Fig. 4 Propagation of two-dimensional finite energy Airy beam in nonlocal nonlinear media. Iso-surface plot of the intensity during propagation. **(a–d)** Intensity distribution when the beam propagates from $z = 0$, to $z = T/4$, $z = T/2$, and $z = 3T/4$, respectively. The other parameters are $\sigma = 15$ and $\omega = 5$.

a spatial inversion at an odd integer multiple of halves of the period. However, the maximum intensity point of the beam is anharmonic. Our investigation may lead to potential applications in particle manipulation, signal processing, and other fields.

Acknowledgements This work was supported by the fund from the Science and Technology Department of Henan Province (Grant Nos. 144300510018 and 17A140003) and the Natural Project in Shaanxi Province (No. 2016JM1006).

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