AIMS Mathematics, 11(10): 32347–32368.
DOI: 10.3934/math.20261271
Received: 21 May 2026
Revised: 26 July 2026
Accepted: 03 August 2026
Published: 08 October 2026

https://www.aimspress.com/journal/Math

Research article

Existence results of fractional delay differential equations with multi-point boundary conditions via $\mathbb{L}$–intuitionistic fuzzy mappings

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Abstract: This paper introduces a novel concept termed $\mathbb{L}$–intuitionistic fuzzy mapping and studies the corresponding $\mathbb{L}$–intuitionistic fuzzy fixed point theorems within the context of a $(\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}})$–level set. These notions are further employed to establish common $\mathbb{L}$–intuitionistic fuzzy fixed point results using hybrid-type contraction on b-metric space. Some important special cases are emphasized and examined as corollaries to indicate the generality of the ideas proposed herein. The hypotheses of the main results are confirmed through nontrivial examples. Additionally, a graphical representation is included to visually depict the newly introduced notion. Furthermore, utilizing one of the outcomes presented, new solvability criteria for a class of fractional delay differential equations subject to multi-point boundary conditions are provided.

Keywords: common fixed point; $\mathbb{L}$–intuitionistic fuzzy mapping; hybrid contractive condition; b–metric space; fractional delay differential equations
Mathematics Subject Classification: 34A12, 46S40, 47H10, 54H25

1. Introduction

The investigation of newly introduced mathematical spaces has been a prominent topic in mathematical research. Among these, b–metric spaces have gained increasing attention, first

introduced through the contributions of Bakhtin [9] and Bourbaki [10]. Czerwik [12] later formulated a condition weaker than the classical triangle inequality and formally defined a b–metric space to broaden the scope of the Banach contraction principle. In the meantime, the theory of b–metric spaces has evolved through numerous generalizations (see, [14, 19, 28]). Interested readers may consult Karapinar [22] for a concise survey on fixed point theory in b–metric spaces. Not long ago, Karapinar and Fulga [21] launched a new notion of b–hybrid contraction in b–metric spaces, together with fixed point results. Shortly, Alansari et al. [3] developed multivalued counterparts of these findings. Interestingly, several applications of hybrid fixed point theory arise in optimization and functional inclusions; refer to [21, 26] and some references therein.

Zadeh [33] introduced fuzzy sets as an advancement beyond the classical idea of crisp sets. Over time, the concept has been extensively refined, with scholars enhancing its scope and applying it across multiple domains of natural sciences and engineering. Heilpern [17] developed a class of fuzzy mappings within the fuzzy framework, serving as a fuzzy counterpart to Nadler's theorem [27]. Thereafter, many authors explored conditions ensuring the existence of fuzzy fixed points (see [7, 8, 25]).

Intuitionistic fuzzy sets (IFS) were introduced by Atanassov [5] and extended the conventional fuzzy set theory by incorporating both membership and non-membership degrees. This framework provides an efficient mathematical approach for handling uncertainty and imprecision arising from inadequate information. Owing to its effectiveness, it has found wide-ranging applications in areas including medical diagnosis, image processing, and decision-making. Meanwhile, ongoing research in this area has increased considerably, producing significant contributions across the multiple disciplines. Not long ago, Azam et al. [7] presented a novel approach for analyzing fixed point results via IFS-valued mappings. In the subsequent studies, Azam and Tabassum [8] derived new criteria for the existence of common coincidence points for three IFS-valued mappings and utilized their findings to examine the existence criteria for solutions of a system of integral equations. More recently, Tabassum et al. [32] developed some common fixed point results in the setting of a $(\mathbb{T}, \mathbb{N}, \alpha)$–level set of IFS.

Atanassov and Stoeva [6] introduced the concept of an $\mathbb{L}$–intuitionistic fuzzy set by replacing the unit interval $[0, 1]$ with a complete bounded lattice $\mathbb{L}$. This generalization provides a richer mathematical structure for representing uncertainty, vagueness, and incomplete information in situations where the classical intuitionistic fuzzy model is inadequate. In an $\mathbb{L}$–intuitionistic fuzzy set, both the membership and non-membership degrees take values in the lattice $\mathbb{L}$, allowing more flexible and expressive modeling of complex systems. Although IFsS have been widely investigated, comparatively little research has been devoted to $\mathbb{L}$–intuitionistic fuzzy sets. Only a few works have investigated such as similarity measures, aggregation operators, and decision-making models.

On the other hand, numerous researchers have employed fixed point theory to obtain the existence and uniqueness of solutions for fractional differential equations under various boundary conditions (see [1, 2, 18]). More recently, fixed point methods have also been applied to stochastic and impulsive fractional differential equations to investigate the existence of mild solutions and approximate controllability (see [11]). Furthermore, fuzzy and intuitionistic fuzzy fixed point techniques have been successfully utilized to study delay and fractional differential equations under uncertainty. In particular, Tabassum et al. [32] applied intuitionistic fuzzy weak contraction mappings to obtain existence results for Riemann-Liouville fractional differential equations.

Motivated by this research gap, The main novelty of this work lies in introducing the notions of

$\mathbb{L}$–intuitionistic fuzzy mappings and $\mathbb{L}$–intuitionistic fuzzy fixed points in b–metric spaces and establishing fixed point theorems under a new $\mathbb{L}$–intuitionistic fuzzy hybrid contraction condition. Unlike the existing literature, which primarily considers classical fuzzy or intuitionistic fuzzy mappings, our approach employs $(\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}})$–level sets together with the Hausdorff b–metric and a comparison function to develop a more general contractive framework. Furthermore, to demonstrate the validity and practical significance of the proposed theory, we have included a non-trivial example together with graphical illustrations. The graphs verify the proposed contractive condition and support the applicability and novelty of the obtained fixed point results. As an application, one of the results obtained herein is employed to analyze more general conditions for the existence of solutions to a class of fractional delay differential equations with multi-points boundary conditions. Our findings, in their crisp forms, recover a variety of fixed point results. The literature appears to lack results on hybrid fixed point theory via $\mathbb{L}$–intuitionistic fuzzy mappings on b–metric spaces. Accordingly, the proposed work contributes a novel research perspective in non-classical analysis.

2. Preliminaries

In this section, we have compiled specific fundamental concepts here that will be needed later.

In what follows, $\Re^+$, $\Re$, and $\aleph$ respectively stand for the sets of non-negative real numbers, real numbers, and natural numbers.

Definition 2.1. [12] Let $\Omega \neq \phi$ and $s \geq 1$ be a fixed constant. Assuming $\sigma : \Omega \times \Omega \to \Re^+$ satisfies all the stated conditions for every $\varrho, \gamma, \omega \in \Omega$,

(i) $\sigma(\varrho, \gamma) = 0$ if and only if $\varrho = \gamma$),
(ii) $\sigma(\varrho, \gamma) = \sigma(\gamma, \varrho)$,
(iii) $\sigma(\varrho, \gamma) \leq s[\sigma(\varrho, \omega) + \sigma(\omega, \gamma)]$.

Then (Ω, σ, s) is referred to as a b–metric space.

Definition 2.2. [10] Let (Ω, σ, s) be a b–metric space. A sequence $\omega_n \subseteq \Omega$ is said to be

(i) convergent if and only if there exists $\omega \in \Omega$ such that $\sigma(\omega_n, \omega) \to 0$ as $n \to \infty$;
(ii) Cauchy if and only if $\sigma(\omega_n, \omega_m) \to 0$ as $n, m \to \infty$;
(iii) complete if every Cauchy sequence in Ω converges to a point of Ω.

Definition 2.3. [20] Let (Ω, σ, s) be a b–metric space. A nonempty subset Ω_1 of Ω is said to be proximal if for each $\omega \in \Omega$, there exists $\gamma \in \Omega_1$ such that $\sigma(\omega, \gamma) = \sigma(\omega, \Omega_1)$.

Let $\mathfrak{N}(\Omega), \mathfrak{CB}(\Omega), \mathcal{P}^r(\Omega)$, $\mathcal{P}^r_b(\Omega)$, and $\mathcal{K}(\Omega)$ denote the collection of all nonempty, closed and bounded, proximal, proximal bounded, and compact subsets of Ω, respectively.

For $\Omega_1, \Omega_2 \in \mathcal{P}^r(\Omega)$ and $\mathcal{H} : \mathcal{P}^r(\Omega) \times \mathcal{P}^r(\Omega) \to \Re^+$, defined by

$$\mathcal{H}(\Omega_1, \Omega_2) = \begin{cases} \max\Big(\sup_{\gamma \in \Omega_1} \sigma(\gamma, \Omega_2), \sup_{\gamma \in \Omega_2} \sigma(\gamma, \Omega_1) \Big), & \text{if it exists,} \\ \\ \infty, & \text{otherwise,} \end{cases}$$

it is said to be the Hausdorff-Pompeiu b–metric on $\mathcal{P}^r(\Omega)$ induced by the b–metric σ, where

$$\sigma(\omega, \Omega_1) = \inf_{\gamma \in \Omega_1} \sigma(\omega, \gamma).$$

Remark 2.4. In view of the fact that every compact set is proximal and every proximal set is closed (see [20]), the following inclusions are valid:

$$\mathcal{K}(\Omega) \subset \mathcal{P}^r(\Omega) \subset \mathcal{CB}(\Omega) \subset \mathfrak{N}(\Omega).$$

Definition 2.5. [21] A nondecreasing function $\breve{\varphi} : \Re^+ \to \Re^+$ is referred to as:

(i) a comparison function of c-type, provided that $\breve{\varphi}^n(t) \to 0$ whenever $\mathfrak{n} \to \infty$ for each $t \in \Re^+$.
(ii) a comparison function of b-type, if there is $n_0 \in \aleph, \hat{\lambda} \in (0, 1)$ and a convergent non-negative series $\sum_{n=1}^{\infty} \omega_n$ satisfying

$$s^{n+1}\breve{\varphi}^{n+1}(t) \leq \hat{\lambda} s^n \breve{\varphi}^n(t) + \omega_n,$$

for $s \geq 1$, $n \geq n_0$, and $t \geq 0$,
where $\breve{\varphi}^n$ stands for the n^{th} iterated form of $\breve{\varphi}$.

Take $\widehat{W}$, and denote the family of $\breve{\varphi}$ that abide by the following:

(i) $\breve{\varphi}$ is a comparison function of b-type;
(ii) $\breve{\varphi}(t) = 0$ if and only if $t = 0$;
(iii) $\breve{\varphi}$ is continuous.

In the following, we present a brief overview of lattices, $\mathbb{L}$–fuzzy and $\mathbb{L}$–intuitionistic fuzzy structures that are essential for the subsequent discussion.

Definition 2.6. [16] A relation $\preceq$ on $\mathbb{L}$ is termed a partial ordering if $\preceq$ satisfies the properties of reflexivity, antisymmetry, and transitivity. The structure $(\mathbb{L}, \preceq)$ is called a partially ordered set.

Such orderings are useful for assigning structure to unordered collections.

Definition 2.7. [16] A partially ordered set $(\mathbb{L}, \preceq)$ is said to be
(i) a lattice if $\omega \vee \gamma \in \mathbb{L}$ and $\omega \wedge \gamma \in \mathbb{L}$ for any $\omega, \gamma \in \mathbb{L}$;
(ii) a complete lattice if $\vee\tilde{\mathrm{E}} \in \mathbb{L}$ and $\wedge\tilde{\mathrm{E}} \in \mathbb{L}$ for any $\tilde{\mathrm{E}} \subset \mathbb{L}$;
(iii) a distributive lattice if $\varrho \vee (\omega \wedge \gamma) = (\varrho \vee \omega) \wedge (\varrho \vee \gamma)$, $\varrho \wedge (\omega \vee \gamma) = (\varrho \wedge \omega) \vee (\varrho \wedge \gamma)$ for any $\varrho, \omega, \gamma \in \mathbb{L}$.

Definition 2.8. [16] A lattice $\mathbb{L}$ is called bounded if there exists a least element $0_{\mathbb{L}} \in \mathbb{L}$ and the greatest element $1_{\mathbb{L}} \in \mathbb{L}$ such that

$$0_{\mathbb{L}} \preceq \omega \preceq 1_{\mathbb{L}}$$

for every $\omega \in \mathbb{L}$.

Definition 2.9. [16] Consider a nonempty set Ω and a bounded complete distributive lattice $\mathbb{L}$ with $0_{\mathbb{L}}$ and $1_{\mathbb{L}}$. An $\mathbb{L}$–fuzzy set $\tilde{F}$ on Ω is a function $\tilde{F}\colon \Omega \to \mathbb{L}$.

Definition 2.10. [29] Let $\tilde{F}$ be an $\mathbb{L}$–fuzzy set in Ω. The $\alpha_{\mathbb{L}}$–level set of $\tilde{F}$ is defined by

$$[\tilde{F}]_{\alpha_{\mathbb{L}}} = \{\omega \in \Omega : \alpha_{\mathbb{L}} \preceq \tilde{F}(\omega)\} \quad if \;\; \alpha_{\mathbb{L}} \in \mathbb{L} \setminus \{0_{\mathbb{L}}\}.$$

Definition 2.11. [29] Consider an arbitrary set Ω and a metric space $\tilde{\Gamma}$. A mapping $\mathfrak{J} : \Omega \to (\tilde{F})^{\tilde{\Gamma}}$ is referred to $\mathbb{L}$ - fuzzy mapping. An element $\omega \in \tilde{\Gamma}$ is termed as a $\mathbb{L}$ - fuzzy fixed point of $\mathfrak{J} : \tilde{\Gamma} \to (\tilde{F})^{\tilde{\Gamma}}$ if $\omega \in [\mathfrak{J}(\omega)]_{\alpha_{\mathbb{L}}}$,
where $(\tilde{F})^{\tilde{\Gamma}}$ stands for the family comprising of all $\mathbb{L}$ - fuzzy subsets of $\tilde{\Gamma}$.

Definition 2.12. [5] Consider a universal set Ω. An IFS Λ in Ω may be expressed as

$$\Lambda = \{(\omega, \mu_\Lambda(\omega), \upsilon_\Lambda(\omega)) : \omega \in \Omega\},$$

where μ_Λ: $\Omega \to [0, 1]$ and υ_Λ: $\Omega \to [0, 1]$ denote the membership and non-membership functions, respectively, such that $0 \leq \mu_\Lambda(\omega) + \upsilon_\Lambda(\omega) \leq 1$ for every $\omega \in \Omega$.

The hesitation degree is given by

$$\pi(\omega) = 1 - \mu_\Lambda(\omega) - \upsilon_\Lambda(\omega).$$

Denote the class of all intuitionistic fuzzy subsets of Ω by $(\text{IFS})^{\Omega}$.

Example 2.1. Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$ denote the set of patients under examination. Assume that $\Lambda =$ {patients diagnosed with influenza} is an IFS defined on Ω.

$$\Lambda = \{(\omega_1, 0.70, 0.15), (\omega_2, 0.50, 0.25), (\omega_3, 0.65, 0.15)\}.$$

Hence, for each patient ω_i,
$\mu_\Lambda(\omega_i)$ represents the degree of evidence supporting the diagnosis of influenza.
$nu_\Lambda(\omega_i)$ represents the degree of evidence against the diagnosis. The degrees of hesitation are given by

$$\pi_\Lambda(\omega_1) = 0.15, \pi_\Lambda(\omega_2) = 0.30, \pi_\Lambda(\omega_3) = 0.20.$$

Here, patient ω_2 is indicating that there is a 50% degree of evidence supporting the diagnosis of influenza, a 25% degree of evidence against the diagnosis, and a 25% hesitation degree reflecting uncertainty due to incomplete medical information. Such uncertainty cannot be represented explicitly using classical fuzzy sets, where only the membership degree is specified.

Definition 2.13. [24] A mapping $\mathbb{T} : [0, 1] \times [0, 1] \to [0, 1]$ is known as a triangular norm (t-norm) for an IFS provided that for each $\omega, \gamma, \varrho \in [0, 1]$, the following requirements are met:
(i) $\mathbb{T}(\omega, \mathbb{T}(\gamma, \varrho)) = \mathbb{T}(\mathbb{T}(\omega, \gamma), \varrho)$;
(ii) $\mathbb{T}(\omega, \gamma) = \mathbb{T}(\gamma, \omega)$;
(iii) if $\omega, \gamma, \varrho \in [0, 1]$ and $\omega \leq \gamma$, then $\mathbb{T}(\omega, \varrho) \leq \mathbb{T}(\gamma, \varrho)$;
(iv) $\mathbb{T}(\omega, 1) = \omega$.

A fuzzy negation for an IFS is a non-increasing mapping $\mathbb{N} : [0, 1] \to [0, 1]$ satisfying $\mathbb{N}(0) = 1$ and $\mathbb{N}(1) = 0$. If $\mathbb{N}(\mathbb{N}(\omega)) = \omega$ for all $\omega \in [0, 1]$, then $\mathbb{N}$ is said to be a strong fuzzy negation.

Definition 2.14. [24] The $(\mathbb{T}, \mathbb{N}, \alpha)$–level set of an IFS Λ is represented by $[\Lambda]_{(\mathbb{T},\mathbb{N},\alpha)}$ and is expressed by

$$[\Lambda]_{(\mathbb{T},\mathbb{N},\alpha)} = \{\omega \in \Omega : \mathbb{T}(\mu_\Lambda(\omega), \mathbb{N}(\nu_\Lambda(\omega))) \geq \alpha\}, \quad \alpha \in [0, 1].$$

Definition 2.15. [31, 32] Let Ω be a nonempty arbitrary set and $\tilde{\Gamma}$ be a metric space. An element $\omega \in \tilde{\Gamma}$ is called an intuitionistic fuzzy fixed point of an intuitionistic fuzzy mapping $\psi : \tilde{\Gamma} \to (\text{IFS})^{\tilde{\Gamma}}$ if there exists $\alpha \in [0, 1]$ such that $\omega \in [\psi(\omega)]_{(\mathbb{T},\mathbb{N},\alpha)}$.

These three lemmas are crucial to the discussion of our main findings:

Lemma 2.1. [21] The following criteria for a comparison function $\breve{\varphi}$ hold:

(i) Each iterate $\breve{\varphi}^n$, $n \in \aleph$ is also a comparison function;
(ii) $\breve{\varphi}(t) < t$ for all $t > 0$.

Lemma 2.2. [21] Let $\breve{\varphi} : \Re^+ \to \Re^+$ be a comparison function of b-type. Then for every $t \in \Re^+$, the series $\sum_{n=0}^{\infty} s^n \breve{\varphi}^n(t)$ converges.

Lemma 2.3. [30] Let (Ω, σ, s) be endowed with a b–metric space. For $\Omega_1, \Omega_2, \Omega_3 \in \mathcal{K}(\omega)$ and $\omega, \gamma \in \Omega$, the following conditions apply:

(i) $\sigma(\omega, \Omega_2) \leq \mathcal{H}(\Omega_1, \Omega_2)$ for any $\omega \in \Omega_1$;
(ii) $\sigma(\omega, \Omega_1) = 0$ holds if and only if in the case $\omega \in \Omega_1$;
(iii) $\mathcal{H}(\Omega_1, \Omega_2) = \mathcal{H}(\Omega_2, \Omega_1)$;
(iv) $\mathcal{H}(\Omega_1, \Omega_2) = 0$ if and only if $\Omega_1 = \Omega_2$;
(v) $\mathcal{H}(\Omega_1, \Omega_2) \leq \varsigma[\mathcal{H}(\Omega_1, \Omega_3) + \mathcal{H}(\Omega_3, \Omega_2)]$;
(vi) $\sigma(\omega, \Omega_1) \leq \varsigma[\sigma(\omega, \gamma) + \sigma(\gamma, \Omega_1)]$.

3. Main results

Definition 3.1. [15] Let Ω be an arbitrary set and $(\mathbb{L}, \preceq)$ be a complete bounded distributive lattice with an involutive order reversing operator $\mathbb{N} : \mathbb{L} \to \mathbb{L}$. An $\mathbb{L}$–intuitionistic fuzzy set $\Lambda_{\mathbb{L}}$ in Ω is represented by

$$\Lambda_{\mathbb{L}} = \{\langle \omega, \mu_{\Lambda_{\mathbb{L}}}(\omega), \nu_{\Lambda_{\mathbb{L}}}(\omega)\rangle : \omega \in \Omega\},$$

where $\mu_{\Lambda_{\mathbb{L}}} : \Omega \to \mathbb{L}$ and $\nu_{\Lambda_{\mathbb{L}}} : \Omega \to \mathbb{L}$ denote the membership and non-membership functions, respectively, such that $\mu_{\Lambda_{\mathbb{L}}}(\omega) \preceq \mathbb{N}(\nu_{\Lambda_{\mathbb{L}}}(\omega))$, for all $\omega \in \Omega$.

Remark 3.1. [15] $\mathbb{L}$–intuitionistic fuzzy sets generalize IFS, and they coincide for $\mathbb{L} = [0, 1]$.

Definition 3.2. [13] Let $(\mathbb{L}, \preceq)$ be a complete bounded lattice. A binary operation $\mathbb{T} : \mathbb{L}^2 \to \mathbb{L}$ is said to be a t-norm for an $\mathbb{L}$–intuitionistic fuzzy set if for each $\omega_1, \omega_2, \omega_3, \omega_4 \in \mathbb{L}$, the following properties hold:
(i) $\mathbb{T}(\omega_1, \omega_2) = \mathbb{T}(\omega_2, \omega_1)$ (commutativity);
(ii) $\mathbb{T}(\omega_1, \mathbb{T}(\omega_2, \omega_3)) = \mathbb{T}(\mathbb{T}(\omega_1, \omega_2), \omega_3)$ (associativity);
(iii) $\mathbb{T}(\omega_1, 1_{\mathbb{L}}) = \omega_1$ (neutral element),
(iv) if $\omega_1 \preceq \omega_2$ and $\omega_3 \preceq \omega_4$, then $\mathbb{T}(\omega_1, \omega_3) \preceq \mathbb{T}(\omega_2, \omega_4)$(monotonicity).

A fuzzy negation for an $\mathbb{L}$–intuitionistic fuzzy set is a decreasing mapping $\mathbb{N} : \mathbb{L} \to \mathbb{L}$ satisfying $\mathbb{N}(0_{\mathbb{L}}) = 1_{\mathbb{L}}$ and $\mathbb{N}(1_{\mathbb{L}}) = 0_{\mathbb{L}}$. If $\mathbb{N}(\mathbb{N}(\omega)) = \omega$ for all $\omega \in \mathbb{L}$, then $\mathbb{N}$ is said to be an involutive negation.

Remark 3.2. If we take the lattice $\mathbb{L} = [0, 1]$, then the notion of the t-norm for $\mathbb{L}$–intuitionistic fuzzy sets coincides with the classical one [24].

Definition 3.3. [13] Let $\Lambda_{\mathbb{L}}$ be an $\mathbb{L}$–intuitionistic fuzzy set in Ω. Then the $(\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}})$–level set of $\Lambda_{\mathbb{L}}$ takes the form

$$[\Lambda_{\mathbb{L}}]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})} = \{\omega \in \Omega : \alpha_{\mathbb{L}} \preceq \mathbb{T}(\mu_{\Lambda_{\mathbb{L}}}(\omega), \mathbb{N}(\nu_{\Lambda_{\mathbb{L}}}(\omega)))\}, \text{ for all } \alpha_{\mathbb{L}} \in \mathbb{L}.$$

Denote by $(\mathrm{IFS})^{\Omega}_{\mathbb{L}}$, the class of all $\mathbb{L}$–intuitionistic fuzzy subsets of Ω.

Definition 3.4. Consider an arbitrary set Ω and a metric space $\tilde{\Gamma}$. A point $\omega \in \Omega$ is said to be an $\mathbb{L}$–intuitionistic fuzzy fixed point of $\mathbb{L}$–intuitionistic fuzzy mapping $\mathrm{Q} : \tilde{\Gamma} \to (\mathrm{IFS})^{\tilde{\Gamma}}_{\mathbb{L}}$ if there is $\alpha_{\mathbb{L}} \in \mathbb{L}$ such that $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})}$.

Definition 3.5. Let (Ω, σ, s) be a b–metric space, and a pair of $\mathbb{L}$–intuitionistic fuzzy mappings $\mathrm{Q}, \mathrm{S} : \Omega \to (\mathrm{IFS})^{\Omega}_{\mathbb{L}}$ is said to form an $\mathbb{L}$–intuitionistic fuzzy b-hybrid contraction if for all $\omega, \gamma \in \Omega$, we have

$$\mathcal{H}([\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}, [\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\gamma)})}) \leq \breve{\varphi}\left(C^{r}_{(\mathrm{Q},\mathrm{S})}(\omega, \gamma, (\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}})\right), \tag{3.1}$$

where $\breve{\varphi} \in \widehat{W}$, $r \geq 0$, and $p_i \geq 0$ with $\sum_{i=1}^{4} p_i = 1$ and

$$C^{r}_{(\mathrm{Q},\mathrm{S})}(\omega, \gamma, (\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}}) = \begin{cases} \Bigg[p_1(\sigma(\omega,\gamma))^r + p_2\left(\sigma\left(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}\right)\right)^r \\ +p_3\left(\sigma\left(\gamma, [\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\gamma)})}\right)\right)^r \\ +p_4\left(\frac{\sigma\left(\gamma,[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}}(\omega))}\right)+\sigma\left(\omega,[\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}}(\gamma))}\right)}{2s}\right)^r\Bigg]^{\frac{1}{r}} \\ \text{for} \quad r > 0, \omega, \gamma \in \Omega, \\ \\ (\sigma(\omega,\gamma))^{p_1}\left(\sigma\left(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}\right)\right)^{p_2} \\ \times\left(\sigma\left(\gamma, [\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}}(\gamma))}\right)\right)^{p_3} \\ \times\left(\frac{\sigma\left(\gamma,[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}\right)+\sigma\left(\omega,[\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\gamma)})}\right)}{2s}\right)^{p_4} \\ \text{for} \quad r = 0, \omega, \gamma \in \Omega \setminus \mathcal{F}_{i\omega}(\mathrm{Q}, \mathrm{S}), \end{cases}$$

where

$$\mathcal{F}_{i\omega}(\mathrm{Q}, \mathrm{S}) = \{\omega, \gamma \in \Omega : \omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}, \gamma \in [\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\gamma)})}\}. \tag{3.2}$$

Theorem 3.1. Let $(\Omega, \sigma, \varsigma)$ be a complete b–metric space, and a pair of $\mathbb{L}$–Intuitionistic fuzzy mappings $\mathrm{Q}, \mathrm{S} : \Omega \to (\mathrm{IFS})^{\Omega}_{\mathbb{L}}$ forms an $\mathbb{L}$–Intuitionistic fuzzy b-hybrid contraction. Assuming for each $\omega \in \Omega$, there exists $\alpha_{\mathbb{L}} \in \mathbb{L}$ such that $[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$ and $[\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$ are nonempty bounded subsets of Ω possessing the proximal property.

Thus, one can find $\omega \in \Omega$ such that $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})} \cap [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$.

Proof. Let $\omega_0 \in \Omega$, then by assumption, there exists $(\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}(\omega_0)})$ such that $[\mathrm{Q}(\omega_0)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_0)})} \in \mathcal{P}^{r}_{b}(\Omega)$. Take $\omega_1 \in [\mathrm{Q}(\omega_0)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_0)})}$ such that $\sigma(\omega_0, \omega_1) = \sigma\left(\omega_0, [\mathrm{Q}(\omega_0)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_0)})}\right)$. Again by hypothesis, there exists $\omega_2 \in [\mathrm{S}(\omega_1)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_1)})}$ such that $[\mathrm{S}(\omega_1)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_1)})} \in \mathcal{P}^{r}_{b}(\Omega)$, and we have $\sigma(\omega_1, \omega_2) = \sigma\left(\omega_1, [\mathrm{S}(\omega_1)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega 1))}}\right)$. Continuing in this way, we can formulate a sequence $\{\omega_n\}_{n\in\aleph}$ of elements of Ω such that

$$\omega_{2q+1} \in [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q}))}}, \quad \omega_{2q+2} \in [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q+1})})}$$

and

$$\sigma(\omega_{2q}, \omega_{2q+1}) = \sigma(\omega_{2q}, [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}),$$

$$\sigma(\omega_{2q+1}, \omega_{2q+2}) = \sigma(\omega_{2q+1}, [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(u_{2q+1})})}).$$

Using Lemma 2.3 and the above expression, one can obtain

$$\sigma(\omega_{2q}, \omega_{2q+1}) \leq \mathcal{H}\left([\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}, [\mathrm{S}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}\right), \tag{3.3}$$

$$\sigma(\omega_{2q+1}, \omega_{2q+2}) \leq \mathcal{H}\left([\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}, [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q+1})})}\right). \tag{3.4}$$

Assume that $\omega_{2q} = \omega_{2q+1}$ for some $q \in \aleph$ and $r > 0$. Thus from inequality 3.1, we get

$$\begin{aligned}
C^r_{(\mathrm{Q},\mathrm{S})}(\omega_{2q}, \omega_{2q+1}) =& \Bigg[p_1(\sigma\left(\omega_{2q}, \omega_{2q+1}\right))^r + p_2\left(\sigma\left(\omega_{2q}, [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}\right)\right)^r \\
&+p_3\left(\sigma\left(\omega_{2q+1}, [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q+1})})}\right)\right)^r \\
&+p_4\left(\frac{\sigma(\omega_{2q+1}, [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})})}{2s}\right)^r \\
&+p_4\left(\frac{\sigma(\omega_{2q}, [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q+1})})})}{2s}\right)^r\Bigg]^{\frac{1}{r}} \\
=& \Bigg[p_1(\sigma(\omega_{2q}, \omega_{2q+1}))^r + p_2(\sigma(\omega_{2q}, \omega_{2q+1}))^r \\
&+p_3(\sigma(\omega_{2q+1}, \omega_{2q+2}))^r \\
&+p_4\left(\frac{\sigma(\omega_{2q+1}, \omega_{2q+1}) + \sigma(\omega_{2q}, \omega_{2q+2})}{2s}\right)^r\Bigg]^{\frac{1}{r}} \\
\leq& \Bigg[p_3(\sigma(\omega_{2q+1}, \omega_{2q+2}))^r \\
&+p_4\left(s\left(\frac{\sigma(\omega_{2q}, \omega_{2q+1}) + \sigma(\omega_{2q+1}, \omega_{2q+2})}{2s}\right)\right)^r\Bigg]^{\frac{1}{r}} \\
=& (p_3 + p_4)^{\frac{1}{r}}\sigma(\omega_{2q+1}, \omega_{2q+2}) = \sigma(\omega_{2q+1}, \omega_{2q+2}) \ \ as \ \ r \to \infty.
\end{aligned}$$

Using inequality 3.4 together with condition (ii) of Lemma 2.1, we have

$$\begin{aligned}
\sigma(\omega_{2q+1}, \omega_{2q+2}) &\leq \mathcal{H}\left([\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}, [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q+1})})}\right) \\
&\leq \breve{\varphi}(\sigma(\omega_{2q+1}, \omega_{2q})) < \sigma(\omega_{2q+1}, \omega_{2q+2}),
\end{aligned} \tag{3.5}$$

a contradiction. Therefore, for all $q \in \aleph$,

$$\omega_{2q} = \omega_{2q+1} \in [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})},$$

$$\omega_{2q} = \omega_{2q+1} = \omega_{2q+2} \in [\mathrm{S}(\omega_{2q+1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q+1})})} = [\mathrm{S}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}.$$

Thus, ω_{2q} is a common $\mathbb{L}$-intuitionistic fuzzy fixed point of Q and S.
Therefore, for $r = 0$ and $\omega_{2q} = \omega_{2q+1}$, for some $q \in \aleph$, we can find $C^r_{(\mathrm{Q},\mathrm{S})}(\omega_{2q}, \omega_{2q+1}, (\mathbb{T}, \mathbb{N}, \alpha_{\mathbb{L}})) = 0$ for all $q \in \aleph$. By property (ii) of $\widehat{W}$, one can obtain $\sigma(\omega_{2q+1}, \omega_{2q+2}) = 0$ for all $q \in \aleph$. In a similar way,

we conclude that $\omega_{2q} \in [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})} \cap [\mathrm{S}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}$. Subsequently, we suppose that for all $q \in \aleph$, $\omega_{q+1} \neq \omega_q$ if and only if $\sigma(\omega_{q+1}, \omega_q) > 0$.
Now, setting $\omega = \omega_{2q}$ and $\gamma = \omega_{2q-1}$ in Eq (3.2), we have

$$C^r_{(\mathrm{Q,S})}(\omega_{2q}, \omega_{2q-1}) = \begin{cases} \Bigg[p_1(\sigma(\omega_{2q}, \omega_{2q-1}))^r + p_2 \left(\sigma(\omega_{2q}, [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}\right)^r \\ + p_3 \left(\sigma(\omega_{2q-1}, [\mathrm{S}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}\right)^r \\ + p_4 \left(\frac{\sigma\left(\omega_{2q-1}, [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}\right) + \sigma\left(\omega_{2q}, [\mathrm{S}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}\right)}{2s} \right)^r \Bigg]^{\frac{1}{r}} \\ \text{for } \ r \geq 0, \\ \\ (\sigma(\omega_{2q}, \omega_{2q-1}))^{p_1} \times \left(\sigma(\omega_{2q}, [\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}\right)^{p_2} \\ \times \left(\sigma(\omega_{2q-1}, [\mathrm{S}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}\right)^{p_3} \\ \times \left(\frac{\sigma(\omega_{2q}, [\mathrm{Q}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}) + \sigma(\omega_{2q-1}, [\mathrm{S}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})}}{2s} \right)^{p_4} \\ \text{for } \ r = 0. \end{cases} \tag{3.6}$$

Thus,

$$C^r_{(\mathrm{Q,S})}(\omega_{2q}, \omega_{2q-1}) = \begin{cases} \Bigg[p_1(\sigma(\omega_{2q}, \omega_{2q-1}))^r + p_2 \left(\sigma(\omega_{2q}, \omega_{2q+1})\right)^r \\ + p_3 \left(\sigma(\omega_{2q-1}, \omega_{2q})\right)^r + p_4 \left(\frac{\sigma(\omega_{2q-1}, \omega_{2q+1}) + \sigma(\omega_{2q}, \omega_{2q})}{2s} \right)^r \Bigg]^{\frac{1}{r}} \text{ for } r > 0, \\ \\ (\sigma(\omega_{2q}, \omega_{2q-1}))^{p_1} \times (\sigma(\omega_{2q}, \omega_{2q+1}))^{p_2} \times (\sigma(\omega_{2q-1}, \omega_{2q}))^{p_3} \\ \times \left(\frac{\sigma(\omega_{2q}, \omega_{2q}) + \sigma(\omega_{2q+1}, \omega_{2q-1})}{2s} \right)^{p_4} \text{ for } r = 0. \end{cases} \tag{3.7}$$

Consider the following two cases:
Case 1. $r > 0$ and assume $\sigma(\omega_{2q}, \omega_{2q+1}) \geq \sigma(\omega_{2q-1}, \omega_{2q})$. Thus, in view of Eq (3.7), we get

$$\begin{aligned} C^r_{(\mathrm{Q,S})}(\omega_{2q}, \omega_{2q-1}) \leq & \Bigg[p_1(\sigma(\omega_{2q+1}, \omega_{2q}))^r + p_2(\sigma(\omega_{2q+1}, \omega_{2q}))^r + p_3(\sigma(\omega_{2q+1}, \omega_{2q}))^r \\ & + p_4 \left(s \left(\frac{\sigma(\omega_{2q+1}, \omega_{2q}) + \sigma(\omega_{2q}, \omega_{2q-1})}{2s} \right) \right)^r \Bigg]^{\frac{1}{r}} \\ \leq & \Bigg[p_1(\sigma(\omega_{2q+1}, \omega_{2q}))^r + p_2(\sigma(\omega_{2q+1}, \omega_{2q}))^r + p_3(\sigma(\omega_{2q+1}, \omega_{2q}))^r \\ & + p_4 \left(s \left(\frac{\sigma(\omega_{2q+1}, \omega_{2q}) + \sigma(\omega_{2q+1}, \omega_{2q})}{2s} \right) \right)^r \Bigg]^{\frac{1}{r}} \\ \leq & \left[p_1(\sigma(\omega_{2q+1}, \omega_{2q}))^r + p_2(\sigma(\omega_{2q+1}, \omega_{2q}))^r + p_3(\sigma(\omega_{2q+1}, \omega_{2q}))^r \right. \\ & \left. + p_4(\sigma(\omega_{2q+1}, \omega_{2q}))^r \right]^{\frac{1}{r}} \\ = & \ \sigma(\omega_{2q+1}, \omega_{2q}) \Big(\sum_{i=1}^{4} p_i \Big)^{\frac{1}{r}} = \sigma(\omega_{2q+1}, \omega_{2q}). \end{aligned} \tag{3.8}$$

Therefore, in view of inequalities (3.2), (3.4), and (3.8), one can obtain

$$\begin{aligned}\sigma(\omega_{2q},\omega_{2q+1}) &\leq \mathcal{H}\left([\mathrm{Q}(\omega_{2q})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q})})},[\mathrm{S}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}\right)\\ &\leq \breve{\varphi}\left(C^{r}_{(\mathrm{Q},\mathrm{S})}(\omega_{2q},\omega_{2q-1})\right)\\ &\leq \breve{\varphi}(\sigma(\omega_{2q+1},\omega_{2q})).\end{aligned} \tag{3.9}$$

By using condition (ii) of Lemma 2.1, inequality (3.9) yields

$$\sigma(\omega_{2q+1},\omega_{2q}) < \sigma(\omega_{2q+1},\omega_{2q}). \tag{3.10}$$

This is a contradiction. Thus from inequality (3.9), we get

$$\sigma(\omega_{2q+1},\omega_{2q}) \leq \breve{\varphi}(\sigma(\omega_{2q},\omega_{2q-1})). \tag{3.11}$$

Substituting $n = 2q \in \aleph$ in inequality (3.11),

$$\begin{aligned}\sigma(\omega_{n+1},\omega_n) &\leq \breve{\varphi}(\sigma(\omega_n,\omega_{n-1})) \leq \breve{\varphi}^2(\sigma(\omega_{n-1},\omega_{n-2}))\\ &\leq \breve{\varphi}^3(\sigma(\omega_{n-2},\omega_{n-3}))..... \leq \breve{\varphi}^n(\sigma(\omega_1,\omega_0)).\end{aligned} \tag{3.12}$$

From inequality (3.12) and the triangular inequality of b–metric space for all $k \geq 1$, we have

$$\begin{aligned}\sigma(\omega_{n+k},\omega_n) &\leq s(\sigma(\omega_{n+k},\omega_{n+1}) + \sigma(\omega_{n+1},\omega_n))\\ &\leq \frac{1}{s^{n-1}}\sum_{i=n}^{n+k-1} s^k\sigma(\omega_i,\omega_{i+1})\\ &\leq \frac{1}{s^{n-1}}\sum_{i=n}^{n+k-1} s^k\breve{\varphi}^k(\sigma(\omega_1,\omega_0))\\ &\leq \frac{1}{s^{n-1}}\sum_{i=n}^{\infty} s^i\breve{\varphi}^i(\sigma(\omega_1,\omega_0)).\end{aligned} \tag{3.13}$$

Setting $n \to \infty$ and applying Lemma 2.2, one can find $\lim_{n\to\infty}\sigma(\omega_{n+k},\omega_n) = 0$. Therefore we have a Cauchy sequence $\{\omega_n\}_{n\in\aleph}$ in $(\Omega,\sigma,\varsigma)$. As the space is complete, therefore there exists $\omega \in \Omega$ such that

$$\lim_{n\to\infty}\sigma(\omega_n,\omega) = 0. \tag{3.14}$$

Now, we have to prove that ω is the common $\mathbb{L}$–intuitionistic fuzzy fixed point of Q and S. First, suppose that $\omega \notin [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$ so $\sigma(\omega,[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) > 0$. Thus, using condition (i) of Lemma 2.3

and the contractive inequality (3.1) for $r > 0$, one can find

$$\begin{aligned}
\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) &\leq s\sigma(\omega, \omega_n) + s\sigma(\omega_n, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) \\
&\leq s\sigma(\omega, \omega_n) + s\mathcal{H}([\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}, [\mathrm{S}(\omega_{n-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{n-1})})}) \\
&\leq s\sigma(\omega, \omega_n) + s\breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega, \omega_{n-1})\right) \\
&= s\sigma(\omega, \omega_n) + s\breve{\varphi}\Big[p_1(\sigma(\omega, \omega_{n-1}))^r + p_2(\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(z)})}))^r \\
&+ p_3(\sigma(\omega_{n-1}, [\mathrm{S}(\omega_{n-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{n-1})})})^r \\
&+ p_4\left(\frac{\sigma(\omega_{n-1}, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) + \sigma(\omega, [\mathrm{S}(\omega_{n-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{n-1})})})}{2s}\right)^r\Big]^{\frac{1}{r}} \\
&= s\sigma(\omega, \omega_n) + s\breve{\varphi}\Big[p_1(\sigma(\omega, \omega_{n-1}))^r + p_2(\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}))^r \\
&+ p_3(\sigma(\omega_{n-1}, \omega_n))^r \\
&+ p_4\left(\frac{\sigma(\omega_{n-1}, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) + \sigma(\omega, \omega_n)}{2s}\right)^r\Big]^{\frac{1}{r}}.
\end{aligned} \tag{3.15}$$

Using condition (ii) of Lemma 2.1 and assuming $n \to \infty$ in inequality (3.15), we have

$$\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) < s\sigma(\omega, [\mathbb{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})})(p_2 + p_4)^{\frac{1}{r}}. \tag{3.16}$$

As $r \to \infty$,

$$\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) < s\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}). \tag{3.17}$$

This is a contradiction for $s = 1$. Consequently, $\sigma(\omega, [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) = 0$, and this implies that $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$.
In a similar way, it can be shown that $\omega \in [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$. Hence for $r > 0$, there is a point $\omega \in \Omega$ such that $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})} \cap [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$.

Case 2. Consider $r = 0$ and use inequality (3.7) and condition (ii) of Lemma 2.1,

$$\begin{aligned}
\sigma(\omega_{2q}, \omega_{2q-1}) \leq& \mathcal{H}\left([\mathrm{Q}(\omega_{2q-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-1})})}, [\mathrm{S}(\omega_{2q-2})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{2q-2})})}\right) \\
\leq& \breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega_{2q-1}, \omega_{2q-2})\right) \\
<& (\sigma(\omega_{2q-1}, \omega_{2q-2}))^{p_1}(\sigma(\omega_{2q-1}, \omega_{2q}))^{p_2}(\sigma(\omega_{2q-2}, \omega_{2q-1}))^{p_3} \\
& \left(\frac{\sigma(\omega_{2q-1}, \omega_{2q-1})) + \sigma(\omega_{2q-2}, \omega_{2q})}{2s}\right)^{p_4} \\
=& (\sigma(\omega_{2q-1}, \omega_{2q-2}))^{p_1+p_3}(\sigma(\omega_{2q-1}, \omega_{2q}))^{p_2} \\
& \left(\frac{\sigma(\omega_{2q}, \omega_{2q-1})) + \sigma(\omega_{2q-1}, \omega_{2q-2})}{2}\right)^{p_4}.
\end{aligned} \tag{3.18}$$

Suppose that $\sigma(\omega_{2q-1}, \omega_{2q-2}) \leq \sigma(\omega_{2q}, \omega_{2q-1})$, therefore inequality (3.18) yields

$$\begin{aligned}
\sigma(\omega_{2q}, \omega_{2q-1}) \leq& (\sigma(\omega_{2q}, \omega_{2q-1}))^{p_1+p_2+p_3}(\sigma(\omega_{2q}, \omega_{2q-1}))^{1-p_1-p_2-p_3} \\
=& \sigma(\omega_{2q}, \omega_{2q-1}),
\end{aligned} \tag{3.19}$$

a contradiction. Thus,

$$\sigma(\omega_{2q}, \omega_{2q-1}) \leq \sigma(\omega_{2q-1}, \omega_{2q-2}). \tag{3.20}$$

Using inequalities (3.18) and (3.20), one can obtain

$$\sigma(\omega_{2q}, \omega_{2q-1}) \leq \breve{\varphi}(\sigma(\omega_{2q-1}, \omega_{2q-2})). \tag{3.21}$$

Similarly, $\{\omega_n\}_{n\in\aleph}$ is a Cauchy sequence in (Ω, σ, s). As Ω is complete, therefore we have $\sigma(\omega_n, \omega) \to 0$ as $n \to \infty$ for some $\omega \in \Omega$.

Now, by applying Lemma 2.3 and inequality (3.2), we have

$$\begin{aligned}\sigma(\omega, [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) \leq & s\sigma(\omega, \omega_n) + s\sigma(\omega_n, [\mathrm{S}(z)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}) \\ & s\sigma(\omega, \omega_n) + s\mathcal{H}\left([\mathrm{Q}(\omega_{n-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{n-1})}}, [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}\right) \\ & s\sigma(\omega, \omega_n) + s\breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega_{n-1}, \omega)\right),\end{aligned} \tag{3.22}$$

where

$$\begin{aligned}C^r_{(\mathrm{Q},\mathrm{S})}(\omega_{n-1}, \omega) = & (\sigma(\omega_{n-1}, \omega))^{p_1}(\sigma(\omega_{n-1}, [\mathrm{Q}(\omega_{n-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{n-1})})})^{p_2} \\ & \times (\sigma(\omega, [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}))^{p_3} \\ & \times \left(\frac{\sigma(\omega_{n-1}, [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)}})) + (\omega, [\mathrm{Q}(\omega_{n-1})]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega_{n-1})})}}{2s}\right)^{p_4}.\end{aligned} \tag{3.23}$$

It can be seen that $\lim_{n\to\infty} C^r_{(\mathrm{Q},\mathrm{S})}(\omega_{n-1}, \omega) = 0$. Thus, inequality (3.22) and $\breve{\varphi} \in \widetilde{W}$ implies that $\omega \in [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$.

Similarly, it can be shown that $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$. Hence $\omega \in \Omega$ is a common $\mathbb{L}$–intuitionistic fuzzy fixed point of Q and S. □

In the following, we explore the additional implications of our key findings. By taking single mapping in Theorem 1, we can derive the following result as a corollary.

Corollary 3.1. Let (Ω, σ, s) be a complete b–metric space and $\mathrm{Q} : \Omega \to (\mathrm{IFS})^{\Omega}_{\mathbb{L}}$ be an $\mathbb{L}$–intuitionistic fuzzy mapping. Assume that for each $\omega \in \Omega$, there exists $\alpha_{\mathbb{L}} \in \mathbb{L}$ such that $[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$ is a nonempty proximal bounded subset of Ω. If the $\mathbb{L}$–intuitionistic fuzzy mapping Q satisfies a b-hybrid contraction property, then one can obtain $\omega \in \Omega$ for which $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})}$.

By considering $\mathbb{L} = [0, 1]$ in Theorem 3.1, the following corollary can be deduced for a pair of intuitionistic fuzzy mappings.

Corollary 3.2. Let (Ω, σ, s) be a complete b–metric space and $\mathrm{Q}, \mathrm{S} : \Omega \to (\mathrm{IFS})^{\Omega}$ be a pair of intuitionistic fuzzy mappings. Assume that for each $\omega \in \Omega$, there exists $\alpha \in [0, 1]$ such that $[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{(\omega)})}$ and $[\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{(\omega)})}$ are nonempty proximal bounded subsets of Ω. If the pair of intuitionistic fuzzy mappings Q and S satisfy a b-hybrid contraction property, then one can find $\omega \in \Omega$ for which $\omega \in [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{(\omega)})} \cap [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{(\omega)})}$.

The following corollary is derived from the main theorem for a pair of $\mathbb{L}$–fuzzy mappings.

Corollary 3.3. Let (Ω, σ, s) be a complete b–metric space and $\mathrm{Q}, \mathrm{S} : \Omega \to (\tilde{F})^{\Omega}_{\mathbb{L}}$ be $\mathbb{L}$–fuzzy mappings. Assume that for each $\omega \in \Omega$, one can find $\alpha_{\mathbb{L}} \in \mathbb{L} \setminus \{0\}$ satisfying $[\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}$ and $[\mathrm{S}(\omega)]_{\alpha_{\mathbb{L}(\omega)}} \in \mathcal{P}^r_b(\Omega)$. If for all $\omega, \gamma \in \Omega$, we have

$$\mathcal{H}([\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}, [\mathrm{S}(\gamma)]_{\alpha_{\mathbb{L}(\gamma)}} \leq \breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega, \gamma, \alpha_{\mathbb{L}})\right),$$

where $\breve{\varphi} \in \widehat{W}$, $r \geq 0$, and $p_i \geq 0$ with $\sum_{i=1}^{4} p_i = 1$ and

$$C^r_{(\mathrm{Q},\mathrm{S})}(\omega,\gamma,\alpha_{\mathbb{L}}) = \begin{cases} \left[p_1(\sigma(\omega,\gamma))^r + p_2\left(\sigma\left(\omega,[\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}\right)\right)^r \right. \\ +p_3\left(\sigma\left(\gamma,[\mathrm{S}(\gamma)]_{\alpha_{\mathbb{L}(\gamma)}}\right)\right)^r \\ \left. +p_4\left(\frac{\sigma\left(\gamma,[\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}\right)+\sigma\left(\omega,[\mathrm{S}(\gamma)]_{\alpha_{\mathbb{L}(\gamma)}}\right)}{2s}\right)^r \right]^{\frac{1}{r}} \\ \quad \text{for} \quad r > 0, \omega, \gamma \in \Omega, \\ \\ (\sigma(\omega,\gamma))^{p_1}\left(\sigma\left(\omega,[\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}\right)\right)^{p_2} \\ \times\left(\sigma\left(\gamma,[\mathrm{S}(\gamma)]_{\alpha_{\mathbb{L}(\gamma)}}\right)\right)^{p_3} \\ \times\left(\frac{\sigma\left(\gamma,[\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}\right)+\sigma\left(\omega,[\mathrm{S}(\gamma)]_{\alpha_{\mathbb{L}(\gamma)}}\right)}{2s}\right)^{p_4} \\ \quad \text{for} \quad r = 0, \omega, \gamma \in \Omega \setminus \mathcal{F}_{i\omega}(\mathrm{Q},\mathrm{S}), \end{cases}$$

where

$$\mathcal{F}_{i\omega}(\mathrm{Q},\mathrm{S}) = \{\omega,\gamma \in \Omega : \omega \in [\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}, \gamma \in [\mathrm{S}(\gamma)]_{\alpha_{\mathbb{L}(\gamma)}}\},$$

then one can obtain $\omega \in \Omega$ satisfying $\omega \in [\mathrm{Q}(\omega)]_{\alpha_{\mathbb{L}(\omega)}} \cap [\mathrm{S}(\omega)]_{\alpha_{\mathbb{L}(\omega)}}$.

Corollary 3.4. Let (Ω,σ,s) be a complete b–metric space and $\mathrm{Q},\mathrm{S} : \Omega \to \mathcal{P}^r_b(\Omega)$ be a pair of multivalued mappings. Assume that for each $\omega \in \Omega$, Q and $\mathrm{S} \in P^r_b(\Omega)$. For all $\omega,\gamma \in \Omega$, we have

$$\mathcal{H}(\mathrm{Q}(\omega),\mathrm{S}(\gamma)) \leq \breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega,\gamma)\right),$$

where $\breve{\varphi} \in \widehat{W}$, $r \geq 0$, and $p_i \geq 0$ with $\sum_{i=1}^{4} p_i = 1$ and

$$C^r_{\mathrm{Q},\mathrm{S}}(\omega,\gamma) = \begin{cases} \left[p_1(\sigma(\omega,\gamma))^r + p_2(\sigma(\omega,\mathrm{Q}(\omega))^r + p_3(\sigma(\gamma,\mathrm{S}(\gamma))^r + \right. \\ \left. p_4(\frac{\sigma(\gamma,\mathrm{Q}(\omega))+\sigma(\omega,\mathrm{S}(\gamma)}{2s})^r \right]^{\frac{1}{r}} \quad \text{for} \quad r > 0, \omega, \gamma \in \Omega, \\ \\ (\sigma(\omega,\gamma))^{p_1}(\sigma(\omega,\mathrm{Q}(\omega))^{p_2}(\sigma(\gamma,\mathrm{S}(\gamma))^{p_3} \\ \times(\frac{\sigma(\gamma,\mathrm{Q}(\omega))+\sigma(\omega,\mathrm{S}(\gamma)}{2s})^{p_4} \quad \text{for} \quad r = 0, \omega, y \in \Omega \setminus \mathcal{F}_{ix}(\mathrm{Q},\mathrm{S}), \end{cases}$$

where

$$\mathcal{F}_{iu}(\mathrm{Q},\mathrm{S}) = \{\omega,\gamma \in \Omega : \omega \in \mathrm{Q}(\omega), \gamma \in \mathrm{S}(\gamma)\}.$$

Therefore, there exists $\omega \in \Omega$ such that $\omega \in \mathrm{Q}(\omega) \cap \mathrm{S}(\omega)$.

Remark 3.1. If the mapping is assumed to be a self-mapping on a complete b–metric space (Ω,σ,s), then Corollary 3.4 reduces to the result given in [21].

Example 3.1. Let $\Omega = [0,1]$ and $\sigma(\omega,\gamma) = |\omega-\gamma|^2$ for all $\omega,\gamma \in \Omega$ then (Ω,σ,s) is a complete b–metric space with $s = 2$. Let $\mathbb{L} = \{\tau_1,\tau_2,\tau_3,\tau_4\}$ with bottom element $0_{\mathbb{L}}$ and top element $1_{\mathbb{L}}$. Assume that $\tau_1 \preceq \tau_2 \preceq \tau_4$ and $\tau_1 \preceq \tau_3 \preceq \tau_4$. Note that τ_2 and τ_3 are not comparable, and thus $(\mathbb{L},\preceq)$ is a

complete distributive lattice. Define a pair of $\mathbb{L}$–intuitionistic fuzzy mappings $\mathrm{Q} =< \mu_{\mathrm{Q}}, \nu_{\mathrm{Q}} >, \quad \mathrm{S} =< \mu_{\mathrm{S}}, \nu_{\mathrm{S}} >: \Omega \to (IFS)^{\Omega}_{\mathbb{L}}$ as below:

$$\mu_{\mathrm{Q}(\rho)}(t) = \begin{cases} \tau_4 & \text{if} \quad 0 \leq t \leq \frac{\rho}{10}, \\ \tau_2 & \text{if} \quad \frac{\rho}{10} < t \leq \frac{\rho}{8}, \\ \tau_1 & \text{if} \quad \frac{\rho}{8} < t \leq \frac{\rho}{6}, \\ \tau_3 & \text{if} \quad \frac{\rho}{6} < t \leq 1, \end{cases}$$

$$\nu_{\mathrm{Q}(\rho)}(t) = \begin{cases} \tau_1 & \text{if} \quad 0 \leq t \leq \frac{\rho}{10}, \\ \tau_2 & \text{if} \quad \frac{\rho}{10} < t \leq \frac{\rho}{7}, \\ \tau_4 & \text{if} \quad \frac{\rho}{7} < t \leq \frac{\rho}{5}, \\ \tau_3 & \text{if} \quad \frac{\rho}{5} < t \leq 1, \end{cases}$$

$$\mu_{\mathrm{S}(\rho)}(t) = \begin{cases} \tau_2 & \text{if} \quad 0 \leq t \leq \frac{\rho}{14}, \\ \tau_4 & \text{if} \quad \frac{\rho}{14} < t \leq \frac{\rho}{10}, \\ \tau_1 & \text{if} \quad \frac{\rho}{10} < t \leq \frac{\rho}{5}, \\ \tau_3 & \text{if} \quad \frac{\rho}{5} < t \leq 1, \end{cases}$$

$$\nu_{\mathrm{S}(\rho)}(t) = \begin{cases} \tau_1 & \text{if} \quad 0 \leq t \leq \frac{\rho}{10}, \\ \tau_2 & \text{if} \quad \frac{\rho}{10} < t \leq \frac{\rho}{5}, \\ \tau_4 & \text{if} \quad \frac{\rho}{5} < t \leq \frac{\rho}{2}, \\ \tau_3 & \text{if} \quad \frac{\rho}{2} < t \leq 1. \end{cases}$$

Then,

$$[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})} = \{t \in \Omega : \alpha_{\mathbb{L}} \preceq \mathbb{T}(\mu_{\mathrm{Q}(\omega)}(t), \mathbb{N}(\nu_{\mathrm{Q}(\omega)}(t)))\}.$$

If we take $\alpha_{\mathbb{L}} = \tau_4$ and $\alpha_{\mathbb{L}} = \tau_2$, then

$$\begin{aligned} [\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})} &= \{t \in \Omega : \tau_4 \preceq \mathbb{T}(\tau_3, \mathbb{N}(\tau_1))\} \\ &= \{t \in \Omega : \tau_4 \preceq \mathbb{T}(\tau_3, \mathbb{N}(0_{\mathbb{L}}))\} \\ &= \{t \in \Omega : \tau_4 \preceq \mathbb{T}(\tau_4, 1_{\mathbb{L}})\} \\ &= \{t \in \Omega : \tau_4 \preceq \tau_4\} \\ &= \left[0, \frac{\omega}{10}\right]. \end{aligned}$$

$$\begin{aligned} [\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})} &= \{t \in \Omega : \tau_2 \preceq \mathbb{T}(\tau_4, \mathbb{N}(\tau_1))\} \\ &= \{t \in \Omega : \tau_2 \preceq \mathbb{T}(\tau_4, \mathbb{N}(0_{\mathbb{L}}))\} \\ &= \{t \in \Omega : \tau_2 \preceq \mathbb{T}(\tau_4, 1_{\mathbb{L}})\} \\ &= \{t \in \Omega : \tau_2 \preceq \tau_4\} \\ &= \left[0, \frac{\omega}{10}\right]. \end{aligned}$$

Thus,

$$\mathcal{H}\left([\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})},[\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})}\right)=\begin{cases}0 & \text{if} \quad \omega=\gamma,\\ \left|\frac{\omega-\gamma}{10}\right|^2 & \text{if} \quad \omega\neq\gamma.\end{cases}$$

Let $\breve{\varphi}(t)=\frac{t}{2}$ for all $t\geq 0$. Clearly, there are two cases in the verification of inequality 3.1.

Case 1. For $\omega=\gamma$, we obtain

$$\mathcal{H}\left([\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})},[\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})}\right)=0=\breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega,\gamma)\right).$$

Case 2. For $\omega\neq\gamma$, we have

$\mathcal{H}\left([\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})},[\mathrm{S}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})}\right)=\frac{1}{100}|\omega-\gamma|^2$

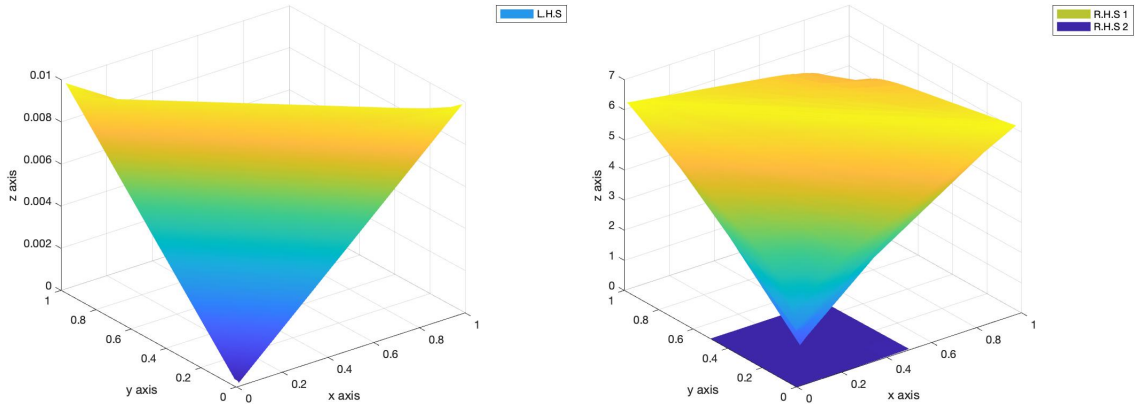
$$<\begin{cases}\left[p_1(|\omega-\gamma|^2)^r+p_2(|\omega-\frac{\gamma}{10}|^2)^r+p_3(|\gamma-\frac{\omega}{10}|^2)^r+p_4\left(\frac{|\gamma-\frac{\omega}{10}|^2+|\omega-\frac{\gamma}{10}|^2}{4}\right)^r\right]^{\frac{1}{r}}\\ \quad \text{for} \quad r>0 \ \text{ and } \ \omega,\gamma\in\Omega,\\ (|\omega-\gamma|^2)^{p_1}\times(|\omega-\frac{\gamma}{10}|^2)^{p_2}\times(|\gamma-\frac{\omega}{10}|^2)^{p_3}\times\left(\frac{(|\omega-\frac{\gamma}{10}|^2)+(|\gamma-\frac{\omega}{10}|^2)}{4}\right)^{p_4}\\ \quad \text{for} \quad r=0 \ \text{ and } \ \omega,\gamma\in\Omega\setminus\mathcal{F}_{i\omega}(Q,S).\end{cases}$$

Therefore, on combining both cases, we have

$$\mathcal{H}\left([\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\omega)})},[\mathrm{S}(\gamma)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}(\gamma)})}\right)\leq\breve{\varphi}\left(C^r_{(\mathrm{Q},\mathrm{S})}(\omega,\gamma)\right).$$

Consequently, all the assumptions of Theorem 3.1 are verified for $p_1=p_4=\frac{1}{2}$ and $p_2=p_3=0$, hence ω is a common fixed point of Q and S.

Figure 1(a) illustrates the three-dimensional surface corresponding to the left-hand side (LHS) of the contractive inequality for a pair of $\mathbb{L}$–intuitionistic fuzzy mappings, whereas Figure 1(b) depicts the three-dimensional surface corresponding to its right-hand side (RHS). It can be observed that the surface representing the left-hand side lies entirely below (or coincides with) the surface representing the right-hand side over the prescribed domain. This provides a graphical confirmation of the validity of the $\mathbb{L}$–intuitionistic fuzzy b-hybrid contractive condition established analytically in Example 3.1.



(a) LHS (b) RHS

Figure 1. Three-dimensional graphical illustration of the $\mathbb{L}$–intuitionistic fuzzy b-hybrid contraction.

4. Applications: A class of delay differential equations with multi-point boundary conditions

A delay differential equation involves derivatives that depend on earlier times and incorporate continuous, discrete, as well as proportional delays. In some circumstances, the chaotic oscillations are represented by non-linear first order delay differential equations. Moreover, a well-known proportional delay differential equation in electrodynamics is the pantograph equation, given by

$$\Psi'(t) = a\Psi(t) + b\Psi(\beta t).$$

Applications of fractional delay differential equations arise in diverse dynamical systems, such as economic models, biosystems, communication channels, and automatic regulation systems.

In this section, we establish the existence of a solution for a fractional delay differential equation with multi-point boundary conditions, using the concept of $\mathbb{L}$–intuitionistic fuzzy mappings.

We consider that the fractional delay differential equation equipped with conditions prescribed at multiple boundary points is of the form

$$ {}^{C}_{0}D^{\lambda}_{t}\Psi(t) + \Upsilon(t, \Psi(t), \Psi(at)) = 0, \quad n-1 \leq \lambda \leq n, \quad t \in \mathbb{J}, \quad a \in (0,1), \tag{4.1}$$

$$\Psi(0) = h(\Psi), \quad \Psi^{(j)}(0) = 0, \quad \Psi(1) = \sum_{j=1}^{m-2} \vartheta_j \Psi(\xi_j) + g(\Psi), \quad j = 1, 2, ...n-2, \tag{4.2}$$

where ${}^{C}_{0}D^{\lambda}$ is the standard Caputo derivative of order λ and $\mathbb{J} = [0,1]$. The nonlinear functions $\Upsilon : \mathbb{J} \times [0,\infty) \times [0,\infty) \to [0,\infty)$ and $\Psi, h, g : \mathbb{J} \to \Re$ are continuous functions, and $\vartheta_j, \xi_j \in (0,1)$ such that $\sum_{j=1}^{m-2} \vartheta_j \xi_j^{\lambda-1} < 1$.

Definition 4.1. [23] The integral for non–integer order $\lambda > 0$ of the function f is given by

$$ {}_{0}I^{\lambda}_{t} f(t) = \frac{1}{\Gamma(\lambda)} \int_0^t (t-u)^{\lambda-1} f(u) du.$$

Definition 4.2. [23] The Caputo fractional derivative for $\lambda > 0$ is defined by

$$ {}^{C}_{0}D^{\lambda}_{t} f(t) = \frac{1}{\Gamma(n-\lambda)} \int_0^t (t-u)^{n-\lambda-1} f^{(n)}(u) du, \quad n-1 \leq \lambda \leq n, \quad n \in \aleph,$$

where $n = [\lambda] + 1$ with $[\lambda]$ denotes the integer part of a positive real number λ and the right side is pointwise defined on $\Re^+$.

Lemma 4.1. [4] Let $\Psi, \Phi : \mathbb{J} \to \Re$ be continuous functions. Then the boundary value problem (BVP)

$$ {}^{C}_{0}D^{\lambda}_{t}\Psi(t) + \Phi(t) = 0, \quad n-1 \leq \lambda \leq n, \quad t \in \mathbb{J},$$

$$\Psi(0) = (\Psi), \quad \Psi^{(j)}(0) = 0, \quad \Psi(1) = \sum_{j=1}^{m-2} \vartheta_j \Psi(\xi_j) + (\Psi), \quad j = 1, 2, ...n-2$$

has at most one solution given by

$$\Psi(t) = \left(1 - \frac{t^{n-1}\vartheta}{\kappa}\right)(\Psi) + \frac{t^{n-1}}{\kappa}(\Psi) + \frac{t^{n-1}}{\kappa\Gamma(\lambda)}\int_0^1 (1-\omega)^{\lambda-1}\Phi(\omega)du$$
$$- \frac{t^{n-1}\sum_{j=1}^{m-2}\vartheta_j}{\kappa\Gamma(\lambda)}\int_0^{\xi_j}(\xi_j-\omega)^{\lambda-1}\Phi(\omega)d\omega - \frac{1}{\Gamma(\lambda)}\int_0^t (t-\omega)^{\lambda-1}\Phi(\omega)d\omega.$$

Note that $\kappa = 1 - \sum_{j=1}^{m-2}\vartheta_j\xi_j^{n-1}$ and $\vartheta = 1 - \sum_{j=1}^{m-2}\vartheta_j$.

On account of Lemma 4.1, the solution of problems (4.1) and (4.2) is given by

$$\Psi(t) = \left(1 - \frac{\vartheta_j t^{n-1}}{\kappa}\right)h(\Psi) + \frac{t^{n-1}}{\kappa}g(\Psi) + \frac{t^{n-1}}{\kappa\Gamma(\lambda)}\int_0^1 (1-\omega)^{\lambda-1}\Upsilon(\omega,\Psi(\omega),\Psi(a\omega))d\omega$$
$$- \frac{t^{n-1}\sum_{j=1}^{m-2}\vartheta_j}{\kappa\Gamma(\lambda)}\int_0^{\xi_j}(\xi_j-\omega)^{\lambda-1}\Upsilon(\omega,\Psi(\omega),\Psi(a\omega))d\omega$$
$$- \frac{1}{\Gamma(\lambda)}\int_0^t (t-\omega)^{\lambda-1}\Upsilon(\omega,\Psi(\omega),\Psi(a\omega))d\omega.$$

Consider for the universal set Ω, the operators $\Theta, \Theta_1, \Theta_2 : \Omega \to \Omega$ are defined as

$$\Theta_1\Psi(t) = \left(1 - \frac{\vartheta_j t^{n-1}}{\kappa}\right)h(\Psi) + \frac{t^{n-1}}{\kappa}g(\Psi),$$

$$\Theta_2\Psi(t) = \frac{t^{n-1}}{\kappa\Gamma(\lambda)}\int_0^1 (1-\omega)^{\lambda-1}\Upsilon(\omega,\Psi(\omega),\Psi(a\omega))d\omega$$
$$- \frac{t^{n-1}\sum_{j=1}^{m-2}\vartheta_j}{\kappa\Gamma(\lambda)}\int_0^{\xi_j}(\xi_j-\omega)^{\lambda-1}\Upsilon(\omega,\Psi(\omega),\Psi(a\omega))d\omega$$
$$- \frac{1}{\Gamma(\lambda)}\int_0^t (t-\omega)^{\lambda-1}\Upsilon(\omega,\Psi(\omega),\Psi(a\omega))d\omega.$$

Thus, $\Theta\Psi = \Theta_1\Psi + \Theta_2\Psi$.

Since the functions g, h, and Υ are continuous, Θ is a well-defined operator. Thus, the solution of problems (4.1) and (4.2) is a fixed point of Θ.

Theorem 4.1. Let $\Omega = C(\mathbb{J}, \Re)$ be the space of all continuous functions endowed with the metric

$$\sigma(\omega_1, \omega_2) = \max_{t\in\mathbb{J}} |\omega_1(t) - \omega_2(t)|^2. \tag{4.3}$$

Suppose that for any $\omega_1, \omega_2 \in \Omega$ and $(t, \omega_1, \omega_2) \in \mathbb{J} \times \Omega \times \Omega$, the following conditions hold:
(i) For the given constant $b_h \in [0, 1)$, we have $|h(\omega_1) - h(\omega_2)| \le b_h|\omega_1 - \omega_2|$.
(ii) For the given constant $b_g \in [0, 1)$, we have $|g(\omega_1) - g(\omega_2)| \le b_g|\omega_1 - \omega_2|$.
(iii) For each $\omega_1, \bar{\omega_1}, \omega_2, \bar{\omega_2} \in \Re$, there exists a constant $L_\Upsilon > 0$ such that $|\Upsilon(t, \omega_1, \omega_2) - \Upsilon(t, \bar{\omega_1}, \bar{\omega_2}| \le L_\Upsilon(|\omega_1 - \bar{\omega_1}| + |\omega_2 - \bar{\omega_2}|)$, and $L_\Upsilon < \frac{\kappa\Gamma(\lambda+1)}{6}$.

Thus, the BVPs (4.1) and (4.2) have a solution.

Proof. It follows from Eq (4.3) that Ω is a complete b–metric space with coefficient $s = 2$. Assume that $F_{\mathbb{L}} : \Omega \to \mathbb{L} \setminus \{0_{\mathbb{L}}\}$ is an arbitrary mapping and $\mathrm{Q} = \langle \mu_{\mathrm{Q}}, \upsilon_{\mathrm{Q}} \rangle : \Omega \to (\mathrm{IFS})^{\Omega}_{\mathbb{L}}$ is an $\mathbb{L}$–intuitionistic fuzzy mapping defined as

$$\mu_{\mathrm{Q}(\omega)}(\omega^*) = \begin{cases} F_{\mathbb{L}}(\omega) & \text{if} \quad \omega^*(t) = \Theta_\omega(t) \quad \text{for all} \quad t \in \Omega, \\ 0_{\mathbb{L}} & \text{otherwise} \end{cases}$$

and

$$\upsilon_{\mathrm{Q}(\omega)}(\omega^*) = \begin{cases} 0_{\mathbb{L}} & \text{if} \quad \omega^*(t) = \Theta_\omega(t) \quad \text{for all} \quad t \in \Omega, \\ F_{\mathbb{L}}(\omega) & \text{otherwise,} \end{cases}$$

$$[\mathrm{Q}(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})} = \{\omega^* \in W : \alpha_{\mathbb{L}} \preceq \mathbb{T}\left(\mu_{\mathrm{Q}(\omega)}(\omega^*), \mathbb{N}(\upsilon_{\mathrm{Q}(\omega)}(\omega^*))\right)\}.$$

If we take $\alpha_{\mathbb{L}} = F_{\mathbb{L}}(\omega)$, then

$$\begin{aligned} [Q(\omega)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})} =& \{\omega^* \in W : F_{\mathbb{L}}(\omega) \preceq \mathbb{T}(F_L(\omega), \mathbb{N}(0_{\mathbb{L}}))\} \\ =& \{\omega^* \in W : F_{\mathbb{L}}(\omega) \preceq \mathbb{T}(F_{\mathbb{L}}(\omega), 1_{\mathbb{L}})\} \\ =& \{\Theta_\omega\}. \end{aligned}$$

However,

$$\mathcal{H}\Big([\mathrm{Q}(\omega_1)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})}, [\mathrm{Q}(\omega_2)]_{(\mathbb{T},\mathbb{N},\alpha_{\mathbb{L}})}\Big) = \max_{t\in\Omega} |\Theta_{\omega_1}(t) - \Theta_{\omega_2}(t)|^2. \tag{4.4}$$

For any ω_1 and ω_2, using the assumptions (i)–(iii), $\sum_{j=1}^{m-2} \vartheta_j \xi_j^{\lambda} < 1$, and $\kappa < 1$, we can obtain

$$\begin{aligned} &|\Theta_{\omega_1}(t) - \Theta_{\omega_2}(t)|^2 \\ &= \Bigg(\Bigg|\left(1 - \frac{\vartheta_j t^{n-1}}{\kappa}\right)(h(\omega_1) - h(\omega_2)) + \left(\frac{t^{n-1}}{\kappa}\right)(g(\omega_1) - g(\omega_2)) \\ &\quad + \frac{t^{n-1}}{\kappa\Gamma(\lambda)} \int_0^1 (1-u)^{\lambda-1} \left(\Upsilon(u, \omega_1(u), \omega_1(au)) - \Upsilon(u, \omega_2(u), \omega_2(au))\right) du \\ &\quad + \frac{t^{n-1} \sum_{j=1}^{m-2} \vartheta_j}{\kappa\Gamma(\lambda)} \int_0^{\xi_j} (\xi_j - u)^{\lambda-1} \left(\Upsilon(u, \omega_1(u), \omega_1(au)) - \Upsilon(u, \omega_2(u), \omega_2(au))\right) du \\ &\quad + \frac{1}{\Gamma(\lambda)} \int_0^t (t-u)^{\lambda-1} \left(\Upsilon(u, \omega_1(u), \omega_1(au)) - \Upsilon(u, \omega_2(u), \omega_2(au))\right) du\Bigg|\Bigg)^2 \\ &\leq \Bigg(\left|1 - \frac{\vartheta_j t^{n-1}}{\kappa}\right| |h(\omega_1) - h(\omega_2)| + \left|\frac{t^{n-1}}{\kappa}\right| |g(\omega_1) - g(\omega_2)| \\ &\quad + \left|\frac{t^{n-1}}{\kappa\Gamma(\lambda)}\right| \left|\int_0^1 (1-u)^{\lambda-1} \left(\Upsilon(u, \omega_1(u), \omega_1(au)) - \Upsilon(u, \omega_2(u), \omega_2(au))\right) du\right| \\ &\quad + \left|\frac{t^{n-1} \sum_{j=1}^{m-2} \vartheta_j}{\kappa\Gamma(\lambda)}\right| \left|\int_0^{\xi_j} (\xi_j - u)^{\lambda-1} \left(\Upsilon(u, \omega_1(u), \omega_1(au)) - \Upsilon(u, \omega_2(u), \omega_2(au))\right) du\right| \\ &\quad + \left|\frac{1}{\Gamma(\lambda)}\right| \left|\int_0^t (t-u)^{\lambda-1} \left(\Upsilon(u, \omega_1(u), \omega_1(au)) - \Upsilon(u, \omega_2(u), \omega_2(au))\right) du\right|\Bigg)^2 \\ &\leq \left(\left(\frac{1+\lambda}{\lambda}\right) b_h |\omega_1 - \omega_2| + \left(\frac{1}{\lambda}\right) b_g |\omega_1 - \omega_2| + \left(\frac{2L_\Upsilon}{\kappa\lambda\Gamma(\lambda)}\right) |\omega_1 - \omega_2| \right. \end{aligned}$$

$$
\begin{aligned}
&+\left(\frac{2L_\Upsilon \sum_{j=1}^{m-2}\vartheta_j\xi_j^\lambda}{\kappa\lambda\Gamma(\lambda)}\right)|\omega_1-\omega_2|+\left(\frac{2L_\Upsilon}{\lambda\Gamma(\lambda)}\right)|\omega_1-\omega_2|\Bigg)^2\\
\leq\ &\left(B_h+B_g+\frac{6L_\Upsilon}{\kappa\Gamma(\lambda+1)}\right)^2|\omega_1-\omega_2|^2\\
\leq\ &\left(\bar{B}+\frac{6L_\Upsilon}{\kappa\Gamma(\lambda+1)}\right)^2|\omega_1-\omega_2|^2\\
=\ &B_\Upsilon^2\sigma(\omega_1,\omega_2),
\end{aligned}
$$

where, $B_h=(\frac{1+\lambda}{\lambda})b_h$, $\quad B_g=(\frac{1}{\lambda})b_g$ and $\quad \bar{B}=B_h+B_g$, and thus one can see that $B_\Upsilon=\left(\bar{B}+\frac{6L_\Upsilon}{\kappa\Gamma(\lambda+1)}\right)<1$.

Note that $\breve{\varphi}(t)=B_\Upsilon^2 t$ for all $t\geq 0$. Therefore, in view of Lemma 2.3 and Eq (4.4), all the hypotheses of Theorem 3.1 for $p_1=r=1$ and $p_2=p_3=p_4=0$ in the setting of Q = S are satisfied to obtain the solution of BVPs (4.1) and (4.2). □

Example 4.1. Consider the fractional delay differential equation under the muti-point boundary condition as

$$
{}_0^C D_t^{\frac{5}{2}}\omega(t)=\frac{|\omega(\frac{t}{3})|^{\frac{1}{4}}+|\omega(t)|^{\frac{1}{4}}}{(0.3-e^t)|\omega(t)|^{\frac{1}{4}}},\qquad 0<t<1, \tag{4.5}
$$

$$
\omega(0)=h(\omega),\quad \omega^{(1)}(0)=0,\quad \omega(1)=\sum_{j=1}^{m-2}\varrho_j\omega(\gamma_j)+g(\omega),\ \ \sum_{j=1}^{m-2}\varrho_j\omega(\gamma_j)=\frac{1}{5}. \tag{4.6}
$$

Taking $\lambda=\frac{5}{2}$, $b_g=b_h=\frac{1}{8}$ *and* $B_\psi^*=\frac{1}{2}$, all the assumptions of Theorem 4.1 hold.

Therefore, the multi-point BVP (4.5) has a solution in $C(\Omega,\Re)$ given by

$$
\omega(t)=(1-t)h(\omega)+t\left(\frac{2}{5}+g(\omega)\right)-_o I^{\frac{5}{2}}f\left(t,\omega(t),\omega\left(\frac{t}{3}\right)\right)+_o I^{\frac{5}{2}}f\left(1,\omega(1),\omega\left(\frac{1}{3}\right)\right).
$$

So, $S_1\omega(t)=(1-t)h(\omega)+t\left(\frac{2}{5}+g(\omega)\right)$ and $S_2\omega(t)=-_oI^{\frac{5}{2}}f\left(t,\omega(t),\omega\left(\frac{t}{3}\right)\right)+_o I^{\frac{5}{2}}f\left(1,\omega(1),\omega\left(\frac{1}{3}\right)\right)$.

Here, $S=S_1+S_2$ as S_1 and S_2 are continuous and bounded functions.

Moreover,

$$
|S_\xi-S_\eta|\leq\frac{1}{4}|\xi-\eta|.
$$

5. Conclusions

This manuscript has proposed a new non-classical idea under the name $\mathbb{L}$–intuitionistic fuzzy mapping. Sufficient conditions for the existence of fixed points for such operators have been presented. It has been observed that many results in the existing literature can be followed using the notion established herein. Consequently, a few important corresponding results in the setting of fuzzy, $\mathbb{L}$–fuzzy, and crisp multivalued mappings have been obtained as corollaries. Nontrivial examples have been constructed to support the genuineness of the assumptions forming our theorems. As an application, one of the obtained results has been utilized to analyze novel conditions for the existence of a solution to a class of fractional differential equations with a multi-point boundary condition.

Author contributions

Rehana Tabassum, Mohammed Shehu Shagari, Marriam Anayat, Akbar Azam and Faryad Ali: Conceptualization; Mohammed A. Al-Kadhi, Faryad Ali and Rehana Tabassum: Formal analysis; Rehana Tabassum, Faryad Ali, Mohammed A. Al-Kadhi and Mohammed Shehu Shagari: Investigation; Rehana Tabassum, Faryad Ali and Marriam Anayat: Writing – review and editing; Rehana Tabassum, Mohammed Shehu Shagari and Marriam Anayat; Methodology.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgement

The author, Faryad Ali, would like to thank Imam Mohammad Ibn Saud Islamic University for paying the APC.

Funding

Not applicable.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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