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A Novel Non-smooth Controller of the Second Order System with Disturbances and Its Application

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Abstract: A novel finite time non-smooth controller of the second system with disturbances is proposed. Its stability is proved by homogeneous system theory, its strong disturbances rejection ability is studied by numerical simulation. The azimuth non-smooth compounded controller of the small float satellite antenna including the non-smooth feed-forward controller, non-smooth feedback controller and the non-smooth proportional and differential (PD) controller is applied to control the azimuth attitude of the antenna. The accurate disturbances estimation of the non-smooth feed-forward controller, the strong disturbances rejection of the non-smooth feedback controller and the rapid response of the non-smooth PD controller are studied. Experiments validate the azimuth non-smooth compounded controller of the small float satellite antenna can control the azimuth attitude of the antenna in time and guarantee the communication between the antenna and the target satellite with high quality.

Key words: non-smooth controller; non-smooth feed-forward controller; non-smooth feedback controller; non-smooth proportional and differential (PD) controller; antenna control

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The non-smooth control is a nonlinear control method lying between the smooth control and discontinuous control, which has been widely studied by reason of its good convergence property and strong disturbance rejection at the balance position of the system. For most nonlinear systems that met the Lipschitz continuous condition, they converged to the balance position at the exponential form in an infinite settling time with a low control efficiency, since 1963 Ryan studied a finite time control law of a linear saturated second order system by isochrones on the base of root locus of the phase place[1], scholars have being studied the non-smooth control. But it is difficult to design a non-smooth finite time controller by reason of the uniqueness of the non-smooth controller for the same control system. Bhat and Besnstein respectively studied global finite time stable controller (1) and (2) of the second order integrator system without a disturbance based on the homogeneity theory in 1997 and 1998[2].

$$u = -k_1 \mathrm{sign}(x_1)|x_1|^{\alpha_1} - k_2 \mathrm{sign}(x_2)|x_2|^{\alpha_2} \quad (1)$$

where $k_1>0$, $k_2>0$, $0<\alpha_1<1$, $0<\alpha_2<1$, $\alpha_2=2\alpha_1/(1+\alpha_2)$.

$$u = -\mathrm{sign}(y)|y|^{\alpha} - \mathrm{sign}(O_0(x,y))|O_0(x,y)|^{\frac{\alpha}{2-\alpha}} \quad (2)$$

where y is the output of the double integrator system, $0<\alpha<1$,

$$O_0(x,y) = x + \frac{1}{2-\alpha}\mathrm{sign}(y)|y|^{2-\alpha}$$

Ding and Li designed a finite time stable

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controller (3) of the second order system with a bounded disturbance to make the system coverage to the zone Q_1 in 2007[3].

$$Q_1 = \Bigg\{(x_1, x_2) : |x_1| \leq \left(\frac{c_1 k_3}{c_2 k_1}\right)^{\frac{q}{1+q}} \left(\frac{2^{\frac{1}{q}} l_1}{2c_1(2q-1)k_3 k_1^{1+q}}\right)^{\frac{q}{2-q}}, \; x_2 \leq \left(1 + k_1\left(\frac{c_1 k_3}{c_2 k_1}\right)^{\frac{q}{1+q}}\right)\left(\frac{2^{\frac{1}{q}} l_1}{2c_1(2q-1)k_3 k_1^{1+q}}\right)^{\frac{q}{2-q}}\Bigg\}$$

$$u = -\left(2 - \frac{1}{q}\right) 2^{1-\frac{1}{q}} k_1^{1+q} k_2 (x_2^q + k_1^q x_1) \frac{2}{q-1} \quad (3)$$

where

$$k_2 > \frac{2^{1-\frac{1}{q}}}{1+q}\left(\frac{2^{3-\frac{1}{q}} q}{(1+q)k_1}\right)^q + \frac{1}{k_1} + \frac{q}{1+q}\left(\frac{4}{(1+q)k_1}\right)^{\frac{1}{q}}$$

$k_1>0$, $1<q=q_1/q_2<2$, q_1 and q_2 are positive odd. From then on they designed three finite time continuous feedback controllers of the extern second order system without a disturbance respectively based on the homogeneity theory, the power integration method and the terminal sliding mode control method[4].

$$u = -k_1 \text{sign}(x_1)|x_1|^{\alpha_1} - k_2 \text{sign}(x_2)|x_2|^{\alpha_2} \quad (4)$$

where $k_1>0$, $k_2>0$, $0<\alpha_1<1/m$, $\alpha_2=(m+1)\alpha_1/(1+\alpha_1)$, m is the degree of the extern system.

$$u = -l(x_2^q + \alpha^{\frac{q}{m}} x_1)^{\frac{1-\tau}{q}} \quad (5)$$

where

$$l \geqslant \left(2 - \frac{1}{q}\right)\alpha^{1+q/m}\left(2^{1-1/q}\frac{q}{m+q} + \frac{2^{1-m/q}m}{m+q} + \frac{2^{2-(m+1)/q}}{\alpha} + l'\right)$$

$$\alpha = 2^{1-q/m}\frac{q}{m+q} + 2^{1-1/q}\frac{q}{m+q} + l'$$

$l' > 0$, $q=m+\tau$, $0<\tau<1$, $\tau=\tau_1/\tau_2$, τ_1 is a positive even.

$$u = -l x_2^{l+m-mp} - \gamma(x_1 + c x_2^{mp})^q \quad (6)$$

where $l=1/(cmp)$, p and q can be expressed as the divisor of two co-prime odds, $1/m<p<2/m$, $0<q<1$, $c>9$, $\gamma>0$. The above studies are helpful for non-smooth control, but these studies could not consider the disturbance of system, only focus on the non-smooth feedback controller, and not include non-smooth feedforward controller and non-smooth PID controller.

In the paper a novel non-smooth controller of the second order system with disturbances is proposed. The azimuth non-smooth compounded controller based on the novel non-smooth controller including the non-smooth feed-forward controller, the non-smooth feedback controller and the non-smooth PD controller is applied to control the azimuth attitude of the small floating satellite antenna. Experiments validated the azimuth non-smooth compounded controller can well control the azimuth attitude of the small float antenna and guarantee the communication between the antenna and the target satellite with high quality.

1 A Novel Non-smooth Controller and Its Characters

1.1 A novel non-smooth controller

In the paper the standard second order integrator system with a bounded disturbance is expressed as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dot{u} + d(t) \\ y = x_1 \end{cases} \quad (7)$$

where $d(t)$ is a disturbance and $0\leq d(t)\leq l$, l is constant and $l>0$, u is an input and y is an output of the system. The vector $\boldsymbol{x}=[x_1,x_2]^{\mathrm{T}}$ is the state of the system. A novel non-smooth controller of the second order integrator system with a bounded disturbance expressed as (7), and different from other controllers in the published references is proposed by reason of the non-uniqueness of the non-smooth controller for the same class system.

$$u = -k_1 x_1^{2\alpha_1+1} - k_2 \text{sign}(x_2)|x_2|^{\alpha_2} \quad (8)$$

where $k_1>0$, $k_2>0$, $0<2\alpha_1+1<1$ and $2\alpha_1+1=n/m$, m is a positive odd, n is a positive integer, $0<\alpha_2<1$. The controller (8) is a finite time stable controller and has strong disturbance rejection ability.

1.2 Stability of the novel non-smooth controller

We shall prove the stability of the novel non-smooth controller (8) by homogeneous dynamic system theory and Barbalat lemma. The related theorem and lemmas are as follows.

Barbalat lemma 1 Suppose $x:[0,+\infty)\to R$ is continuous, differentiable, time-varying the first order system, when $t\to\infty$, there is limit of the system. If $x(t), t\in[0,+\infty)$ is uniformly continuous, then $\lim_{t\to\infty}x(t)=0$ [5].

Barbalat lemma 2 Suppose $x:[0,+\infty)\to R$ is continuous, differentiable, time-varying the first order system, when $\to\infty$, there is limit of the system. If there is $\dot{x}(t)$, $t\in[0,+\infty)$, then $\lim_{t\to\infty}x(t)=0$ [5].

Lemma 3 If $x:[0,+\infty)\to R$ is uniformly continuous, and $\left|\lim_{t\to\infty}\int_0^t x(\tau)\,\mathrm{d}\tau\right|\le M$, M is a constant, then $\lim_{t\to\infty}x(t)=0$[5].

Barbalat lemma 4 If $x:[0,+\infty)\to R$ is uniformly continuous, and $x\in L_p, p\in[1,\infty)$, $L_p:=\{x|\,x:[0,\infty)\to R\quad,(\int_0^\infty|x(t)|^p\mathrm{d}t)^{\frac{1}{p}}<\infty\}$, then $\lim_{t\to\infty}x(t)=0$[6].

Barbalat lemma 5 Suppose: $x:[0,+\infty)\to R$ is L_p, $p\in[1,+\infty)$ and $\dot{x}(t), t\in[0,+\infty)$ is bounded, then $\lim_{t\to\infty}x(t)=0$.

Barbalat lemma 6 Suppose $x:[0,+\infty)\to R$ is absolutely continuous. If $x(t)\in L_p, p\in[1,+\infty)$, and for a compact set $C\in[0,+\infty)$ is uniformly locally integrable, then $\lim_{t\to\infty}x(t)=0$[6-7].

Barbalat lemma 7 Suppose $\alpha:R^+\to R$ is continuous and not decreasing, and only when $x=0$, $\alpha(0)=0$, if $x:[0,+\infty)\to R$ is uniformly continuous and $\alpha(|x(t)|)=0$, then $\lim_{t\to\infty}x(t)=0$[7].

Barbalat lemma 8 Suppose $f(t):R^+\to R$ is a derivative function, and when $t\to\infty$ it has limit, if its derivative is made up of two function: $f(t)=g_1(t)+g_2(t)$, where $g_1(t)$ is uniformly continuous function, and $\lim_{t\to\infty}g_2(t)=0$, then $\lim_{t\to\infty}g_1(t)=0$, $\lim_{t\to\infty}f(t)=0$[7].

Theorem 1 If the system is globally asymptotically stable and has a negative homogeneity, then the system is globally stable in finite time[8-9].

According to Barbalat lemmas, homogeneous system theory, only if we prove the system globally asymptotically stable and it has a negative homogeneity. The system is globally stable in finite time.

Proof Consider a Lyapunov function candidate

$$\mathrm{V}(x_1,x_2)=\frac{x_2^2}{2}+\frac{c}{2\alpha_1+2}x_1^{2\alpha_1+2} \tag{9}$$

where c is constant and $c>0$, $c\in R$. Obviously $V(x_1,x_2)$ is a continuously differentiable function defined on R, positive definite and radially unbounded by reason of $c>0$ and $2\alpha_1+1>0$. The derivative of $V(x_1,x_2)$ is

$$\begin{aligned}\dot{V}(x_1,x_2)=&x_2\dot{x}_2+cx_1^{2\alpha_1+1}\dot{x}_1=\\&x_2(-k_1x_1^{2\alpha_1+1}-k_2|x_2|^{\alpha_2}\mathrm{sign}(x_2)+\\&d(t))+cx_1^{2\alpha_1+1}x_2=\\&(-k_1+c)x_1^{2\alpha_1+1}x_2-k_2|x_2|^{\alpha_2+1}+d(t)x_2\end{aligned} \tag{10}$$

Let $c=k_1$, then

$$\begin{aligned}&\dot{V}(x_1,x_2)=-k_2|x_2|^{\alpha_2+1}+|x_2|\,d(t)\le\\&-k_2|x_2|^{\alpha_2+1}+l\,|x_2|\le 0\end{aligned} \tag{11}$$

Obviously $V(x_1,\ x_2)$ is nonincreasing and bounded. Of course x_1 and x_2 are bounded. The derivative of $\dot{V}(x_1,\ x_2)$ is

$$\ddot{V}(x_1,x_2)=-\frac{k_2}{1+\alpha_2}|x_2|\,\mathrm{sign}(x_2) \tag{12}$$

$\because 1+\alpha_2>0, \therefore \ddot{V}(x_1,x_2)$ is bounded, and $\dot{V}(x_1,x_2)$ is uniformly continuous. Based on Barbala lemma $\because \dot{V}(x_1,x_2)\to 0$, then $x_2(t)\to 0$. From (7) and (8) x_1 and x_2 are respectively bounded, so x_1 and x_2 are uniformly continuous. In order to prove stability of the novel non-smooth controller, the derivative of x_1x_2 is

$$\begin{aligned}\frac{\mathrm{d}(x_1x_2)}{\mathrm{d}t}=&x_2\dot{x}_1+x_1\dot{x}_2=x_2^2-k_1x_1^{2\alpha_1+2}-\\&k_2x_1|x_2|\,\alpha_2\mathrm{sign}(x_2)+x_1d(t)\le\end{aligned}$$

$$x_2^2-k_1x_1^{2\alpha_1+2}-k_2x_1|x_2|\,\alpha_2\mathrm{sign}(x_2)+|x_1|l$$

Set $g_1(t)=-k_1x_1^{2\alpha_1+2}+|x_1|\,l\le|x_1|\,l$, $g_2=x_2^2-k_2x_1|x_2|\,\alpha_2\mathrm{sign}(x_2)$.

Obviously $g_1(t)$ is bounded and uniformly continuous. When $x_1(t)\to 0$, $g_1(t)\to 0$. Based on Barbala lemma 8 when $g_1(t)\to 0$, $x_1(t)\to 0$. So

the closed system (8) is globally asymptotically stable.

Based on homogeneous system, set r_1=1, $r_2=\dfrac{2+2\alpha_1}{2}, 0<2\alpha_1+1<1, \alpha_2=\dfrac{2(2\alpha_1+1)}{2+2\alpha_1}$, then the vector field of the system (11) satisfies

$$f(\varepsilon^{r_1}x_1,\varepsilon^{r_2}x_2)=\begin{pmatrix}\varepsilon^{r_2}x_2\\ -k_1\varepsilon^{r_1\alpha_1}\alpha_1^{2\alpha_1}-k_2\varepsilon^{r_2\alpha_2}\mathrm{sign}(x_2)|x_2|^{\alpha_2}\end{pmatrix}=\begin{pmatrix}\varepsilon^{r_1+k} & 0\\ 0 & \varepsilon^{r_2+k}\end{pmatrix}\begin{pmatrix}x_2\\ -k_1x_1^{2\alpha_1+1}-k_2\mathrm{sign}(x_2)|x_2|^{\alpha_2}\end{pmatrix}=\begin{pmatrix}\varepsilon^{r_1+k}f_1(x_1,x_{2)}\\ \varepsilon^{r_2+k}f_2(x_1,x_{2)}\end{pmatrix} \tag{13}$$

$$k=-\frac{1-(1+2\alpha_1)}{2}=\alpha_1<0 \tag{14}$$

where homogeneity of the system is based on Theorem 1, the system (8) is a globally stable in finite time.

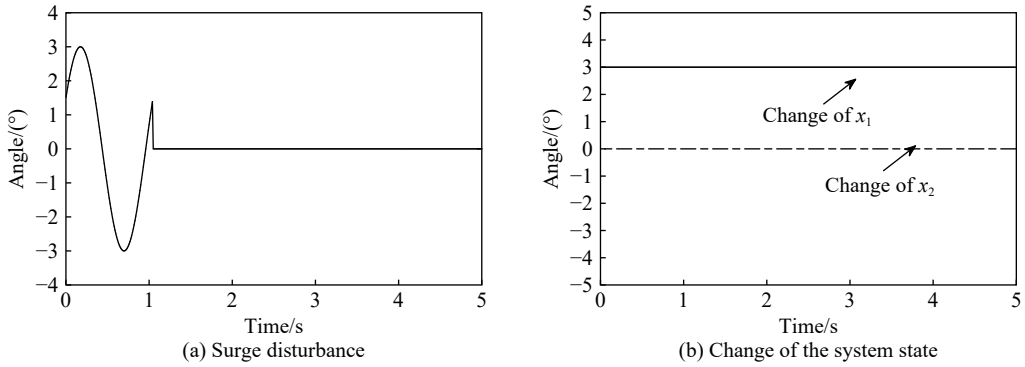
1.3 Disturbance rejection ability of the novel non-smooth controller

The disturbance rejection ability of the novel non-smooth controller (8) is studied by numerical simulation. The disturbances from environment are composed of a surge disturbance and a random disturbance individually or in combination.

① Suppose the initial state: $(x_1,x_2)^{\mathrm{T}}=(3,1)^{\mathrm{T}}$, when a surge disturbance expressed as (15) is added to the system (7), then the state change of the system is shown in Fig. 1 when k_1=150, k_2=1.5, $2\alpha_1$+1=10/11, α_2=0.1. From Fig. 1, the novel non-smooth controller (8) can rapidly and well reject the surge disturbance.

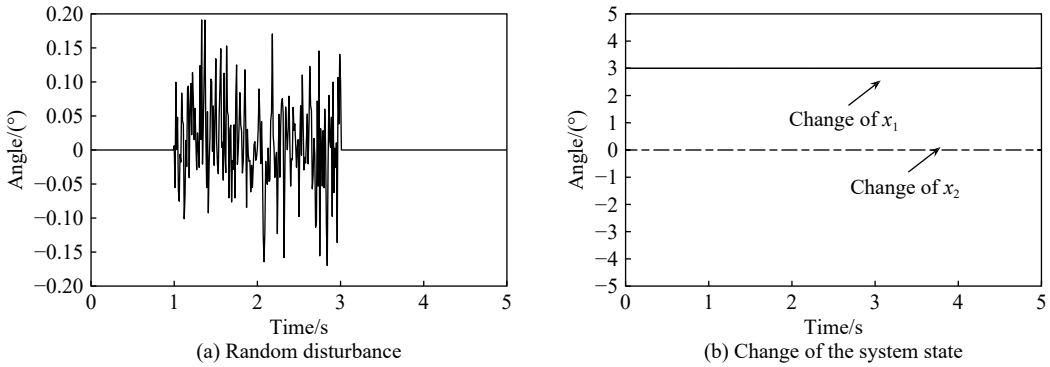
$$d(t)=\begin{cases}3\cos(-6t+\pi/3), & 0\le t\le \pi/3\\ 0, & \text{others}\end{cases} \tag{15}$$

② Suppose there is a random disturbance whose distribution density conforms to the standard normal distribution and its amplitude is not more than 2%. When the random disturbance shown in Fig. 2 is added to the system (7), the state change of the system is shown in Fig. 2 when k_1=150, k_2=1.5, $2\alpha_1$+1=10/11, α_2=0.1. Obviously the novel non-smooth controller (8) can well reject the random disturbance.



(a) Surge disturbance (b) Change of the system state

Fig. 1 Disturbance rejection of the surge disturbance



(a) Random disturbance (b) Change of the system state

Fig. 2 Disturbance rejection of the random disturbance

③ Suppose a complex disturbance expressed as follows is added to the system (7)

$$u = A(t)\cos(\omega(t)t) \tag{16}$$

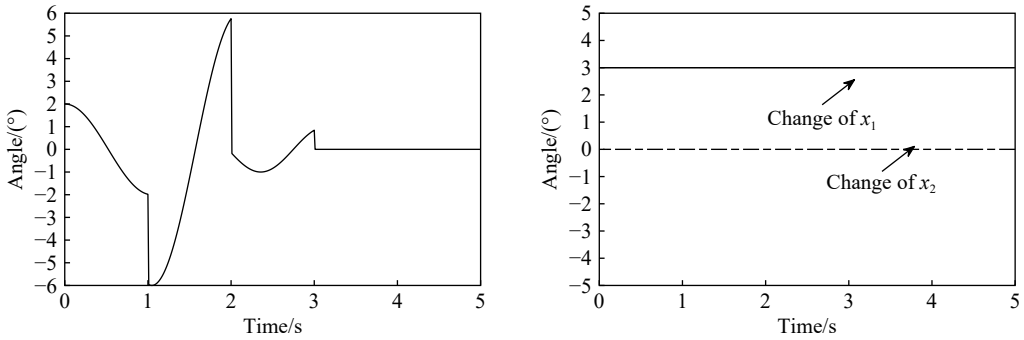
where

$$A(t) = \begin{cases} 2, 0 \le t \le 1 \\ 6, 1 < t \le 2 \\ 1, 2 < t \le 3 \\ 0, t > 3 \end{cases}, \quad \omega(t) = \begin{cases} 3, 0 \le t \le 2 \\ 4, 2 < t \le 3 \\ 0, t > 3 \end{cases}$$

The state change of the system is shown in Fig. 3 when k_1=150, k_2=1.5, $2\alpha_1+1=10/11$, α_2=0.1. From Fig. 3 the novel non-smooth controller (8) can also rapidly and well reject the complex disturbance.

It is clear that the novel non-smooth controller (8) of the system (7) can well reject all kinds of disturbances and the system (7) can rapidly converge to the balance in less than 0.05 s.

$$(x_1, x_2)^{\mathrm{T}} = (3, 0)^{\mathrm{T}} \tag{17}$$



(a) Complex surge disturbance (b) Change of the system state

Fig. 3 Disturbance rejection of the complex surge disturbance

2 Azimuth Compounded Non-smooth Controller of the Small Float Satellite Antenna

2.1 Small float satellite antenna

The small float satellite antenna in the paper is a novel non-stationary antenna, which has been studied in Ref. [10] and will float at China coast within some zones. For a good communication quality between satellite and antenna, the antenna must always point to the best zone. When the antenna deviates from the best zone, the communication quality will degrade, in order to ensure the communication quality it is necessary to control the antenna to point to the best zone in real time. In application the small float antenna is easily deviated from the best zone due to the influence of marine environmental disturbance, so the controller is applied to reject environmental disturbances and compensate the bias between the attitude of the antenna and the best zone of the communication to ensure the communication with high quality. The attitude tracking of the small float antenna is realized by the motion of the servo actuators. The servo actuator mode is expressed as[11]

$$G(s) = \frac{0.6}{(0.3s+1)} \frac{8.696}{(0.001\,7s+1)} \tag{18}$$

2.2 Non-smooth feed-forward controller

In the novel non-smooth controller (8) x_1 is the state variable of the system and x_2 is the derivative of x_1. The derivative of the disturbance d need to be calculated in time when the non-smooth feed-forward controller is designed. In order to calculate the derivative of the disturbance we design tracking derivative(TD) which can output the smooth original input signal and its derivative.

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -aR^2\left[(x_1(t) - v(t) + \dfrac{x_2(t)}{R}\right] - \\ \qquad bR^2\left[(x_1(t) - v(t))^m + \left(\dfrac{x_2(t)}{R}\right)^m\right] \end{cases} \tag{19}$$

where R is the constant time scale and R>0, a>0, b>0, m is odd and m>2, $v(t)$ is the origin-

al input, x_1 is the estimation of $v(t)$, x_2 is the derivative of x_1. The non-smooth feed-forward controller can be expressed as

$$u_1 = -k_{1f} d^{2\alpha_{1f}+1} - k_{2f}\,\mathrm{sign}(\dot{d})\left|\dot{d}\right|^{\alpha_{2f}} \tag{20}$$

where $k_{1f}>0$, $0<2\alpha_{1f}+1=n_f/m_f<1$, m_f is odd, n_f is a positive integer, $k_{2f}>0$, $0<2\alpha_{2f}<1$. The disturbances from marine environment are mainly composed of a surge disturbance, and a random disturbance individually and in combination.

① Suppose a surge disturbance expressed as follows is added to the system

$$d(t) = \begin{cases} \cos t, & 2 \le t \le 2+2\pi \\ 0, & \text{others} \end{cases} \tag{21}$$

The surge disturbance estimation is shown in Fig. 4 with the parameters: k_{1f}=1.2, $2\alpha_{1f}+1=n_f/m_f$=1/11, k_{2f}=0.95, α_{2f}=0.4, a=5, R=10, b=5, m=7. From Fig. 4, the surge disturbance estimation relative error is less than 0.2%.

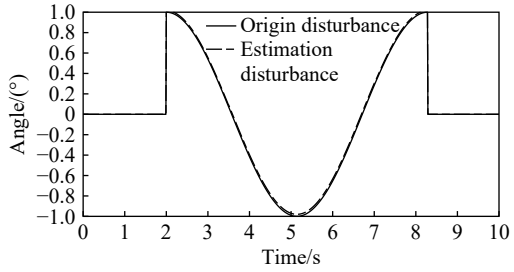


Fig. 4 Estimation of the surge disturbance

② Suppose a random disturbance in Fig. 5 is added to the system, the random disturbance estimation is shown in Fig. 5 with the parameters: k_{1f}=1.2, $2\alpha_{1f}+1=n_f/m_f$=1/11, k_{2f}=0.95, α_{2f}=0.4, a=5, R=10, b=5, m=7. From Fig. 5, the random disturbance estimation is nearly equal to the original. The relative error is less than 0.1%.

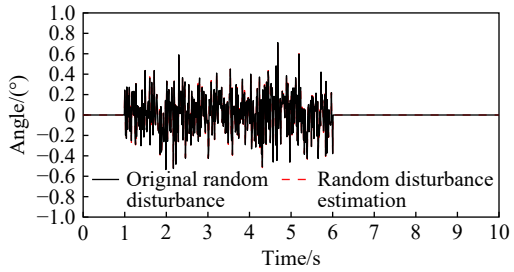


Fig. 5 Estimation of the random disturbance

③ Suppose a complex disturbance is added to the system. The complex disturbance estimation is shown in Fig. 6 with the parameters: k_{1f}=1.2, $2\alpha_{1f}+1=n_f/m_f$=1/11, k_{2f}=0.95, α_{2f}=0.4, a=5, R=10, b=5, m=7. From Fig. 6, the maximum absolute error is nearly 0.1.

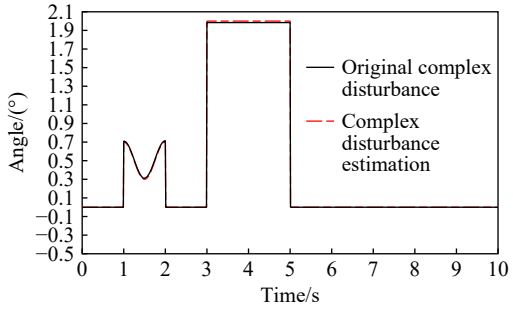


Fig. 6 Estimation of the complex disturbance

$$d(t) = \begin{cases} 0.5 + 0.2\cos 2\pi(t-1), & 1 \le t \le 2 \\ 2, & 3 \le t \le 5 \\ 0, & \text{others} \end{cases} \tag{22}$$

Obviously the non-smooth feed-forward controller can accurately estimate all kinds of disturbances on sea.

2.3 Non-smooth feedback controller

The non-smooth feedback controller is designed as follows with the help of TD (19)

$$u_2 = -k_{1fd} y^{2\alpha_{1fd}+1} - k_{2fd}\left|\dot{y}\right|^{\alpha_{2fd}}\mathrm{sign}(\dot{y}) \tag{23}$$

where $k_{1fd}>0$, $0<2\alpha_{1fd}+1=n_{fd}/m_{fd}<1$, m_{fd} is odd, n_{fd} is a positive integer, $k_{2fd}>0$, $0<\alpha_{2fd}<1$, y is the system output. In order to validate the disturbance rejection ability of the non-smooth feedback controller we do some experiments as follows.

① After a step signal is input to the system, a surge disturbance expressed as follows is added to the input of the system.

$$d(t) = \begin{cases} 0.2\cos 4t, & 3 \le t \le 3+\pi/2 \\ 0, & \text{others} \end{cases} \tag{24}$$

The step response of the non-smooth feedback controller with the surge disturbance with the parameters k_{1fd}=382, $2\alpha_{1fd}+1$=10/11, k_{2fd}=53.5, α_{2fd}=0.3 is shown in Fig. 7. From Fig. 7, the disturbance rejection ratio is larger than 0.95.

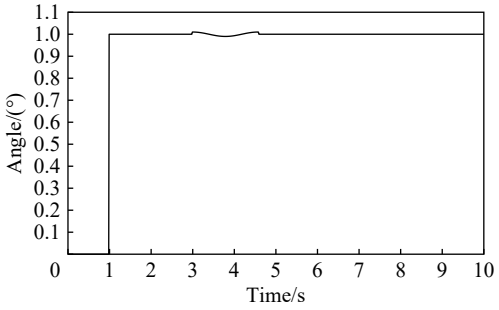


Fig. 7 Step response of the non-smooth feedback controller with the surge disturbance

② After a step signal is input to the system, a random disturbance shown in Fig. 3. a is added to the input of the system. The step response of the non-smooth feedback controller with the random disturbance with the parameters k_{1fd}=382, $2\alpha_{1fd}+1=10/11$, k_{2fd}=53.5, α_{2fd} =0.3 is shown in Fig. 8. From Fig. 8, the disturbance rejection ratio is near to 1.

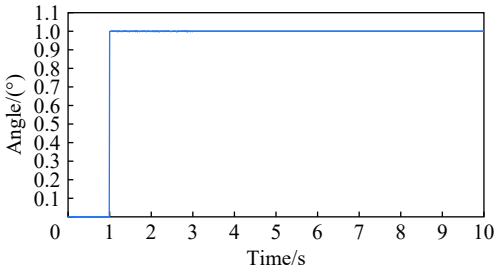


Fig. 8 Step response of the non-smooth feedback controller with the random disturbance

③ After a step signal is input to the system, a complex disturbance expressed as (22) is added to the input of the system. The step response of the non-smooth feedback controller with the complex disturbance with the parameters k_{1fd}=382, $2\alpha_{1fd}+1=10/11$, k_{2fd}=53.5, α_{2fd} =0.3 is shown in Fig. 9. From Fig. 9, the minimum disturbance rejection ratio is 0.975.

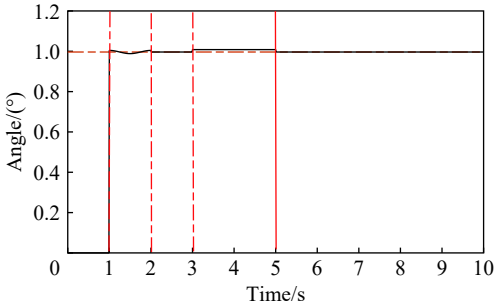


Fig. 9 Step response of the non-smooth feedback controller with the complex disturbance

Obviously the non-smooth feedback controller has strong disturbance rejection ability and can reject all kinds of disturbance on sea.

2.4 Non-smooth PD controller

The control variable of the traditional PID control law expressed as follows is the sum of the proportion, the integrator and the differential of the deviator.

$$u(t)=k_{\mathrm{p}}e(t)+k_{\mathrm{i}}\int e(t)\mathrm{d}t+k_{\mathrm{d}}\frac{\mathrm{d}e(t)}{\mathrm{d}t} \qquad (25)$$

where $u(t)$ is the input of the controller, $e(t)$ is the deviator of the output and the target of the system, k_{p} is the proportional coefficient, k_{i} is the integration coefficient, k_{d} is the differential coefficient. In the real application the PID control law was made up of the partial and total sum of the proportion, the differential and the integrator of system deviator[12]. In the paper the non-smooth PD controller is designed as follows with the help of TD (19)

$$u(t)=k_{pn}e(t)^{2\alpha_p+1}+k_{dn}|\dot{e}(t)|^{\alpha_d}\mathrm{sign}(\dot{e}(t)) \qquad (26)$$

where $k_{pn}>0$, $k_{dn}>0$, $0<2\alpha_p+1=n_{pn}/m_{pn}<1$, m_{pn} is odd, n_{pn} is a positive integer, $0<\alpha_d<1$. In order to validate the performance of the non-smooth PD controller, with the help of the linear feedback controller with the feedback coefficient k_f=0.925, the step response curves of the non-smooth PD controller with the parameters k_{pn}=4, $2\alpha_p+1=n_{pn}/m_{pn}=3/25$, k_{dn}=1.18, α_d=0.98, a=5, R=10, b=5, m=7 and the traditional PID controller with the parameters k_p=2.5, k_d=0.02, k_i=0.01 are shown in Fig. 10.

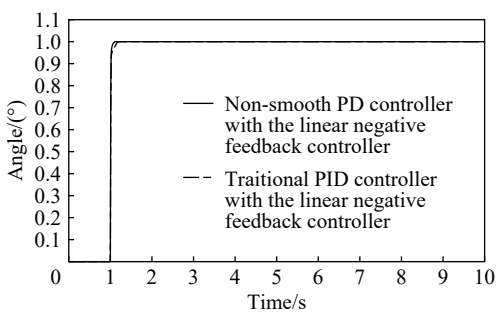


Fig. 10 Step response of the non-smooth PD controller and the traditional PID controller

From Fig.10, the response speed of the non-smooth PD controller is faster than that of the traditional PID controller; the overshot of the non-smooth PD controller is nearly zero. Obviously the performance of the non-smooth PD controller is better than that of the traditional PID controller.

3 Experiments Research

3.1 MATLAB simulation

The azimuth non-smooth compounded controller of the small float satellite antenna is modeled by MATLAB according to its components. The parameters of the azimuth non-smooth compounded controller are listed in Tab.1.

Tab. 1 Parameters of the azimuth non-smooth compounded controller

Parameter	Value	Parameter	Value
k_{1f}	0.8	$2\alpha_{1f}+1$	2/9
k_{2f}	11.6	α_{2f}	0.9
A	5	R	10
B	5	m	7
k_{1fd}	1.025	$2\alpha_{1fd}+1$	1/11
k_{2fd}	0.95	α_{2fd}	0.4
k_{pn}	1	$2\alpha_{pn}+1$	1/19
k_{dn}	0.5	α_{dn}	0.5

The step response curve of the azimuth non-smooth controller is shown in Fig. 11. From Fig. 11, the response of the azimuth non-smooth compounded controller without overshot is rapider than the compounded controller including the PID controller and linear feedback controller with parameters k_p=2.5, k_d=0.02, k_i=0.01, k_f=0.925.

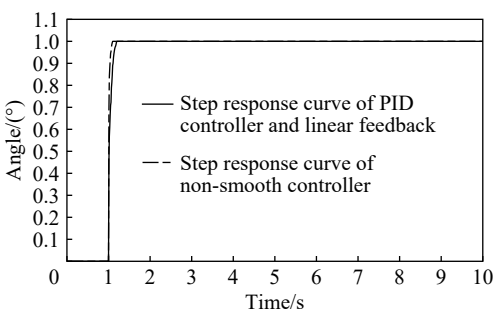


Fig. 11 Step response of the azimuth non-smooth compounded controller vs the traditional PID controller

In order to validate the disturbance rejection ability of the azimuth non-smooth compounded controller we do some experiments as following.

① After a step signal is input to the system, a surge disturbance expressed as follows is added to the input of the system

$$d(t)=\begin{cases}2\cos 4(t-2), & 3\leqslant t\leqslant 3+\pi/2\\ 0, & \text{others}\end{cases} \tag{27}$$

The step response curve of the azimuth non-smooth compounded controller with the surge disturbance is shown in Fig. 12. From Fig. 12, we can find the disturbance rejection ratio of the surge disturbance is larger than 0.99.

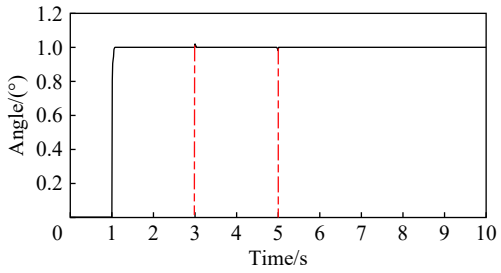


Fig. 12 Step response of the azimuth non-smooth compounded controller with the surge disturbance

② After a step signal is input to the system, a random disturbance shown in Fig. 5 is added to the input of the system, the step response curve of the azimuth non-smooth compounded controller with the random disturbance is shown in Fig. 13. From Fig. 13, we can find the disturbance rejection ratio of the random disturbance is larger than 0.998.

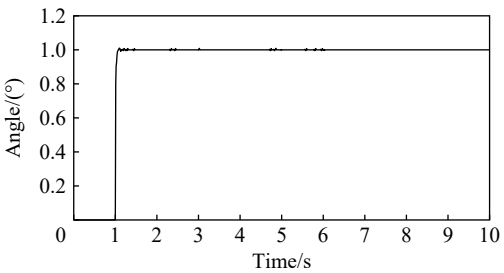


Fig. 13 Step response of the azimuth non-smooth compounded controller with the random disturbance

③ After a step signal is input to the system, a complex disturbance expressed as (22) is added to the input of the system, the step response

curve of the azimuth non-smooth compounded controller with the complex disturbance is shown in Fig. 14. From Fig. 14, we can find the disturbance rejection ratio of the complex disturbance is larger than 0.988.

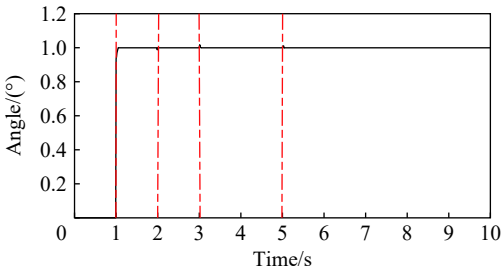


Fig. 14 Step response of the azimuth non-smooth compounded controller with the complex disturbance

Obviously the azimuth non-smooth compounded controller can well reject disturbance and has a good control effect to realize the azimuth attitude control of the small float satellite antenna.

3.2 Hardware in-the-loop simulation

The small float satellite antenna is put on the six degree of freedom swaying platform, and the swaying platform is put on the flat car. The hydrostatic flow is simulated by the translation of the flat car and the surge disturbance is simulated by the swing of the platform. The hardware in-the-loop simulation simulated the condition of the small antenna floating on sea to validate the control effect of the azimuth non-smooth compounded controller with the parameters in Tab. 1. Firstly the small float antenna located the satellite ChinaStar-10, then the flat car started translation at the speed of 0.2 m/s and the six-degree freedom swaying platform swayed at the trail $\xi \approx \cos(t)$, in the process of experiment the electromagnetic acquisition component acquired the auto gain control (AGC) for an hour. In the process of Experiment the transmitting frequency of the antenna is 1132 MHz, the transmitting bandwidth is 2 M, the transmitting power is –19 dB, the receiving frequency is 1134 MHz, the power amplifier is 40 W. The curve of the AGC is shown in Fig. 15. In fact when AGC> 4.2 dB. The small float antenna can communicate with the target satellite with high quality. From Fig. 15, the AGC lies between 4.5 dB and 8 dB, obviously the small float satellite antenna can communicate with ChinaStar-10 satellite with high quality. It is proved that the azimuth non-smooth compounded controller of the small float antenna can well realize the antenna stable tracking the target satellite and ensure the communication between the antenna and the target satellite with high quality.

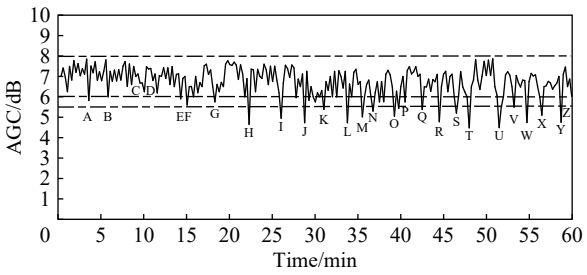


Fig. 15 Curve of AGC in-the-loop simulation

4 Conclusion

A novel non-smooth controller of the second order system with disturbances is proposed in the paper. Its stability is proved by homogeneous system theory. Its strong disturbance rejection ability is validated by numerical simulation. The azimuth non-smooth compounded controller of the small float satellite antenna including the non-smooth feed-forward controller, the non-smooth feedback controller and the non-smooth PD controller is applied to control the azimuth attitude of the small float satellite antenna. The precise disturbance estimation of the non-smooth feed-forward controller, the rapidly response and strong disturbance rejection of the non-smooth feedback controller and the rapid response of the non-smooth PD controller are verified by the numerical simulation. It is validated the azimuth non-smooth compounded controller can reject the surge disturbance well and control the azimuth attitude of the small float satellite antenna to ensure the communication with high quality between the antenna and the target satellite by

the numerical simulation and the hardware in-the-loop simulation.

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