

Improved ensemble Kalman filter algorithm based on GNSS/SINS integrated navigation

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Abstract: The ensemble Kalman filter (EnKF) has emerged as a popular data fusion filtering method in vehicle-mounted global navigation satellite system/strapdown inertial navigation system (GNSS/SINS) integrated navigation systems. It employs Monte Carlo methods based on sample estimates to approximate the system's state distribution. However, the EnKF typically assumes a Gaussian distribution for the state distribution, and this assumption may fail in non-Gaussian scenarios. To address this issue, this paper proposes a Cauchy robust ensemble Kalman filter (CREnKF) that dynamically identifies and suppresses outliers through the Cauchy weighting function, and reduces the impact of non-Gaussian noise by combining residual direct weighting and observation covariance reconstruction dual-path robustness strategies. The algorithm was applied to a GNSS/SINS integrated navigation system and tested through simulation experiments and in-vehicle experiments. The experimental results show that the position RMSE of this scheme in a non-Gaussian noise environment is decreased by 82%, 81%, and 63% relative to EKF, EnKF, and EnKF robust with Huber Kernel function, respectively, effectively enhancing the positioning accuracy of the integrated navigation system.

Key words: EnKF; non-Gaussian noise; integrated navigation; robust filter; Monte Carlo methods; Cauchy function

0 Introduction

In an integrated navigation and positioning system, high-precision filtering algorithms have a significant impact on the accuracy of navigation and positioning calculations. The Kalman filter (KF) is a commonly used navigation filtering algorithm^[1-4], which is based on the linear Gaussian assumption and achieves optimal state estimation through a two-stage predictive-update recursive process. However, during the actual movement of the vehicle, sudden state changes and other behaviors can significantly increase the system's nonlinearity. At this point, process noise and measurement noise are often time-varying, which can affect the filter's accuracy^[5-7]. To address these issues, researchers have proposed nonlinear algorithms, such as the extended Kalman filter (EKF)^[8-9], sage-husa adaptive filter (SHAKF)^[10-11], and volume Kalman filter (CKF)^[12-13]. The EKF locally linearizes the nonlinear system model using a first-order Taylor expansion and updates the covariance matrix using the Jacobian matrix. However, it is important to note that the EKF performs poorly under strong nonlinearity and struggles to handle non-Gaussian

measurement noise. The SHAKF dynamically adjusts the covariance matrices of process noise and measurement noise based on system updates, adapting to changes in measurement noise. However, SHAKF is sensitive to the initial value settings. If the initial values are improperly set, the filter may require a long time to converge, or even diverge due to numerical issues, before it converges. CKF performs a third-order unbiased approximation of Gaussian integrals by placing symmetric cube points in the state space, eliminating the need to calculate the Jacobian matrix. As a result, it provides higher accuracy than EKF in weakly nonlinear systems and exhibits good numerical stability. However, CKF assumes that noise follows a Gaussian distribution, making it sensitive to heavy-tailed non-Gaussian noise, and it lacks the adaptive noise covariance change mechanism of SHAKF. There is still a risk of residual errors or insufficient convergence in strongly nonlinear or highly non-Gaussian environments.

When the measurement system is subject to heavy-tailed non-Gaussian noise interference, performing the above filters often deteriorates significantly^[14-15]. Heavy-tailed non-Gaussian noise is commonly present in practical engineering

applications, particularly when the system is subjected to pulse noise, which exhibits a non-Gaussian, heavy-tailed distribution. Compared to other filters, the EnKF employs Monte Carlo sampling to generate a state set and performs recursive estimation by calculating the mean and covariance of the state set, demonstrating excellent performance when the system is nonlinear^[16]. Fan et al.^[17] proposed a method combining EnKF with particle filter (PF) for optimal estimation. The two proposed filtering frameworks ensure that the update process for system states and parameters no longer strictly relies on the Gaussian distribution assumption, thereby enhancing the robustness and accuracy of the filter in non-Gaussian environments. However, because EnKF and PF must run separately at each time step, they significantly increase the computational overhead. Additionally, the algorithm suffers from PF sample degradation issues, making it difficult to apply in practice. Lin et al.^[18] proposed a unscented Kalman filtering (UKF) algorithm that introduced the maximum correlation criterion (MCC), combining the nonlinear processing capability of the UKF and the ability of the MCC to capture higher-order moments, reducing the impact of non-Gaussian noise on the filtering system, and using immobile-point iteration to improve the convergence speed. However, the performance of the algorithm is highly sensitive to the selection of parameters, such as kernel bandwidth; too large or too small will lead to performance degradation and even cause filter divergence, and the performance is not as good as that of the traditional UKF in Gaussian noise environments, which makes the practical application challenging. Jiang et al.^[19] proposed a method to robustify the EnKF using the Huber function, reshaping the measurement update process with a robust function to reduce the impact of outliers. However, the Huber function may assign excessive weights to moderately outlying values, and its thresholds and other hyperparameters require empirical manual tuning, making it difficult to meet the practical application needs of different scenarios.

To address the impact of outliers caused by non-Gaussian noise, this paper proposes a Cauchy robust ensemble Kalman filter (CREnKF). The algorithm is based on the EnKF and introduces a Cauchy robust Kernel function to achieve adaptive robust processing. Through the Cauchy robust Kernel function, an adaptive robust mechanism reduces the weight of outlier observation particles and dynamically reconstructs the observation covariance via a dual-path mechanism to address observation anomalies of varying degrees. Additionally, a multi-channel approach is used to estimate the scale parameters online to address issues from inconsistent noise

intensities across dimensions, which effectively suppresses the negative impact of outlier observations on state estimation. The improved algorithm considers the dual requirements of real-time performance and robustness in navigation and positioning applications, employing simple yet effective optimization methods to significantly enhance positioning reliability in complex environments while maintaining computational efficiency.

1 Robust adaptive ensemble kalman filter

1.1 Ensemble Kalman filter

EnKF performs simulation forecasting and analysis updates in the form of a set, using a set of model states to represent error covariance information, and subject to the constraint of minimizing the error covariance between observed and simulated values, to perform an optimal estimation of the target^[20]. Consider a class of discrete nonlinear systems, as shown in Eqs. (1) and (2).

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k) + \mathbf{F}_k \mathbf{w}_k, \quad (1)$$

$$\mathbf{y}_{k+1} = h(\mathbf{x}_{k+1}) + \mathbf{v}_{k+1}, \quad (2)$$

where $f(\cdot)$ denotes a nonlinear function, $h(\cdot)$ denotes a measurement equation, the discrete-time index is k , the system state vector is $\mathbf{x}_k$, the observation vector is $\mathbf{y}_k$, the noise drive matrix is $\mathbf{F}_k$, the system process noise is $\mathbf{w}_k$, the system observation noise is $\mathbf{v}_k$. The statistical characteristics of the noise are shown as

$$E[\mathbf{w}_k] = 0, \quad (3)$$

$$E[\mathbf{w}_k \mathbf{w}_j^T] = \mathbf{Q}_k \delta_{kj}, \quad (4)$$

$$E[\mathbf{v}_k] = 0, \quad (5)$$

$$E[\mathbf{v}_k \mathbf{v}_j^T] = \mathbf{R}_k \delta_{kj}, \quad (6)$$

$$E[\mathbf{w}_k \mathbf{v}_j^T] = 0, \quad (7)$$

where $\mathbf{Q}_k$ is the process noise covariance matrix, representing model uncertainty, typically a non-negative definite matrix, and $\mathbf{R}_k$ is the measurement noise covariance matrix, representing measurement uncertainty, typically a positive definite matrix. δ_{kj} indicates that the noise is independent at different moments. EnKF addresses the state estimation problem for nonlinear systems through ensemble approximation. As shown in Eqs. (8) – (12), optimal estimation is achieved by updating the states and covariances of a set of ensemble members in real time.

$$\mathbf{x}_{k|k-1}^{(i)} = f(\mathbf{x}_{k-1}^{(i)}) + \mathbf{w}_k^{(i)}, \quad i = 1, \dots, N, \quad (8)$$

$$\bar{\mathbf{x}}_{k|k-1} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_{k|k-1}^{(i)}, \quad (9)$$

$$P_{k|k-1} = \frac{1}{N-1} \sum_{i=1}^N (x_{k|k-1}^{(i)} - \bar{x}_{k|k-1})(x_{k|k-1}^{(i)} - \bar{x}_{k|k-1})^T, \quad (10)$$

$$K = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R)^{-1}, \quad (11)$$

$$x_k^{(i)} = x_{k|k-1}^{(i)} + K(Z_k - H_k x_{k|k-1}^{(i)} + v^{(i)}). \quad (12)$$

Eqs. (8) – (10) describe the prediction step, in which each ensemble member is propagated independently through the model operator and model noise is added. Eq. (11) and Eq. (12) describe the update step, in which the Kalman gain is calculated using the observation data and ensemble covariance to adjust the state of the ensemble members. This design ensures that the EnKF can meet the minimum mean square error estimation performance criteria while ensuring that the residual sequence approximately maintains white noise characteristics.

1.2 Adaptive Cauchy function

For traditionally EnKF, the observation innovation for the i sample is defined as

$$r_k^{(i)} = y_k + v_k^{(i)} - H_k x_k^{(i)}, \quad (13)$$

where y_k is the actual observed value, $v_k^{(i)}$ is the observed noise disturbance of the i sample, H_k is the observation matrix, and $x_k^{(i)}$ is the predicted sample. This definition performs well when the observed noise follows a Gaussian distribution and there are no outliers. However, once occasional heavy-tailed noise or outliers appear in the observations, the large residual $r_k^{(i)}$ is overly amplified by the Kalman gain, causing the filtering results to deviate or even diverge^[21]. To address this issue, we introduce the Cauchy Kernel function while retaining the parallelism and random disturbance framework of EnKF. The Cauchy Kernel function corresponds to the heavy-tailed distribution assumption, which is more in line with the impulse noise and heavy-tailed non-Gaussian noise characteristics that are prevalent in practical engineering environments. The Huber Kernel function, on the other hand, is equivalent to a quadratic loss when the residuals are small, and turns to a primary loss when it exceeds the threshold, and thus still has a large weight for medium-amplitude anomalies, which is not enough to weaken the effect of extreme anomalies. The Tukey Kernel is too aggressive in its truncation, which may cause valuable information to be discarded prematurely, leading to an increase in the estimation variance, while the Cauchy Kernel balances the two, providing better robustness to occasional anomalies of large magnitude. The weighting curves of

the three Kernel functions are shown in Fig.1.

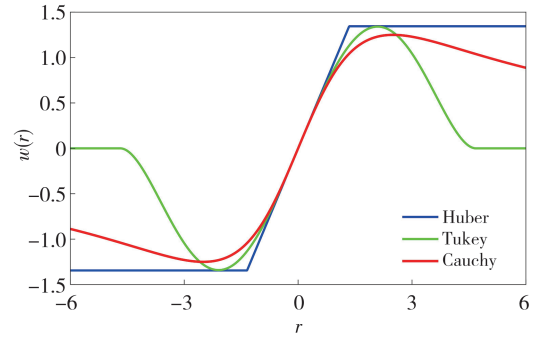


Fig. 1 Weight curves of three Kernel functions

The Cauchy Kernel function is given by

$$\rho(x) = \ln\left(1 + \frac{x^2}{\gamma^2}\right). \quad (14)$$

The corresponding weight function is defined as

$$w_k^{(i)} = \frac{1}{1 + \left(\frac{r_k^{(i)}}{b}\right)^2}. \quad (15)$$

The scale parameter b is used to control the suppression strength of large deviations. When $|r| \ll b$, the weight $w \approx 1$, and the update are approximately standard EnKF. When $|r| \gg b$, the weight $w \approx b^2/r^2 \ll 1$, and the abnormal residuals are quickly suppressed. To automatically switch between stable and abnormal environments, b is adjusted according to the median of the residuals.

$$\hat{b}_k = \text{median}(|r_k|). \quad (16)$$

However, considering that the intensity of non-Gaussian noise may differ across different observation dimensions, it is necessary to adjust the b value for each channel separately, rewriting Eq. (16) as

$$\hat{b}_k^j = \sum_{j=1}^{N_e} \text{median}(|r_k^j|), \quad (17)$$

where N_e is the measurement dimension, $\hat{b}_k^j$ is the estimated scale parameter of dimension j at time k , and to prevent excessive jumps in the b value at similar times, a forgetting factor γ is introduced to perform recursive updates.

$$b_k^j = (1 - \gamma)b_{k-1}^j + \gamma\hat{b}_k^j, \quad (18)$$

where $\gamma \in (0, 1)$, b_k^j is the scale parameter of the j dimension at the k moment.

1.3 Robust adaptive ensemble Kalman filter

Using the raw residuals $r_k^{(i)}$ directly to calculate the weights can lead to excessive suppression or poor suppression effects due to the different scales of the

observation dimensions. Therefore, the observation covariance P_{yy} is calculated, and the residuals of each component are normalized by the standard deviation.

$$P_{yy} = \frac{1}{N-1} \sum_{i=1}^N (\mathbf{y}_{k|k-1}^{(i)} - \bar{\mathbf{y}}_{k|k-1})(\mathbf{y}_{k|k-1}^{(i)} - \bar{\mathbf{y}}_{k|k-1})^T + \mathbf{R}_k, \quad (19)$$

$$\begin{cases} \sigma_j = \sqrt{[P_{yy}]_j}, \\ \tilde{r}_{k,j}^{(i)} = \frac{r_{k,j}^{(i)}}{\sigma_j}, \end{cases} \quad (20)$$

where $\mathbf{y}_{k|k-1}^{(i)} = h(\mathbf{x}_{k|k-1}^{(i)})$, so that all components have the same measure of unit variance. Next, Cauchy weights are computed on the normalized residuals $\tilde{r}_{k,j}^{(i)}$ component by component and loaded into the diagonal matrix.

$$\mathbf{w}_{k,j}^{(i)} = \frac{1}{1 + (\tilde{r}_{k,j}^{(i)}/b)^2}, \quad (21)$$

$$\mathbf{W}_k^{(i)} = \text{diag}(\mathbf{w}_{k,1}^{(i)}, \dots, \mathbf{w}_{k,j}^{(i)}). \quad (22)$$

After calculating the weight matrix using robust Kernel functions, the residual and measurement noise matrices can be reconstructed using the weight matrix. If an anomaly occurs in a certain dimension, the weight of that dimension will be suppressed and will not affect the normal updating of other dimensions. On this basis, weighted new information is constructed, and the observed covariance is reconstructed.

$$\hat{\mathbf{r}}_k^{(i)} = \mathbf{W}_k^{(i)} \mathbf{r}_k^{(i)}, \quad (23)$$

$$\tilde{\mathbf{R}}_k = \text{diag}\left(\frac{1}{N} \sum_{j=1}^N (\hat{\mathbf{r}}_k^{(i)})^2\right). \quad (24)$$

The flowchart of the CREnKF algorithm is presented in Fig.2.

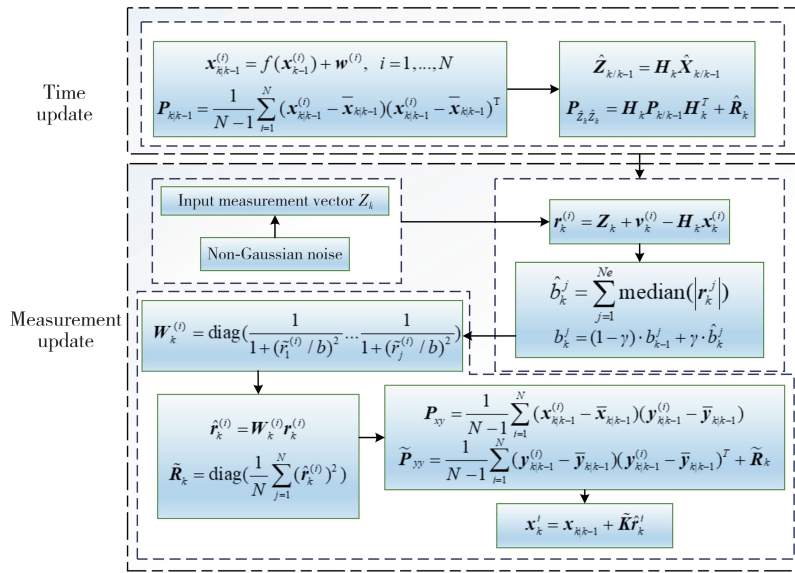


Fig. 2 Flowchart of CREnKF

At this point, the equation for the robust set Kalman filter of the Cauchy function is

$$\mathbf{x}_{k|k-1}^{(i)} = f(\mathbf{x}_{k-1}^{(i)}) + \mathbf{w}_{k-1}^{(i)}, \quad \mathbf{w} \sim N(0, \mathbf{Q}), \quad (25)$$

$$\bar{\mathbf{x}}_{k|k-1} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_{k|k-1}^{(i)}, \quad (26)$$

$$\mathbf{P}_{xy} = \frac{1}{N-1} \sum_{i=1}^N (\mathbf{x}_{k|k-1}^{(i)} - \bar{\mathbf{x}}_{k|k-1})(\mathbf{y}_{k|k-1}^{(i)} - \bar{\mathbf{y}}_{k|k-1}), \quad (27)$$

$$\tilde{\mathbf{P}}_{yy} = \frac{1}{N-1} \sum_{i=1}^N (\mathbf{y}_{k|k-1}^{(i)} - \bar{\mathbf{y}}_{k|k-1})(\mathbf{y}_{k|k-1}^{(i)} - \bar{\mathbf{y}}_{k|k-1})^T + \tilde{\mathbf{R}}_k, \quad (28)$$

$$\tilde{\mathbf{K}} = \mathbf{P}_{xy} \tilde{\mathbf{P}}_{yy}^{-1}, \quad (29)$$

$$\mathbf{x}_k^i = \mathbf{x}_{k|k-1} + \tilde{\mathbf{K}} \hat{\mathbf{r}}_k^i. \quad (30)$$

To analyze the convergence of the filter in this paper, the estimation error $\mathbf{e}_k$ is first considered. After local linearization of the system and incorporating the prediction-update structure of the CREnKF, the error dynamics can be written as

$$\mathbf{e}_{k+1} = (\mathbf{I} - \mathbf{K}_k \mathbf{W}_k \mathbf{H}_k) \mathbf{A}_k \mathbf{e}_k + \mathbf{F}_k \mathbf{w}_k + \mathbf{A}_k, \quad (31)$$

where $\Phi_k = (\mathbf{I} - \mathbf{K}_k \mathbf{W}_k \mathbf{H}_k) \mathbf{A}_k$ denotes the closed-loop error transfer matrix, $\mathbf{F}_k \mathbf{w}_k$ represents the noise term, and $\mathbf{A}_k$ signifies the linearization error. The analytical expression for the observation gain is

$$\mathbf{K}_k = \mathbf{P}_k \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k \mathbf{H}_k^T + \mathbf{W}_k \mathbf{R}_k \mathbf{W}_k)^{-1}. \quad (32)$$

The introduction of a forgetting factor in the recursive derivation of the scale parameter b ensures that the weight sequence $\mathbf{w}_{k,j}^{(i)}$ remains within consistent upper and lower bounds, thereby guaranteeing that $\tilde{\mathbf{R}}_k$ remains positive definite and non-degenerate. The following Lyapunov candidate function is selected.

$$\mathbf{V}_k = E[\mathbf{e}_k^T \mathbf{P}_k^{-1} \mathbf{e}_k]. \quad (33)$$

Substituting and expanding Eq. (31) yields

$$\mathbf{V}_{k+1} = E[\mathbf{e}_k^T \Phi_k^T \mathbf{P}_{k+1}^{-1} \Phi_k \mathbf{e}_k] + E[\mathbf{w}_k^T \mathbf{F}_k^T \mathbf{P}_{k+1}^{-1} \mathbf{F}_k \mathbf{w}_k] + O(\|\mathbf{e}_k\|^2). \quad (34)$$

Since $\tilde{\mathbf{R}}_k$ remains bounded under the upper and lower bounds on its weights, its inverse matrix provides a lower bound on the amount of information in the observations. That is, there is a constant $\lambda_{\min} > 0$ such that

$$\mathbf{H}_k^T (\tilde{\mathbf{R}}_k)^{-1} \mathbf{H}_k \geq \lambda_{\min} \mathbf{I}. \quad (35)$$

Combining Eq. (32) with the uniform boundedness of $\mathbf{P}_k$, we can derive a norm bound for the closed-loop operator.

$$\|\mathbf{I} - \mathbf{K}_k \mathbf{W}_k \mathbf{H}_k\| < 1. \quad (36)$$

Therefore, when $\|\mathbf{A}_k\|$ is controlled, $0 < \rho < 1$ exists, leading to

$$\Phi_k^T \mathbf{P}_{k+1}^{-1} \Phi_k \leq \rho \mathbf{P}_k^{-1}. \quad (37)$$

At this point, the filter covariance recursively becomes

$$\mathbf{P}_{k+1} = \mathbf{A}_k (\mathbf{I} - \mathbf{K}_k \mathbf{W}_k \mathbf{H}_k) \mathbf{P}_k (\mathbf{I} - \mathbf{K}_k \mathbf{W}_k \mathbf{H}_k)^T \mathbf{A}_k^T + \mathbf{A}_k \mathbf{K}_k \tilde{\mathbf{R}}_k \mathbf{K}_k^T \mathbf{A}_k^T + \mathbf{Q}_k + \boldsymbol{\psi}_k, \quad (38)$$

where $\boldsymbol{\psi}_k$ denotes the contribution from linearized higher-order terms. It follows that $\mathbf{P}_k$ is always bounded, rendering $\mathbf{v}_k$ equivalent to the error's L2 norm. Substituting Eq. (37) into Eq. (35) and uniformly capping both noise and higher-order residuals with a constant β yields the recursive inequality

$$\mathbf{v}_{k+1} \leq \rho \mathbf{v}_k + \beta, \quad (39)$$

$$\mathbf{v}_k \leq \rho \mathbf{v}_0 + \frac{\beta}{1 - \rho}. \quad (40)$$

Therefore, the error converges at a geometric rate to a bounded neighborhood proportional to the noise level. As the noise approaches zero, the error converges to zero at an exponential rate.

2 SINS/GNSS integrated navigation system model

To validate the effectiveness of the algorithm proposed in this paper, we selected a GNSS/SINS navigation system model, adopted a loose integration strategy, and used the "East-North-Up (E-N-U)" geographic coordinate system as the navigation reference coordinate system for the inertial navigation system. The main errors of the SINS include attitude error angles, velocity errors, position errors, and inertial device errors. A 15-dimensional system state vector is constructed as

$$\mathbf{X}(t) = [\boldsymbol{\varphi}^T \ (\delta \mathbf{v})^T \ (\delta \mathbf{p})^T \ (\boldsymbol{\varepsilon}^b)^T \ (\nabla^b)^T]^T, \quad (41)$$

where $\boldsymbol{\varphi}^T = (\varphi_E, \varphi_N, \varphi_U)^T$, $(\delta \mathbf{v})^T = (\delta v_E, \delta v_N, \delta v_U)^T$, $(\delta \mathbf{p})^T = (\delta p_L, \delta p_H)^T$, $\boldsymbol{\varepsilon}^b = (\varepsilon_x, \varepsilon_y, \varepsilon_z)^T$, and $\nabla^b = (\nabla_x, \nabla_y, \nabla_z)^T$ correspond to attitude error, velocity error, position error, gyroscope random constant error, and accelerometer constant error, respectively. Fig.3 illustrates

the structural diagram of the GNSS/SINS integrated navigation system.

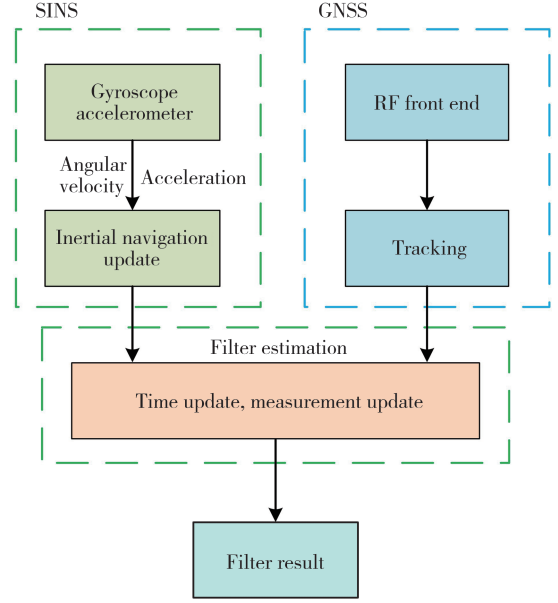


Fig. 3 Structure diagram of GNSS/SINS integrated navigation system

3 Experimental analysis

3.1 Simulation study

To verify the robustness and accuracy of the proposed filtering algorithm under strong nonlinearity and heavy-tailed noise conditions, this paper selects the classic UNGM (univariate nonlinear growth model) as the simulation test^[22]. The UNGM model is described as

$$x_k = 0.5x_{k-1} + \frac{25x_{k-1}}{1 + x_{k-1}^2} + 8\cos(1.2(k-1)) + w_k, \quad (42)$$

$$y_k = \frac{x_k}{20} + v_k, k = 1, 2, \dots, K, \quad (43)$$

where $w_k \sim N(0, 1)$, $v_k \sim N(0, 1)$. We generated data using simulated initial values of $x_0 = 0$ and initial covariance $\mathbf{P}_0 = \mathbf{I}$. The sample size of the ensemble filter is set to 50, with a simulation step size $K=500$ and 50 Monte Carlo experiments conducted for simulation validation. To demonstrate the performance of the proposed filter across different scenarios, we compare its behavior against the EnKF, Huber Kernel robust EnKF (HREnKF), and CREnKF. The Huber Kernel threshold is set to $c=1.345$ in this paper, balancing estimation efficiency under Gaussian noise with robustness under non-Gaussian noise. This value ensures approximately 95% asymptotic efficiency under a Gaussian distribution.

This study examines two scenarios. Scenario 1 primarily evaluates the performance of CREnKF under Gaussian

noise. Scenario 2 aims to validate the performance of CREnKF when measurements contain non-Gaussian noise. The heavy-tailed non-Gaussian noise in Scenario 2 follows the distribution

$$V_k = \begin{cases} N(0, R), & 90\%, \\ \chi^2(0, 100R), & 10\%, \end{cases} \quad (44)$$

where χ^2 is the chi-square distribution, R is the initial noise covariance matrix, and its initial value is set to 1.

Here, we compare the estimation accuracy of EnKF, HREnKF, and CREnKF based on two metrics: the root-mean-square error (RMSE) per Monte Carlo run and the mean normalized error squared (MNES). RMSE characterizes the stability and consistency of the filtering process. MNES value closer to the dimension of the estimated state indicates better stability and consistency in the filtering algorithm, as it assesses whether the estimation error is reasonable in terms of covariance. The formulas for calculating RMSE and MNES are

$$\text{RMSE}(m) = \frac{1}{K} \sum_{k=1}^K (x_k - \hat{x}_k)^2, m = 1, \dots, M, \quad (45)$$

$$\text{MNES}(k) = \frac{1}{K} \sqrt{(x_k - \hat{x}_k)^T P^{-1} (x_k - \hat{x}_k)}. \quad (46)$$

To ensure that the selection of the scale parameter b and the forgetting factor γ in the algorithm is grounded in both theory and experimentation, this paper employs a combined approach of robust online estimation and systematic sensitivity analysis for their selection. The scale parameter b is estimated at time step t using the mean absolute deviation (MAD) of normalized residuals to obtain an instantaneous scale, which is then smoothed via exponential smoothing to derive the final scale applied to Cauchy weights. To determine the specific initial values for scale parameter b and forgetting factor γ , this paper performs a grid search on the simulation dataset for γ and several amplification factors based on the initial MAD. The RMSE and MNES are calculated for each parameter set, and performance heatmaps are plotted.

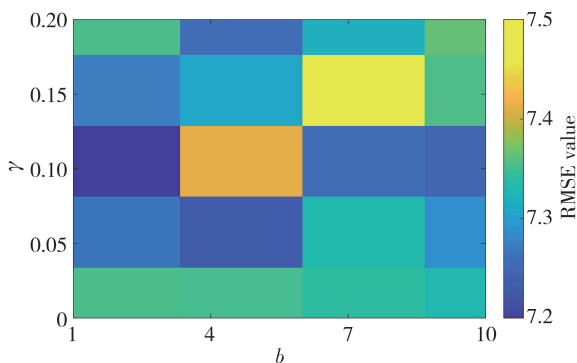


Fig. 4 Sensitivity heatmap of γ and b to RMSE

Experimental results indicate that under the noise intensity of the experimental setup, setting the initial value of γ to 0.05 and the initial value of b to 4 achieves the optimal compromise between RMSE and MNES while demonstrating good robustness.

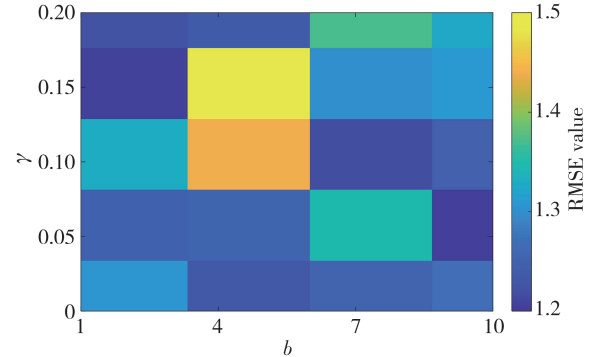


Fig. 5 Sensitivity heatmap of γ and b to MNES

The estimation accuracies of EnKF, HREnKF, and CREnKF are compared by calculating the average of 50 Monte Carlo simulations to obtain the RMSE and MNES of the three algorithms. Figs. 6 and 7 show the RMSE and MNES results for case 1. Figs. 8 and 9 show the RMSE and MNES results for the three algorithms in case 2. For a visual comparison of the three algorithms, Figs. 10 and 11 show the RMSE boxplots of the three algorithms for case 1 and case 2. In order to quantitatively analyze the filtering performance of EnKF and CREnKF, the RMSE and MNES values of the three filtering algorithms for both cases are given in Table 1.

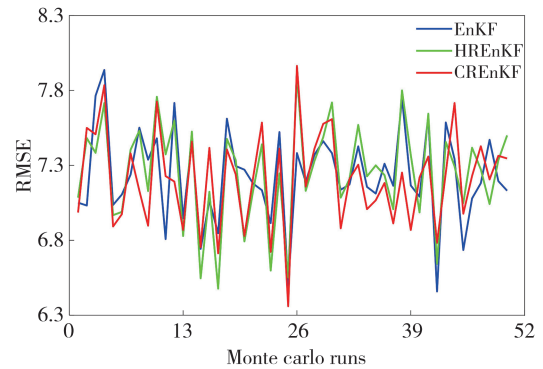


Fig. 6 RMSE results in case 1

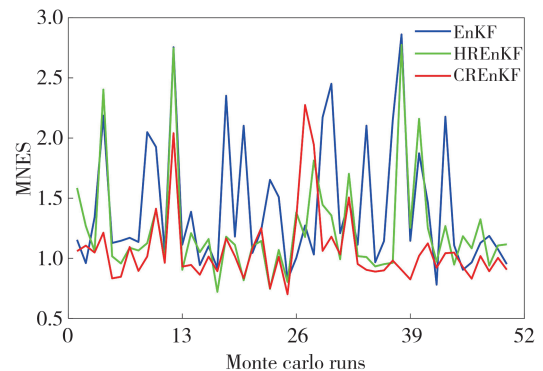
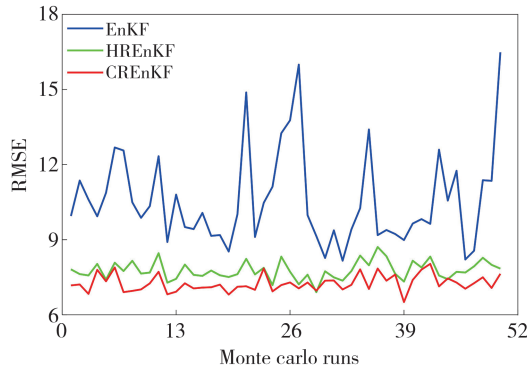
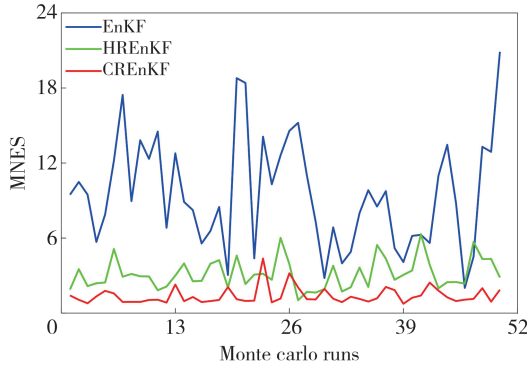


Fig. 7 MNES results in case 1

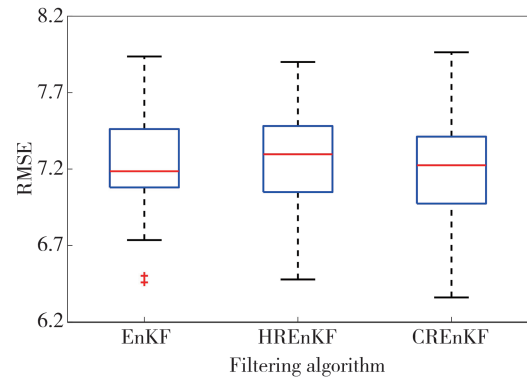
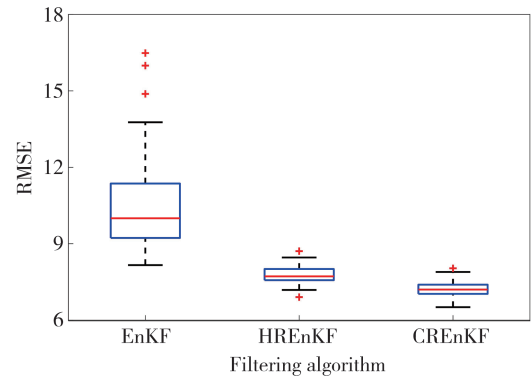
**Fig. 8 RMSE results in case 2****Fig. 9 MNES results in case 2****Table 1 Mean and variance-based error metrics**

Algorithm	Case 1		Case 2	
	RMSE	MNES	RMSE	MNES
EnKF	7.232 7	1.421 6	10.599 5	9.562 9
HREnKF	7.250 0	1.237 1	7.776 1	3.132 6
CREnKF	7.211 3	1.068 1	7.258 9	1.384 5

As can be seen in Fig. 6, the RMSE of the three algorithms are basically the same in the case of Gaussian noise, indicating that the estimation accuracy of the three algorithms is basically comparable. Since the state dimension is 1, the closer the MNES of the filter is to 1, the more stable it is. In Fig. 7, the MNES of CREnKF is closest to 1, so it has the best stability. The stability of HREnKF is moderate, and the stability of EnKF is the worst. This indicates that CREnKF outperforms HREnKF and EnKF in terms of consistency of covariance prediction and coverage of confidence intervals in case 1. In Fig. 10, for case 1, the RMSE quartile is 7.18 for EnKF, 7.29 for HREnKF, and 7.22 for CREnKF. The RMSE and MNES values indicate that CREnKF improves the performance of EnKF in the case of Gaussian noise with better filtering consistency.

As shown in Figs. 8 and 9, when the measurement data containing non-Gaussian noise is introduced in case 2, the robustness of CREnKF is more obvious. Because the two-path robust strategy of weighted new interest and adaptive adjustment of covariance matrix is introduced, CREnKF is more capable of suppressing the

influence of non-Gaussian noise in the measurement data. And the adaptive ability of EnKF is not strong, which leads to poor filtering effect. In contrast, although HREnKF can to some extent suppress the influence of non-Gaussian noise, its filtering accuracy and confidence level are inferior to those of CREnKF. In Fig. 11, for case 2, the RMSE interquartile range of CREnKF is smaller, and the true value data is more concentrated. The RMSE and the MNES illustrate the performance of CREnKF in the case of Gaussian noise, and it has a stronger robustness for the case of non-Gaussian noise in the measurement data.

**Fig. 10 RMSE boxplot in case 1****Fig. 11 RMSE boxplot in case 2**

In Table 1, RMSE and MNES are the average values for 50 Monte Carlo simulations. In case 1, the mean values of RMSE of CREnKF compared to HREnKF and EnKF are basically comparable, while the mean value of MNES of CREnKF is the closest to 1, which indicates that it has the best filtering consistency. In case 2, the mean RMSE values of CREnKF are reduced by 31% and 6% and the mean MNES values are reduced by 85% and 55% compared to HREnKF and EnKF, respectively. The experimental results show that CREnKF has better filtering performance and robustness compared to HREnKF and EnKF, and can effectively suppress the interference of non-Gaussian noise on the filtering performance of the system.

Statistical tests conducted on the RMSE and MNES metrics obtained for Scenario 2 reveal p -values < 0.001 in

both one-way analysis of variance analyses (ANOVA), indicating significant performance differences among the three algorithms. Further Tukey post-hoc tests reveal that the p -values for both RMSE and MNES between CREnKF and EnKF are less than 0.001, indicating extremely significant differences. The p -values between CREnKF and HREnKF are both less than 0.05, demonstrating statistical significance. Therefore, it can be confirmed that CREnKF is statistically superior to the other methods. Table 2 shows the significance test results for RMSE and MNES of the three filters under Scenario 2.

Table 2 Significance test for RMSE and MNES of three filters under case 2

Parameter	ANOVA p value	Significant finding
RMSE	$p<0.001$	EnKF-HREnKF: $p<0.001$
		EnKF-CREnKF: $p<0.001$
		HREnKF-CREnKF: $p=0.003\ 4$
MNES	$p<0.001$	EnKFvHREnKF: $p<0.001$
		EnKF-CREnKF: $p<0.001$
		HREnKF-CREnKF: $p=0.000\ 25$

3.2 Vehicle experiment

To further validate the effectiveness of the method described in this paper, we conducted an in-vehicle experiment. The experiment was carried out on a road in Lanzhou City, Gansu Province. The experimental equipment mainly included a GNSS receiver, an inertial navigation system, a portable power supply, and other related devices, with the inertial navigation system fixed inside the vehicle compartment. The total length of the test route was 3 893 m, and the total test time was 770 s. The GNSS receiver model is K823, and the IMU in the SINS is STIM300. Prior to fusion, all sensor outputs undergo preprocessing to ensure unit consistency and data alignment. Gyroscope and accelerometer signals are converted into angular and velocity increments, while GPS data is transformed into position and velocity measurements in the International System of Units for subsequent analysis. Table 3 lists the specifications of the gyroscope and accelerometer sensors used in the experiment.

Table 3 Sensor parameters

Parameter	Value
Zero bias stability/ $((^{\circ})\cdot\text{h}^{-1})$	5
Random angle wandering/ $((^{\circ})\cdot\text{h}^{-\frac{1}{2}})$	0.15
Zero bias instability/mg	0.02
Speed randomised roaming/ $(\mu\text{g}\cdot\text{h}^{-\frac{1}{2}})$	700
Update frequency/Hz	125

Fig.12 shows the vehicle trajectory during the experiment. Fig.13 displays the combined trajectories of the EKF, EnKF, and proposed method alongside the actual driving trajectory under on-board testing.



Fig. 12 Vehicle track diagram

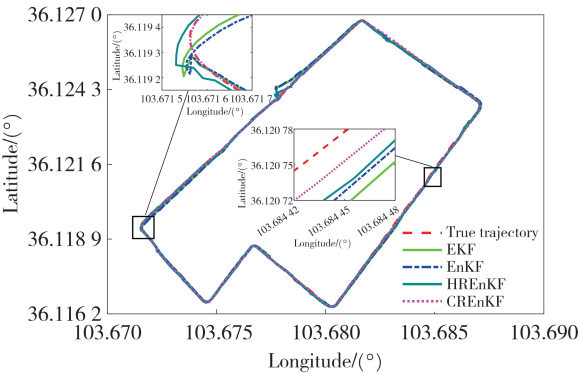


Fig. 13 Comparison of vehicle test trajectories

In Fig.13, the trajectories generated by all four methods closely match the original trajectory during vehicle testing. Figs. 14–17 compare the latitude/longitude errors, eastward velocity errors, and northward velocity errors of the four methods in experiments involving moving vehicles.

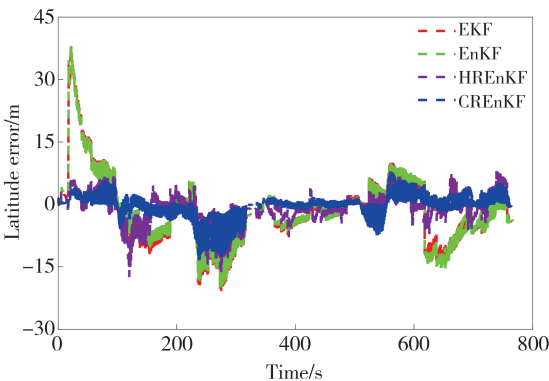


Fig. 14 Latitude error of vehicle-based experiments

As shown in Figs.14 and 15, the proposed method maintains stable errors in the longitudinal, latitudinal, and altitudinal directions, whereas the traditional EKF and EnKF algorithms exhibit significant fluctuations.

The HREnKF algorithm can suppress non-Gaussian noise interference to a certain extent, but its suppression performance is inferior to that of the CREnKF when encountering large outliers. Figs.16 and 17 display the eastward and northward velocity errors from the vehicle-based experiment. The proposed method demonstrates markedly superior suppression of velocity errors compared to the EKF, EnKF, and HREnKF algorithms. The experimental results validate the robustness of the CREnKF algorithm against observation noise caused by external disturbances. To further quantify the consistency of positioning accuracy, Figs.18 and 19 compare the probability density distributions of the longitude and latitude errors for the three methods in the vehicle-based experiment.

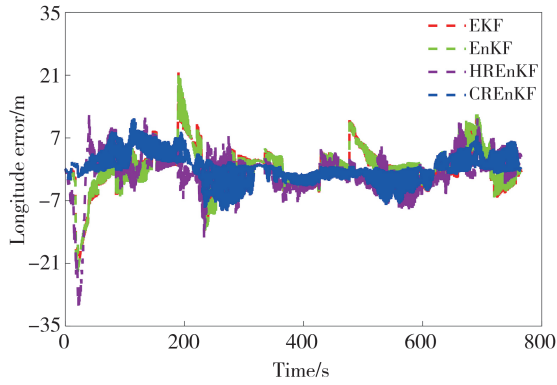


Fig. 15 Longitude error of vehicle-based experiments

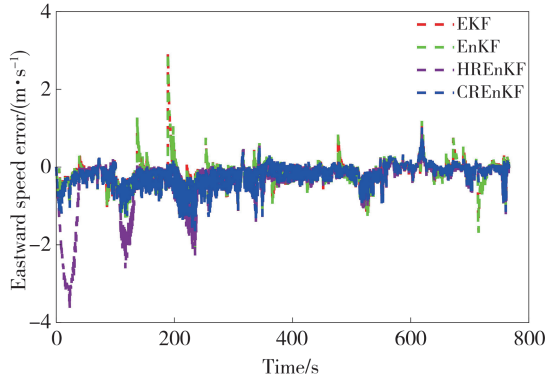


Fig. 16 Eastward velocity error of vehicle-based experiments

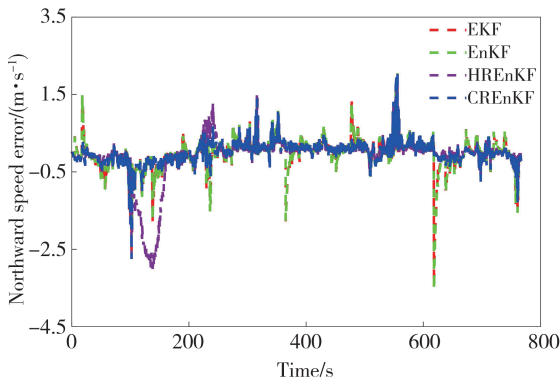


Fig. 17 Northward velocity error of vehicle-based experiments

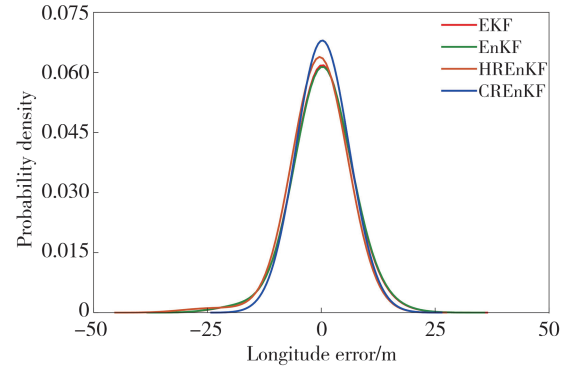


Fig. 18 Probability density of longitude error

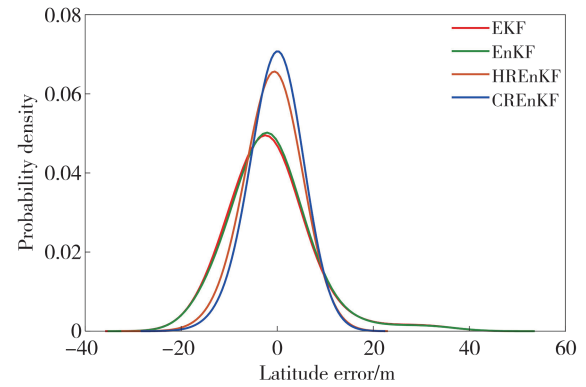


Fig. 19 Probability density of latitude error

As shown in Figs.18 and 19, the probability density distributions of latitude and longitude errors for the traditional EKF and EnKF algorithms exhibit significant dispersion, with both displaying pronounced extreme errors. Notably, the peak values of the error distributions for all four algorithms cluster near zero, confirming that the filters generally provide estimates close to the true values. Among these, the peak of HREnKF is higher than that of the traditional EnKF, indicating its robustness against non-Gaussian noise. However, its peak remains lower than that of CREnKF. CREnKF exhibits the error distribution peak closest to zero for both latitude and longitude, with a steeper distribution curve, smaller error fluctuation range, and more prominent stability. Experiments demonstrated that CREnKF effectively suppressed extreme error propagation through enhanced state sensitivity coefficients and dynamic uncertainty adaptation mechanisms. Table 4 shows RMSE and MAD values for the four methods considered in the moving vehicle experiment.

Table 4 RMSE and MAD of runway experiments

Algorithm	RMSE		MAD	
	Longitude/m	Latitude/m	Longitude/m	Latitude/m
EKF	7.473	3.788	3.441	1.980
EnKF	7.431	3.779	3.617	1.874
HREnKF	2.642	3.247	1.684	1.309
CREnKF	1.053	1.069	0.502	0.534

As shown in Table 4, the RMSE of the longitude and latitude of the proposed method are smaller than those of EKF, EnKF, and HREnKF. Compared with EKF, EnKF, and HREnKF, the RMSE of the longitude of CREnKF is decreased by 6.42 m, 6.378 m, and 1.589 m, respectively, while the MAD is decreased by 2.939 m, 3.115 m, and 1.182 m, respectively. For latitude errors, the RMSE is decreased by 2.719 m, 2.71 m, and 2.177 m, respectively, while the MAD is decreased by 1.446 m, 1.34 m, and 0.775 m, respectively. For elevation errors, the RMSE is decreased by 5.406 m, 5.78 m, and 2.31 m, respectively, while the MAD is decreased by 3.122 m, 2.835 m, and 2.03 m, respectively. It demonstrated that the proposed method achieved high filtering accuracy, maintaining estimates of the true state even when measurement noise included heavy-tailed non-Gaussian noise.

4 Conclusions

To address estimation bias caused by heavy-tailed non-Gaussian noise in dynamic noise environments under traditional ensemble Kalman filtering, this paper proposes an adaptive ensemble Kalman filtering algorithm incorporating Cauchy robust strategies. By introducing a forgetting factor for real-time optimization of robust adjustment parameters and combining covariance dynamic calibration techniques to autonomously adjust measurement noise intensity, the algorithm significantly enhances the system's adaptability to anomalous observations. Vehicle positioning experiments demonstrated that compared to Huber robust ensemble Kalman filtering, this method reduced latitude positioning error by 1.589 m and longitude error by 2.177 m. Experiments demonstrated that this method effectively enhanced the accuracy and reliability of navigation systems in complex scenarios, providing a practical solution for engineering applications. Current limitations include the global noise estimation mechanism's insufficient consideration of heterogeneous signal source characteristics. Future research will introduce multi-channel/multi-scale weighting strategies, implementing dynamic priority allocation by distinguishing frequency-domain features across sensor channels to enhance the environmental adaptability of integrated navigation systems during long-term operation.

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Declaration of conflicting interests

The authors have no conflict of interests related to this publication.

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基于 GNSS/SINS 组合导航的改进集合卡尔曼滤波算法

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摘要: 随着智能交通与自动驾驶技术的迅速发展, 车载全球导航卫星系统/捷联惯性导航系统(global navigation satellite system/strapdown inertial navigation system, GNSS/SINS)组合导航对高精度与强鲁棒性定位性能的需求不断提升。集合卡尔曼滤波(ensemble Kalman filter, EnKF)因其基于样本统计的蒙特卡罗估计方法, 能够有效处理系统非线性问题, 逐渐成为组合导航中的重要数据融合方法。然而, 传统 EnKF 假设噪声服从高斯分布, 当存在非高斯或脉冲噪声时, 其滤波性能显著下降。针对该问题, 本文提出一种基于 Cauchy 函数的鲁棒自适应集合卡尔曼滤波算法(Cauchy robust ensemble Kalman filter, CREnKF), 利用 Cauchy 权重函数动态识别并抑制异常观测值, 引入残差加权与观测协方差重构的双路径鲁棒策略, 设计基于残差中值绝对偏差的尺度参数自适应调整机制, 实现对不同噪声强度与分布特性的在线自适应, 有效降低非高斯噪声对滤波结果的影响。实验结果表明, CREnKF 在非高斯噪声环境下的位置均方根误差分别比 EKF、EnKF 和基于 Huber 核的鲁棒 EnKF 降低 82%、81% 和 63%, 在保持实时性的同时显著提升了系统的稳定性和精度。

关键词: 集合卡尔曼滤波; 非高斯噪声; 组合导航; 鲁棒滤波器; 蒙特卡罗方法; Cauchy 函数

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