

HE Jian-e

The gravity-induced phase shift detected by high- T_c Josephson junctions

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Abstract We derive from the Kaluza-Klein theory a formula for the gravity-induced phase shift around a circuit loop, which amounts to the order of 10^{-6} . We propose experiments to detect this phase shift by using the high- T_c d-wave Josephson junction, which is included in a cuprate superconductor circuit loop. By rotating the loop around the horizontal axis, the gravity-induced phase shift can be detected as a frequency shift. These settings can also be used in turn to determine the gravitational constant. This method will be sensitive and accurate.

Keywords Josephson junction, gravity-induced phase shift, high- T_c superconductor

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Gravitational force is the weakest of the four fundamental forces, which is on the order of 10^{-36} of the electromagnetic force. The gravity-induced phase shift is very small, almost of the order of 10^{-6} . Thus, it will be very difficult to test this small quantity. Since the introduction of the quantum Hall effect and the high T_c superconductors, the situation has changed and the detection of this very small phase shift has become possible. In this paper, we first derive a formula for the gravity-induced phase shift and then propose experiments to detect this phase shift by both the conventional and high- T_c Josephson junctions. To obtain the gravity-induced phase shift, we shall apply the powerful Kaluza-Klein theory [1–3].

We consider the five-dimensional space with cylindrical metric:

$$ds^2 = \gamma_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu = 1, 2, \dots, 5 \quad (1)$$

$$\frac{\partial \gamma_{\mu\nu}}{\partial x^5} = 0 \quad (2)$$

Kaluza-Klein suppose $\gamma_{55} = 1$. In addition to the coordinate transformation of x^k ($k=1, 2, 3, 4$) in general relativity, there is the following transformation group in the special coordinate system above:

$$x'^5 = x^5 + f(x^1, x^2, x^3, x^4) \quad (3)$$

Let's write Eq. (1) as

$$ds^2 = (dx^5 + \gamma_{i5} dx^i)^2 + g_{ik} dx^i dx^k \quad (4)$$

Obviously, under the transformation (3),

$$g'_{ik} = g_{ik}, \quad \gamma'_{i5} = \gamma_{i5} - \frac{\partial f}{\partial x^i}, \quad i = 1, 2, 3, 4 \quad (5)$$

This is of the same form as the gauge transformation. Therefore, we can consider γ_{i5} equivalent to the electromagnetic potential except for a proportional factor. Anti-symmetric tensor

$$f_{ik} = \frac{\partial \gamma_{k5}}{\partial x^i} - \frac{\partial \gamma_{i5}}{\partial x^k} \quad (6)$$

is invariant for the gauge transformation (5), so it is proportional to the field strength.

Now we find the geodesic line of the metric (4). It satisfies

$$\delta \int ds = 0 \quad (7)$$

Therefore,

$$\frac{dx^5}{ds} + \gamma_{i5} \frac{dx^i}{ds} = \text{const} = C \quad (8)$$

The geodesic equation is

$$\frac{d}{ds} \left(g_{ik} \frac{dx^k}{ds} \right) - \frac{1}{2} \frac{\partial g_{rs}}{\partial x^i} \frac{dx^r}{ds} \frac{dx^s}{ds} = C \cdot f_{ik} \frac{dx^k}{ds} \quad (9)$$

Compared with the equation of motion of the charged particle in both the electromagnetic and gravitational fields, we

HE Jian-e (✉)
 Physics Department, Wuhan University, Wuhan 430072, China
 E-mail: jehe@whu.edu.cn

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have

$$C = \frac{q}{m} \quad (10)$$

From Eqs. (4) and (8) we have

$$ds^2 = C^2 ds^2 + g_{ik} dx^i dx^k$$

Therefore,

$$ds = \frac{q}{m} \left(dx^5 + \gamma_{i5} dx^i \right) + g_{ik} \frac{dx^i}{ds} dx^k \quad (11)$$

Now we consider the case of the constant gravitational field,

$$g_{44} = 1 - A(x, y, z)$$

$$g_{11} = g_{22} = g_{33} = -1 - A(x, y, z) \quad (12)$$

and the other g_{ik} all equal zero, where $A(x, y, z)$ is a small quantity. It is known that the equation for the geodesic line is

$$\frac{d^2 x^i}{ds^2} = - \left\{ \begin{matrix} i \\ jk \end{matrix} \right\} \frac{dx^j}{ds} \frac{dx^k}{ds} \quad (13)$$

where $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}$ is the second Christoffel symbol.

Equation (13) gives

$$\frac{d^2 x^i}{dt^2} = \frac{1}{2} \frac{\partial A}{\partial x^i}, \quad i = 1, 2, 3 \quad (14)$$

Therefore,

$$m_0 \frac{d^2 x^i}{dt^2} = - \frac{\partial}{\partial x^i} \left(-\frac{1}{2} m_0 A \right) \quad (15)$$

And then the gravitational potential is $-\frac{1}{2} m_0 A$. It is known that the gravitational potential in Newton mechanics is $m_0 g z$, so

$$A = -2gz \quad (16)$$

or

$$A = -2gz/c^2 \quad (17)$$

such that A is dimensionless.

We have

$$g_{44} = 1 + 2gz/c^2$$

$$g_{11} = g_{22} = g_{33} = -1 + 2gz/c^2 \quad (18)$$

Let a charged particle pass through two paths from A to B as shown in Fig. 1. The upper path 1 and the lower path 2 form a closed loop. Since these two paths are coherent, there must be interference at point B . The interference pattern will be determined by the path integral:

$$\int e^{\frac{i}{\hbar} S} D[\mathbf{r}] \quad (19)$$

where S is the action. If the loop is made of a superconductor wire, the wave function of the particle will be dominantly contained in the wire. So we can make eikonal approximation, i.e., we only take account of the paths S_0 satisfying $\delta S = 0$.

The interference pattern on the screen at B will be determined by

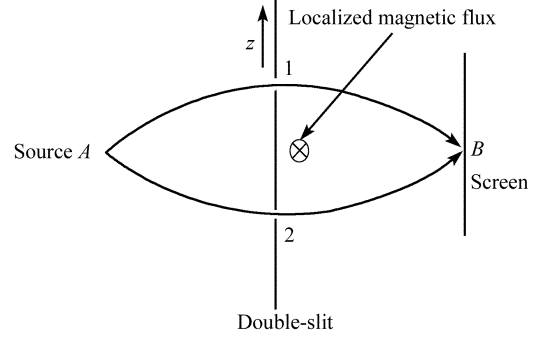


Fig. 1 Schematic diagram of the interference effect.

$$\psi = A_1 e^{\frac{i}{\hbar} S_{01}} + A_2 e^{\frac{i}{\hbar} S_{02}} \quad (20)$$

where

$$S_{01} = -m_0 c^2 \int_{(1)} d\tau_1$$

$$S_{02} = -m_0 c^2 \int_{(2)} d\tau_2 \quad (21)$$

From Eq. (9) we know that f_{ik} is equivalent to the field strength, so from Eq. (6) we can consider γ_{i5} equivalent to the electromagnetic potential A_i ($i = 1, 2, 3, 4$).

Now we return to the usual four-dimensional space from the five-dimensional space, i.e., putting $dx^5 = 0$. From Eq. (11) we have

$$ds = \frac{q}{m_0 c^2} A_i dx^i + g_{ik} \frac{dx^i}{ds} dx^k \quad (22)$$

or

$$d\tau = \frac{q}{m_0 c^3} A_i dx^i + \frac{g_{ik}}{c^2} \frac{dx^i}{d\tau} dx^k \quad (23)$$

From de Broglie relation,

$$P^i = m_0 \frac{dx^i}{d\tau} = \hbar k^i = \hbar(\mathbf{k}, \omega) \quad (24)$$

we have

$$\begin{aligned} \psi &= A_1 \exp \left(-\frac{i}{\hbar c} q \int_{(1)} A_i dx^i - i \int_{(1)} g_{ik} k^i dx^k \right) \\ &+ A_2 \exp \left(-\frac{i}{\hbar c} q \int_{(2)} A_i dx^i - i \int_{(2)} g_{ik} k^i dx^k \right) \\ &= e^{i\varphi} \left\{ A_1 + A_2 \exp \left[-i \frac{q\Phi}{\hbar c} + i \oint \left(1 - \frac{2gz}{c^2} \right) \mathbf{k} \cdot d\mathbf{r} \right] \right\} \end{aligned} \quad (25)$$

where $\Phi = \oint A \cdot d\mathbf{r}$ is the magnetic flux enclosed in the loop.

When the magnetic flux $\Phi = 0$, the interference pattern is determined by $\oint \left(1 - \frac{2gz}{c^2} \right) \mathbf{k} \cdot d\mathbf{r}$. As $\Phi \neq 0$, the phase changes, thus inducing the shift of the interference pattern. This is just the Aharonov-Bohm effect. However, the starting point that we obtain this result is the Kaluza-Klein theory, where the electromagnetic potential $\gamma_{i5} = A_i$ ($i = 1, 2, 3, 4$)

influences the structure of the four-dimensional space-time. This is apparent from Eq. (12). In other words the electromagnetic potential gives rise to Aharonov-Bohm effect through changing the metric of the space-time. This is the reason why the localized magnetic field can influence the behavior of the charged particles in the region where there is no field at all, i.e., even if there is no magnetic field acting on them.

From Eq. (25) we find that the gravitational field can also influence the interference pattern, but this is independent with the influence of the magnetic field. Obviously, the term $-\oint (2gz/c^2) \mathbf{k} \cdot d\mathbf{r}$ can be interpreted as the phase shift due to gravity. If we calculate the $\oint e^{\frac{i}{\hbar} S} D[\mathbf{r}]$ around the loop, we have the same results as before. The value of the gravity-induced phase shift around a loop depends on the shape and size of the loop. For a suitable loop, e.g., a square circuit loop with edge length ≈ 30 cm, it is on the order of 10^{-6} . How to detect this very small phase shift is a difficult problem.

As is known, Josephson junctions have been recently studied extensively for fundamental researches, e.g., studying the behavior of vortex tunneling in Josephson junctions [4], exploring the possibility by using Josephson junctions to test the dark energy which is responsible for the accelerating expansion of the Universe [5], etc.

Now we propose an experiment for the detection of this gravity-induced phase shift by the Josephson junction. Let the loop be a square with edge length L . Let a superconductor wire with n turns along the edges of the square be connected to the electrodes of a Josephson junction which is placed near the middle point of the right-hand edge of the square. We put the square loop perpendicular to the earth first. Then we rotate the loop around the axis which is parallel to the earth surface and passes through the middle points of the opposite edges of the square as shown in Fig. 2. Then the phase difference across the junction will evolve. To avoid interference from the earth's magnetic field, we should make the axis parallel to the earth magnetic field and screen the whole square frame magnetically. The electrodes of the junction should be round and polished such that the contact between the terminals of the wire and the electrodes of the junction is sliding. The relative orientation of the plane of the square winding with that of the link interface of the junction should be such, that as the square plane of the winding is perpendicular to the earth, the contacts should be placed on the link interface of the junction. To make sure this condition being achieved, we should make fine adjustment of the relative angle between the two planes mentioned above, such that the output voltage due to rotation should be maximum. In such a way the polarities of the electrodes of the junction will not change during rotation. Thus we have the frequency ω_1 :

$$\omega_1 = \frac{16\pi f k g L^2}{c^2} \quad (26)$$

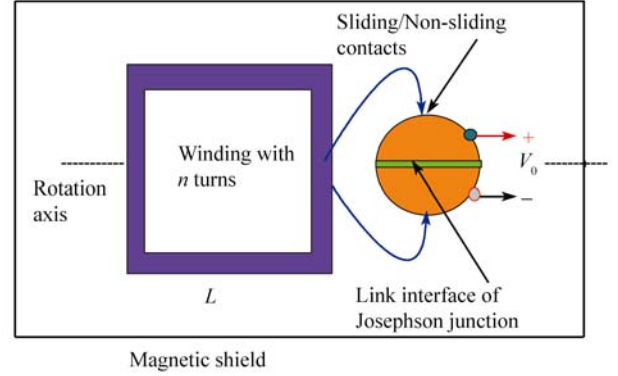


Fig. 2 Schematic diagram of the experiment arrangement.

which gives rise to a dc voltage across the junction

$$V_1 = \frac{\hbar \omega_1}{2e} \quad (27)$$

where n is the number of turns of the superconductor wire loop and f is the rotation frequency of the square frame. If we apply an additional dc voltage V_0 across the junction, we have a frequency

$$\omega = \omega_0 + \omega_1 \quad (28)$$

where $\omega_0 = \frac{2eV_0}{\hbar}$. By measuring the frequency shift we can

obtain the gravity-induced frequency, thus obtaining the gravity-induced phase shift. In this way we can also find the value of k of the Cooper pairs in the superconductor with high precision. We can also detect the gravity-induced phase shift in another way. Here we connect the wire terminals firmly to the electrodes of the junction without any sliding. When we rotate the square frame, there occurs an alternative voltage $V_1 \cos \omega_1 t$ across the junction:

$$V_1 \cos \omega_1 t = (2g\hbar k L^2/e) \omega_1 \cos \omega_1 t \quad (29)$$

where ω_1 is the angular frequency of rotation of the square frame. If we apply a dc voltage V_0 across the junction, the total voltage across the junction would be

$$V = V_0 + V_1 \cos \omega_1 t \quad (30)$$

Thus we have the phase shift

$$\gamma(t) = \gamma_0 + \omega_0 t + \frac{2eV_1}{\hbar \omega_1} \sin \omega_1 t \quad (31)$$

where γ_0 is an integration constant and $\omega_0 = 2eV_0/\hbar$. We apply the Josephson equation $I_s = I_c \sin \gamma$ to get the supercurrent I_s :

$$I_s = I_c \sum (-)^n J_n \left(\frac{2eV_1}{\hbar \omega_1} \right) \sin(\gamma_0 + \omega_0 t - n\omega_1 t) \quad (32)$$

where J_n is the Bessel function of the n th order. Obviously, I_s has no dc component unless $\omega_0 = n\omega_1$ for some n , i.e.,

$$V_0 = V_n = n \frac{\hbar \omega_1}{2e} \quad (33)$$

In this case the I - V curve has steps called Shapiro steps [6], the half-width of which is

$$I_n = I_c J_n \left(\frac{2eV_1}{\hbar\omega_1} \right) \quad (34)$$

The half-width I_n can be determined by experiments. Then from Eq. (34) we can obtain V_1 by numerical calculations, thus obtaining the gravity-induced phase shift. In addition, we can obtain the value of g , and thus the gravitational constant G [12], accurately and sensitively from the phase shift as k can be determined by experiments.

Although the discussions above are mainly on the basis of conventional superconductors, the derivations and the results can be applicable to high T_c superconductors. It is known that the cuprate superconductors have dominantly d -wave pairing [7–11], i.e., the pairing symmetry is $d_{x^2-y^2}$.

The gap parameter $\Delta(\mathbf{k})$ has the symmetry $k_x^2 - k_y^2$ with angular momentum $l=2$ and spin singlet. So $\Delta(\mathbf{k}) \propto \cos 2\theta$, where θ is measured from the x -axis in the ab -plane. That the sign of $\Delta(\mathbf{k})$ changes with the angle may introduce an intrinsic phase π into the Josephson formulas. However, this could not by itself produce observable effects in 0 or π junctions [7]. Only the phase gradient may produce a spontaneous current.

Our previous arguments refer to just a single junction, not two junctions as in SQUIDS where the relative phase shift between the two junctions would produce an interference effect. We also consider only the gravity-induced phase shift due to rotation, so our arguments still hold for high- T_c superconductors. We can do the experiments more easily than

that with conventional superconductors since the high- T_c Josephson junctions can operate under temperatures of about 130 K. This case benefits our experiments more.

In this paper we have derived a formula for gravity-induced phase shift, and proposed experiments to detect this phase shift.

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