

专家介绍



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基于反馈控制的弹性波调控机理与 功能研究进展

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摘 要: 弹性波超构材料因其功能基元的精心设计, 可以实现自然界材料无法具备的波动效应, 在国防装备、精密仪器等领域展现出巨大的应用潜力。近年来, 各种被动式超构材料相继被提出, 实现了诸如低频波动抑制、超分辨率聚焦、负折射等新奇功能, 但其存在可调性差、结构复杂等不足。基于多物理场耦合的主动式超构材料正逐渐被研究者广泛关注, 其中反馈控制型主动式超构材料表现出高效的可调性甚至是自适应调控功能。因此, 对该种超构材料的研究非常重要, 可以拓宽弹性波的调控机理与功能。本研究基于调控机理, 从三方面介绍引入反馈控制对弹性波调控的作用。首先, 反馈机制的引入可以有效地改变材料的等效参数以及增加能带结构的可调性。其次, 反馈控制构造了等效增益与损耗, 使超构材料具有新颖模态并展示出反常波动效应。此外, 反馈控制使得利用干涉作用改变弹性波的幅值、相位和频率成为可能, 进一步实现丰富的波动功能。最后, 总结了基于反馈控制的弹性波超构材料的不足和应用难点, 并展望未来研究方向。本研究为设计反馈式主动超构材料提供了方法策略, 为丰富波动功能提供了设计思路。

关键词: 反馈控制; 弹性波; 非互易; 干涉; 增益损耗; 趋肤效应; 振动抑制

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Review on mechanism and functions of elastic wave control based on feedback control

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Abstract: Because of their careful design of their functional elements, elastic wave metamaterials can realize the wave modulation functions that natural materials cannot have, and show great application potential in the fields of national defense equipment and precision instruments. In recent years, a variety of passive metamaterials have been proposed to achieve novel functions such as low frequency wave suppression, super resolution focusing, and negative refraction, but they have some shortcomings such as poor tunability and complex structure. Researchers gradually pay more and more attention to active metamaterials based on multi-physics coupling, among which the feedback controlled active metamaterials show high efficiency tunability and even adaptive modulation. Therefore, the study of this kind of metamaterials is very important, which can broaden the mechanism and functions of elastic wave modulation. Based on the modulation mechanism, this review introduces the effect of feedback control on the elastic wave control from three aspects. First of all, the introduction of feedback mechanism can effectively change the effective parameters of the material and increase the tunability of the band structure. Secondly, feedback control constructs equivalent gain and loss, which makes the metamaterial have novel modes and exhibit anomalous wave effects. In addition, feedback control makes it possible to use interference to change the amplitude, phase and frequency of elastic waves, and further realize rich wave functions. Finally, the shortcomings and application difficulties of the elastic wave superstructure materials based on feedback control are summarized, and the future research direction is prospected. This review provides a strategy for the design of feedback active metamaterials, and offers a design guidance to enrich wave functions.

Key words: feedback control; elastic wave; nonreciprocity; interference; gain/loss; skin effect; vibration attenuation

弹性波是弹性介质受到扰动或外力作用时,在质点相互作用力下引起的质点振动并传播的波动现象,在各类机械系统中广泛存在。例如,精密仪器或者大型工业产品对振动噪音的要求较高,因此需要对弹性波进行阻隔或衰减^[1-2]。在一些无源的环境能量采集应用场景中,比如自供能设备,需要采集弹性波的机械能并转化为电能^[3-4]。另外,利用弹性波传播过程中遇到不连续边界发生透反射的性质可以实现缺陷检测^[5-6]。这些基于工程实践的需求使得对弹性波调控研究的重要性日益凸显。

弹性波超构材料的出现赋予了弹性波调控更丰富的自由度和更好的功能效果^[7],实现了如负折射^[8]、全吸收^[9]、隐身^[10-11]、聚焦^[12]、波导^[13]等新奇

功能。根据参数是否可调,可以将弹性波超构材料分为被动式和主动式超构材料。新奇的功能往往意味着复杂的微结构设计,使被动式超构材料存在可调性差、高精度制造困难等限制,而且其等效参数和功能往往也受限于结构本身的几何参数和材料性质。外场的引入可以增加调控自由度,拓宽应用场景,在亚波长尺度实现更优异的性能。因此,近年来对于主动超构材料的研究越来越受到人们关注,这些研究利用多场耦合材料例如压电材料^[14-15]、光敏材料^[16]、磁致伸缩材料^[17]等来构建可调甚至可重构的超构材料。主动超构材料的设计中比较重要的一环是如何利用多场耦合进行调控,而将输出信息通过控制律处理再进行输入的反馈控制策略更是增

加了主动超构材料对于波动调控的可能性^[18-19]。一般来说,弹性波反馈控制策略的基本定义为在一个基元某处探测到弹性波的信息,经过控制律处理,再输入反馈给本基元或者其他基元,以实现弹性波功能调控目的。

目前,基于反馈控制策略的弹性波研究,按照调控机理主要有3种:改变材料等效动态参数;在传播介质中构造增益和损耗;以及利用波动的干涉作用来调控弹性波,如图1所示。本研究将按照三大机理来总结相关领域的工作:第一节中阐述了利用反馈控制实现对超构材料等效动态参数和能带的调制,从弹簧-振子模型拓展到梁和板结构,详细总结了相关工作;第二节介绍了反馈控制在复本征值系统中构造等效增益和损耗的机理,从实现的功能出发,总结了模态耦合点、非互易和趋肤效应等方面的应用;第三节讨论了反馈控制在实现干涉作用中的机制,可以对弹性波的幅值、相位和频率进行调控,从而实现吸收、聚焦、转向、频率转换等功能;最后,第四节进行了总结和展望。

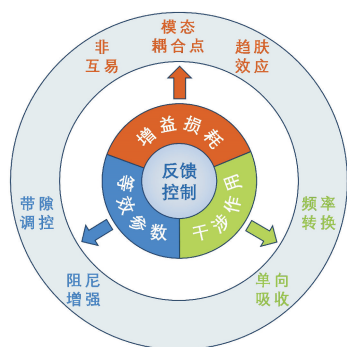


图1 基于反馈控制的弹性波调控机理及功能

Fig. 1 Elastic wave modulation based on the feedback control

1 能带和等效动态参数研究

能带理论来源于固体物理,描述的是晶体中电子的运动及其状态,用于解释不同材料的基本物理性质如电导率、热导率等。而后,KUSHWAHA等^[20]、SIGALAS等^[21]类比光子晶体^[22]提出了声子晶体,把能带这一概念引入到了周期性结构中弹性波传播的特性表征。能带结构中不存在能带曲线的频率范围称为禁带,表示弹性波在该频率范围内无法有效在结构中传播。常见的禁带一般分为两种:Bragg禁带^[20]与局域共振禁带^[23]。Bragg禁带的形

成与结构的周期性密切相关,因此禁带对应的波长与声子晶体的晶格常数处于同一量级。LIU等^[23]提出在环氧树脂中周期性地埋入硅橡胶包裹金属球的局域共振机理则打破了这一限制,使弹性波超构材料的小型化成为可能。后续研究发现这样的结构使谐振子产生偶极共振从而表现出等效负质量密度^[24]。HUANG等^[25]设计实现了负等效模量与负等效质量密度的谐振子,并研究了其禁带特性,将等效动态参数与禁带的产生联系了起来。此后,大量的研究构造了负等效密度、负等效体积模量、负等效杨氏模量等具有极端动态参数的超构材料并研究了其弹性波调控作用。因此,等效参数的计算对于局域共振型声子晶体的研究十分重要。

1.1 弹簧-振子模型中反馈控制的应用

早在1946年,BORN^[26]就详细地计算了周期性弹簧-振子模型的振动特性。在声子晶体这一概念出现后,温激鸿等^[27-29]将集中质量法用于声子晶体的禁带计算,把连续介质例如梁、板简化为了离散的弹簧-振子模型,减小了计算的难度。弹簧-振子模型可以简单清晰地揭示声子晶体禁带产生的原理,对声子晶体的设计提供了较强的指导意义。近年来,有许多研究^[18, 30-38]将反馈控制策略引入弹簧-振子模型并计算了其等效的动态密度或者刚度及其能带结构,证明了该策略在超结构中运用的可能性与有效性。如图2(a)所示,BERA等^[31]在单胞内引入了反馈控制,将 m_1 与 m_2 处测得的位移与速度乘以反馈系数矩阵 C_0 ,得到的结果作为两者之间的相互作用力 A ,并计算了不同结构参数下相应的等效动态质量(图2b)与透射率。进一步研究了该局域反馈控制(反馈控制施加在原胞内)弹簧-振子模型的阻尼增强效应^[32],并利用机器学习量化了随机参数来证明控制过程中的鲁棒性^[33],如图2(c)所示。而GAO等^[34]则利用时滞控制探讨了其在实际中应用的可能性,并结合遗传算法对时滞控制进行优化,实现了宽频带的波动抑制。随后,GAO等^[35]还提出了非局域型反馈控制(反馈控制施加在原胞之间)弹簧-振子模型(图2d),将相邻质量块的位移乘以反馈系数矩阵 G ,并将结果以外力的形式作用在质量块中间的谐振子上,随后计算了不同反馈控制系数下的能带结构(图2e)和等效归一化刚度(图2f)等。此外,BAXY等^[30]在二维弹簧-振子结构中做了类似的研究,实现了宽频带的弹性波衰减。这些研

究及其相关的实验^[36-38]足以证明反馈控制机制的引入可以增加禁带的宽度,并在一定范围内具有可

调性,为反馈控制在连续介质超构材料中的应用奠定了基础。

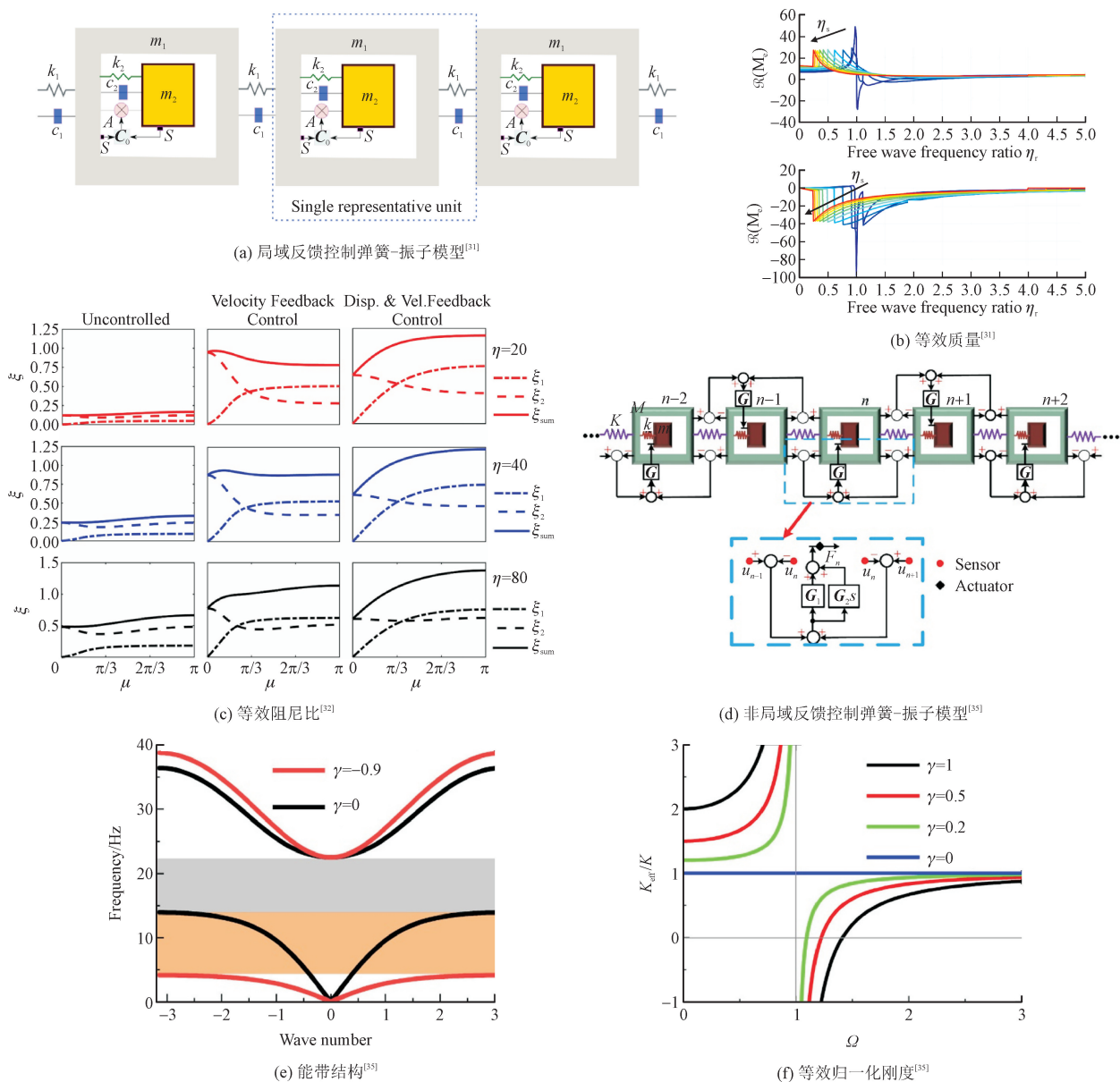


图2 带有反馈控制的周期性弹簧-振子模型及其等效参数和能带结构

Fig. 2 Periodic spring-mass model with feedback control and its effective parameters and band structure

1.2 梁、板声子晶体中反馈控制的应用

在声子晶体梁/板中,反馈控制一般通过在结构上粘贴压电片作为传感与激励的形式实现,探索被动式声子晶体无法实现的功能。这些研究相对于弹簧-振子的理论计算更加复杂但是接近实际应用。为了增加可调性与拓宽禁带宽度,有研究^[39-45]通过在结构上粘贴周期性的压电片,并外接分流电路的形式来改变声子晶体的等效参数,这可以看作是反馈控制的雏形,其传感与激励由同一块压电片实现。

随后,有研究利用两块压电片分别实现传感与激励并在两者之间加入微处理器进行反馈控制来设计声子晶体。LI等^[46]提出了如图3(a)所示的二维声子晶体板,其传感与激励压电片成对粘贴在平板的上下表面(蓝色为传感,红色为激励)。研究详细地讨论了位移反馈、速度反馈、加速度反馈及其不同组合控制下的能带改变(图3b),并利用机器学习优化了反馈控制系数以得到较宽的禁带。对于梁结构,也有类似的研究发表^[47-55],尽管粘贴方式可能不一致,但是其研究目标基本聚焦于反馈控制如何改变

等效材料参数或者能带结构。除了在结构表面粘贴压电片,也有一些研究将压电材料嵌入结构或作为结构本身的形式设计声子晶体,图3(c)为LI等^[56]设计制作的一维声子晶体及其等效模型。利用微控制器实现了可编程的反馈控制系数,建立了反馈系数与等效模型谐振子刚度之间的关系,并拟合了等效刚度与禁带频率之间的曲线(图3d)从而实现了拟合范围内宽频带的弯曲波透射抑制(图3e)。上

述研究详尽地讨论了不同控制系数下能带结构的改变并且通过拟合拓宽了禁带,但是较少阐述其如何改变等效动态参数从而影响能带。此外,HE等^[57-63]在平板上加入具有反馈的面外谐振子,做了一系列弹性波及其声波辐射研究。SIROTA等^[64-65]研究了声子晶体在反馈控制下的拓扑态。ZHOU等^[66-67]将反馈控制应用在声学超构材料中,实现了非互易的传播,构造了可调的等效密度和模量^[68-70]。

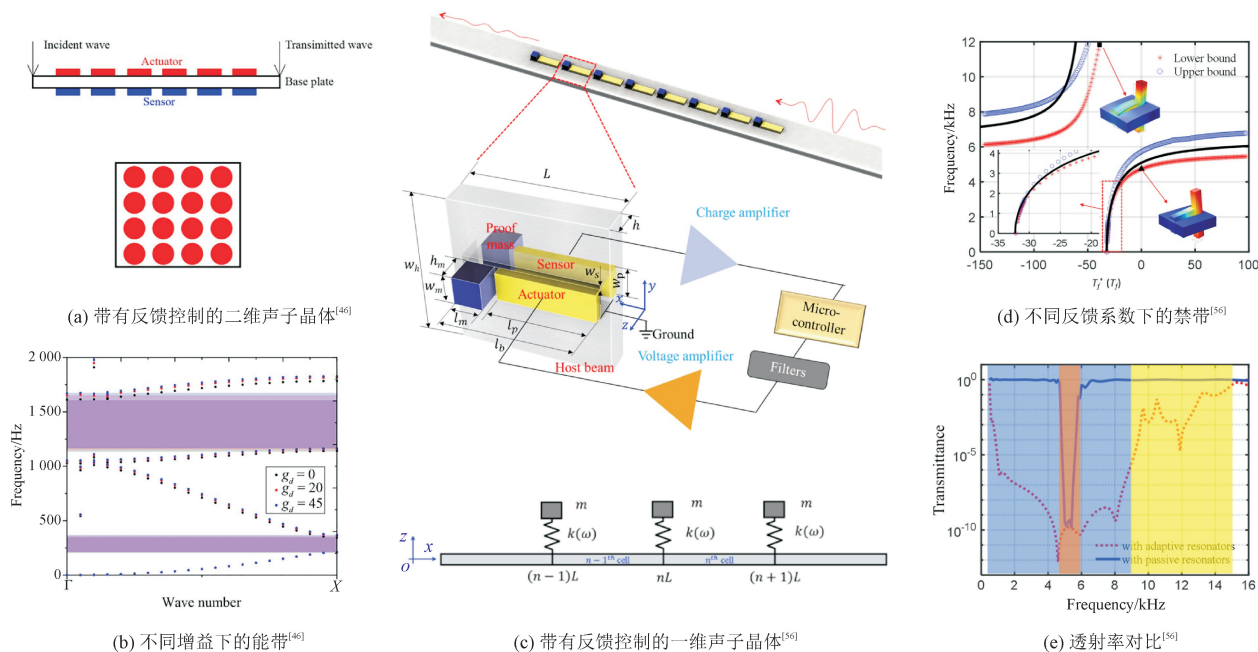


图3 引入反馈控制的声子晶体及其能带结构

Fig. 3 Phononic crystals with feedback control and its band structure

2 增益和损耗

超构材料长期以来是基于能量守恒的封闭系统建立的,其可观测量是实数,对应实的本征能量。而现实物理系统往往是与外界环境发生交互的开放系统,此类系统具有复本征值。BENDER等^[71]发现在宇称-时间反演对称的复本征值系统中依然可以存在纯实的能谱,此类系统可以通过增益损耗基元的调制来等效地构造^[72-73]。得益于光学和声学系统高度可控的材料设计和加工平台,相关研究首先在这些领域开展,比如调制介质折射率虚部的正负可以实现等效损耗和增益,利用损耗材料如材料的黏滞、多孔介质来引入损耗,利用有源器件向系统注入能量来引入增益等^[74-77]。

由实数参数平面到整个复数参数平面的转换增

加了波调控的自由度,因此复本征值系统展示出的一些新颖特性,使得其近年来受到广泛关注,例如相干完美吸收^[78-79]、模态耦合点^[80-81]、非对称散射^[82]等。在弹性波领域,构建复本征值系统所需的增益和损耗在理论上体现在描述系统的哈密顿量对角元素的虚部,其中,正虚部为损耗,负虚部为增益。一般通过材料模量的虚部引入增益和损耗等。由于自然增益材料的缺乏,发展了纯损耗调制下的复本征值系统,如DOMÍNGUEZ-ROCHA等^[83]在铝柱施加阻尼涂层引入损耗,试验测得了损耗增加过程中的频率演化并得到了模态耦合点。LI等^[84]利用阻尼调制的非对称谐振器实现了模态耦合点处的极端非对称弯曲波吸收。在主动弹性波超构材料领域,增益损耗可以由精准控制下的多物理场交互作用来实现,借助于不断发展的压电材料,利用机电耦合效应的压电系统是广泛被使用的手段。

2.1 模态耦合点

模态耦合点是存在于复本征值系统中的一种奇异点,标志着有限结构中本征频率和本征模态的同时简并^[84],波导结构散射矩阵的本征值和本征向量的同时简并^[85],也是宇称-时间反演对称系统中纯实谱和复共轭谱的相变点^[86]。ROSA 等^[87]从理论上探讨了反馈控制的外力能够实现等效的增益和损耗并诱导模态耦合点。从本质来讲,增益损耗体现在系统哈密顿量对角项的共轭虚部。针对弹性波体系,可以在运动方程中的刚度项引入虚部来实现。具体到主动控制方案,通过施加以速度信息作为反馈的外力可以实现等效的阻尼效应(注意到阻尼的量纲,力与速度乘积的量纲相同)。考虑实际的外力加载,压电机主动控制提供了便捷且精准的方法。基于此,ROSA 等^[87]设计了压电等效外力的方案在二维弹性介质中构造了模态耦合点,如图 4(a)所示。外力由表面的压电片产生,电压以压电片对应位置的弹性介质探测到的速度信息进行馈入,构成了反馈回路,其中反馈系数为 γ 。随着反馈系数 γ 增加,如图 4(b)所示频谱图中的本征频率实部逐渐

靠近并最终简并于一点,同时本征模态也在这一点耦合为相同的杂化模态。

模态耦合点具有许多新颖的性质,如单向隐身^[88]、单模激光器^[89]、增强传感灵敏度^[90]等。当系统中出现扰动时该二阶模态耦合点会发生退化,导致原本简并的频率重新分裂,且频率分裂与扰动成平方根关系。当扰动很微小时,这种响应关系优于一般共振结构的线性响应。基于这一特性,CAI 等^[91]设计了如图 4(c)所示的柱体结构梁,通过反馈控制外力对称加载的方式实现了模态耦合点。外力加载于梁上表面,梁下表面对应位置进行速度信息采集并乘以反馈系数来输入外力。以附加质量作为扰动计算了频率响应谱,如图 4(d)所示,频率分裂展现了相对于线性响应的增强灵敏度。JIN 等^[92]在图 4(e)所示的梁中通过反馈外力构造了模态耦合点,并引入三维微裂纹来计算响应谱,如图 4(f)所示,随着裂纹尺寸的改变,频率分裂成平方根趋势扩展。此外,在波导结构中,通过外接电路控制的压电片也能够实现模态耦合点并检测材料模量的变化,同时改变电路参数以实现单向无反射等新颖波动功能^[93]。

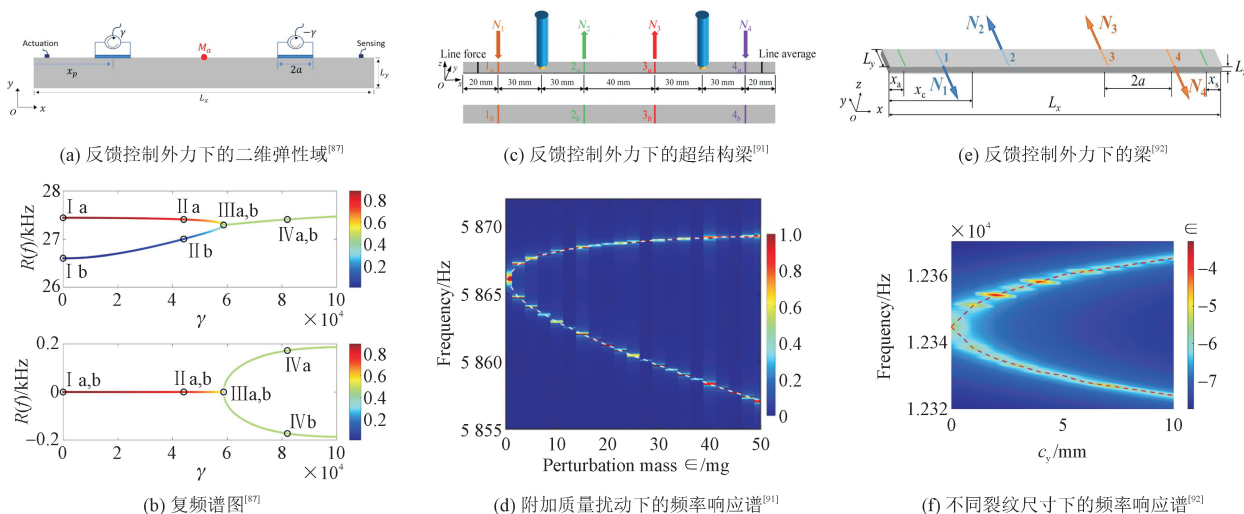


图4 反馈控制的外力作用下构造的模态耦合点以及扰动影响下的频率响应谱

Fig. 4 Exceptional points constructed by the feedback-controlled external forces and the response spectra under the perturbation

2.2 趋肤效应和非互易传播

基于复本征值系统的趋肤效应表现为结构中大量模态局域在边界处,且对边界条件有极端敏感性,即开放边界条件与使用周期性边界条件计算的本征频谱显著不同,这深刻地改变了带拓扑和体边界对

应关系^[73, 94]。在此系统中任意位置输入激励,产生的波总是在一个方向上传播,而在相反方向上的传播受到强烈抑制^[94-96]。因此在振动抑制^[97-98]、传感^[91]和能量采集^[3]等诸多领域展现出了良好的应用前景。复本征值系统的趋肤效应可以通过非局域反馈控制来实现。如图 5(a)所示,ROSA 等^[99]在一

维周期弹簧-质量模型利用非局域反馈相互作用探讨了沿不同方向传播波的衰减和放大相关的非互易性,图5(b)复色散谱中的频率正负虚部分别对应等效损耗和增益。通过在第 n 个晶格的质量上施加附加力控制相互作用,该力与距离为 a 晶格的弹簧伸长率成比例,即 $n-a$ 处与刚度相关的信息馈入了第 n 个晶格。这种方案中,反馈信息采集位置与信息馈入位置不同,此时刚度矩阵为实数且非对称,因而能得到复本征值。

如图5(c)~图5(d)所示,ZHONG等^[100]采用类似的反馈控制机制在二维结构中实现了趋肤效应。 A 和 D 作为传感部分,通过压电片的电压换算得到对应位置的位移信息,乘以比例因子后馈入 C 处压电片电压信号, C 为作动器输出激励。这种反馈回

路中的比例因子定义了反馈强度,作为可调谐的参数使得系统局部状态能够重新配置,因而基于这种反馈控制机制的趋肤效应具有可重构的优势。CAI等^[91]采用了类似于2.1节中的反馈外力方案,在有限梁中实现了趋肤效应。如图5(e)~图5(f)所示,仅在一侧的柱体施加了反馈控制的外力,通过破坏空间对称性来打破互易性,以速度作为反馈信息实现了该系统的等效增益和损耗,原本应该出现在两端开放边界的模态只局域在一侧,且与激发位置无关。此外,可重构的复本征值系统可以应用微机电系统进行灵活控制^[101]。在基于压电片的传感-作动控制系统的奇弹性结构中,针对弯曲变形和剪切力建立非互易耦合,可实现主动的奇弹性模量,能产生非互易和振动模态的局域化^[96, 102]。

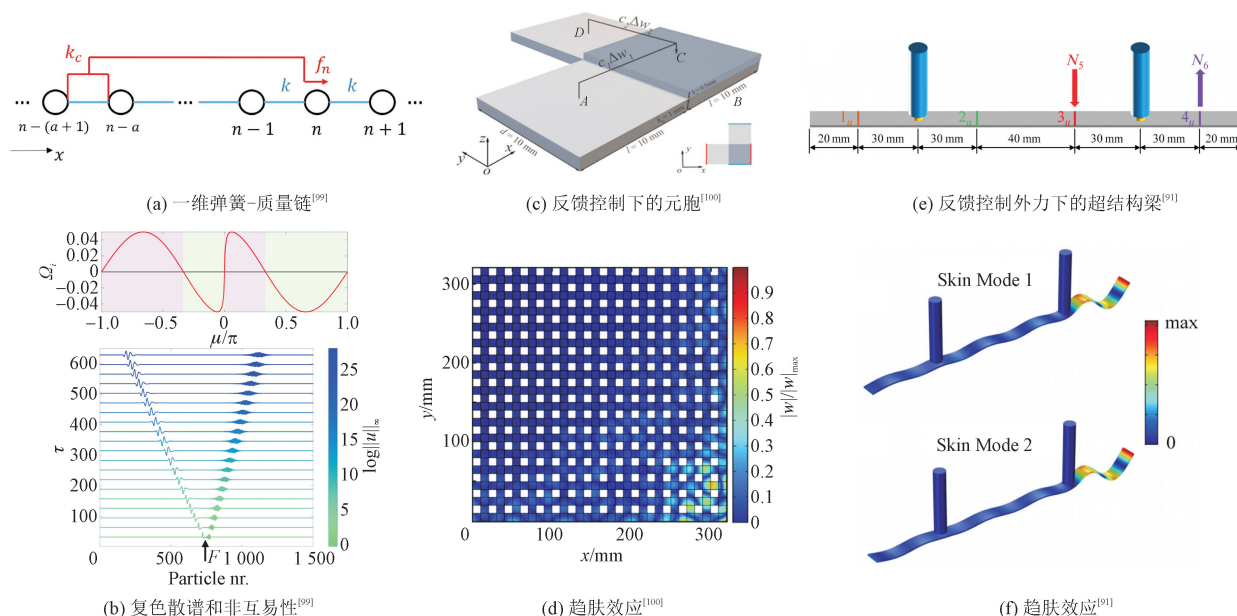


图5 反馈控制下得到的趋肤效应以及非互易性

Fig. 5 Skin mode and nonreciprocity with feedback control

3 利用干涉作用调控弹性波

在前述1.2节中,可编程声子晶体展现了非凡的波动调控可能性。不仅如此,这种可编程方式通过利用波动干涉作用来调控弹性波的幅值、相位甚至频率。并且,这种可编程的反馈控制,对于不同的频率,其输入与输出信号之间的增益是可以经过设计的,这一特点使超构材料可以具有相对于频率的自适应性,使其作用频率范围极大拓宽。此外,该主

动调控方式可以只有一个压电片作为传感器,若干个压电片作为激励,对周期性要求不高,实现结构小型化。类似的控制方法在工程应用、机器人和自动化等领域已经有很多理论基础与实践经验^[103-105]。

当前已有许多主动调控研究实现弹性波幅值与相位的调控。CHEN等^[106]提出了如图6(a)所示的压电超构材料,通过微控制器实现传感与激励之间特定的传递函数,独立控制超构材料层的非对称极化振动,实现了反射波与透射波的独立控制以及非互易的弯曲波抑制(图6b)。对于声波,也有类似的

研究^[107-109],用于产生独特的声波传输特性。LI等^[110]类比电磁学与声学中的因果律不等式,计算了弯曲波吸收的因果律极限(吸收频段宽度受吸收器厚度限制),并设计了如图6(c)所示的宽频带弯曲波吸收压电超构材料。其反馈控制原理如图6(d),频域仿真(图6e)与时域实验(图6f)很好地验证了宽频带弯曲波吸收效果。对于波动相位的调控也一直

是研究的热点。

除了图6(a)所示的超构材料,CHEN等^[111]还设计了用于弹性波相位调制的超构表面(图7a),其单个基元可以实现任意相位的调制(图7b),并展示了转向、聚焦等功能,如图7(d)所示。此外,研究还利用多个基元的非局域反馈控制(图7c),实现了如图7(e)所示的弹性波隐身效果。

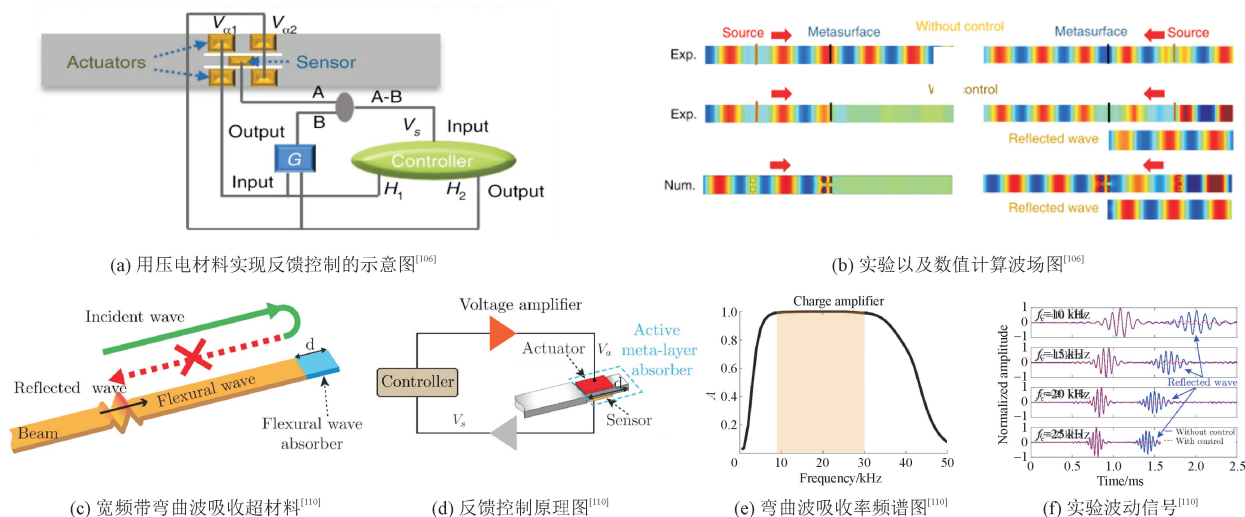


图6 通过反馈控制实现对弹性波幅值的调控

Fig. 6 Modulation of the amplitude of elastic waves by feedback control mechanism

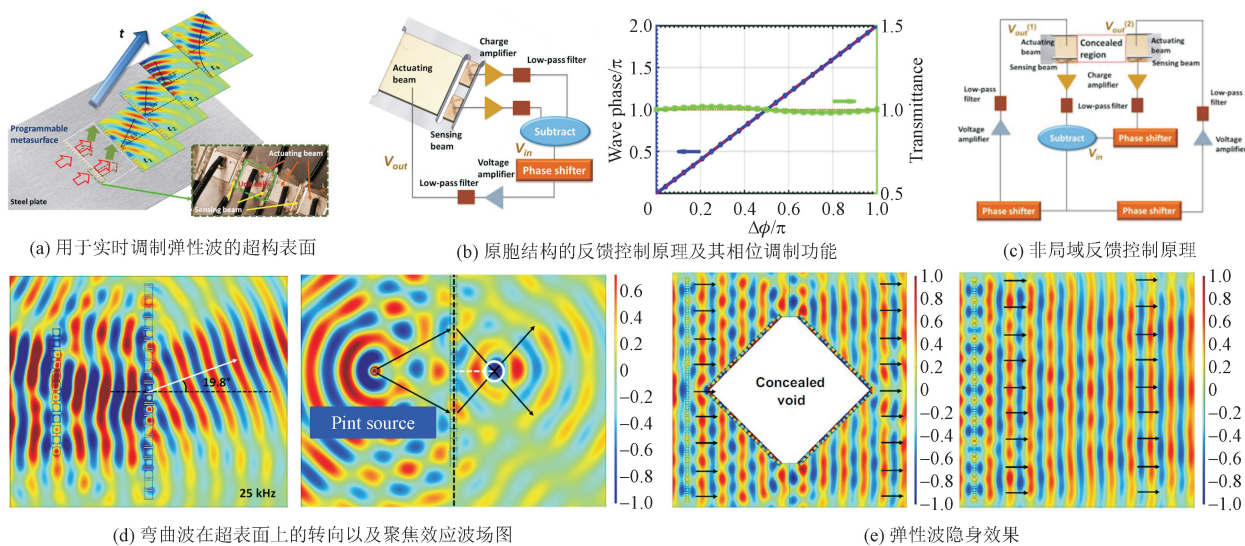
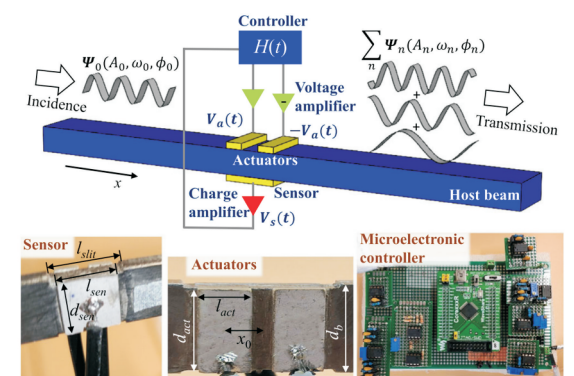


图7 通过反馈控制实现对弹性波相位的调控^[111]

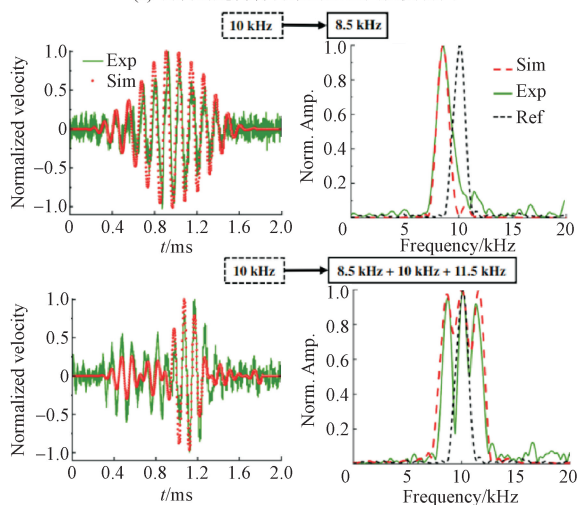
Fig. 7 Modulation of the phase of elastic waves by feedback control mechanism^[111]

除了对弹性波幅值和相位的调控之外,WU等^[112]还利用如图8(a)所示的超构材料梁对弹性波的频率进行调制(图8b),将入射波转换为任意频率的透射波,并且谐波之间是解耦的。这对避开结构

的共振频率有重要的意义。上述研究利用传感接收弹性波信号,再利用激励激发能与入射波产生干涉作用的弹性波从而实现各种各样的功能,其主要的难点在于控制的稳定性以及控制律的编程。



(a) 调制的超构材料梁的原理图及实物图



(b) 频率调制透射波的时域信号及频谱图

图8 通过反馈控制实现对弹性波频率的调控^[112]Fig. 8 Modulation of the frequency of elastic waves by feedback control mechanism^[112]

4 总结和展望

基于反馈控制的弹性波调控在主动型超构材料中应用广泛,不同的反馈交互方案设计和可调谐的反馈参数为波动调控提供了丰富的可能,促进了更高性能、更多功能的实现。本研究主要阐述了弹性波主动型超构材料领域中反馈控制实现的3个机制:利用反馈控制对材料等效动态参数的调制,有助于灵活的禁带调控以及实现低频宽带等性能;通过反馈机制构造的等效增益损耗能够实现复本征值系统,进而拓展了模态耦合点和趋肤效应在多个领域的实际应用,例如对于广泛存在于各种材料与结构中影响使用寿命与安全的微小裂纹,利用模态耦合点可以对其进行高灵敏度检测,而趋肤效应可以用于波动抑制和能量采集等;反馈控制还能够利用干涉作用实现对弹性波幅值、相位和频率的调控,在减

振、信号放大、聚焦、频率转换等领域有良好前景。

结合当前反馈控制的一些难点和最新研究方向,本研究提出了以下3点展望。

1) 反馈控制带来的稳定性问题一直是重点和难点,在一些参数下系统输出无限增长或无法控制。这需要结合基体结构和控制系统,从理论出发对系统参数和控制律合理选择和设计,实现更稳定有效的反馈控制和相应的功能。

2) 反馈控制系统对参数变化和外部干扰的鲁棒性较低,传感数据的实时获取和计算控制输出也是难点所在。可以结合当前快速发展的人工智能、机器学习和优化算法,实现更高鲁棒性和实时性的控制系统。

3) 工业的发展使得工程任务和环境丰富多变,反馈控制系统急需增强适应性以满足更多样的需求。这需要设计自适应控制算法和智能决策机制,使得反馈控制下的弹性波调控自动适应任务和环境的变化,实现更多的功能。

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