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盾构下穿引起上覆管线变形的简化计算方法

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摘 要:盾构开挖易引起上覆地表沉降变形,进一步会引起上覆既有管线的受力变形响应.基于此,提出了一种盾构开挖引起上覆管线变形的简化计算方法.首先采用Loganathan公式获得既有管线在盾构下穿影响下的附加应力,进一步将管线简化成无限长梁放置在Pasternak地基上,引入无限远端侧向土体位移对既有管线的影响,采用力学平衡法获得管线竖向受力变形控制方程,通过有限差分法获得管线变形及其内力数值解析.案例分析表明:与退化解析对比,该方法计算结果更贴近既有文献工程实测数据,验证了其可靠性.进一步参数研究表明:增大隧道与管线的竖向净距会引起既有管线受力变形的非线性减小;管线变形及其内力会随着地层损失率增大而线性增大;管线抗弯刚度的增大会引起管线变形减小,但会大幅增加管线弯矩.

关键词:管-土相互作用;Pasternak地基;侧向土体影响;简化计算方法

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Simplified Calculated Method for Pipeline Deformation Induced by Shield Tunneling

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Abstract: Tunnel excavation can easily cause settlement and deformation of the surface of the overlying surface, which will further cause the stress and deformation response of the existing pipeline. Based on this, a simplified method for calculating the deformation of the overlying pipeline due to shield excavation is proposed. Firstly, the Loganathan formula is selected to solve the additional stress along the pipeline induced by tunneling underlying, and the pipeline is further simplified into an infinitely long beam resting on the Pasternak foundation. The influence of the lateral displacement of the infinite distal soil on the existing pipeline is introduced. Then, the mechanical equilibrium method is used to obtain the control equation of the vertical force and deformation of the pipeline. The finite difference method is selected to obtain the numerical analytical solution of the pipeline deformation and internal force analysis. The case analysis shows that, compared with the degradation analysis of this method, the calculation results are closer to the measured data of an actual project in the existing literature, which verifies the reliability of this method. Further parameter studies show that increasing the vertical clearance between the tunnel and the pipeline causes a nonlinear reduction in the stress and deformation of the pipeline. Increasing the volume loss rate results in a linear increase in the deformation and inner force of the pipeline. Increasing the pipeline bending stiffness reduces the pipeline deformation but greatly increases the pipeline bending moment.

Key words: pipeline-soil interaction; Pasternak foundation; lateral soil effect; simplified calculated method

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城市地下空间的不断开发带动了城市经济的飞速发展,市政管道的建设极大地便利了城市居民的生活.而在城市地下空间不免会出现邻近开挖对既有管线造成安全威胁[1-5],故有必要研究既有管线在邻近开挖影响下受力变形响应.

邻近开挖对既有管线影响的理论文献大部分都是基于两阶段法.第一阶段采用Loganathan等[6]解析获得管线轴线处的附加应力;第二阶段将管线简化为无限长梁搁置在地基模型上,进一步获得管线受力变形解析.Attewell等[7]首次将管线简化成搁置在Winkler地基模型上的弹性梁,获得上覆管线在隧道开挖下的变形解析;张陈蓉、李海丽等[8-9]考虑到既有管线非连续接口的工况,基于Winkler地基和欧拉梁理论获得盾构下穿引起既有管线变形理论.Winkler地基未考虑到土体的剪切变形,其预测结果与实测数据相比偏大[10].随后,可考虑土体剪切变形的Pasternak地基受到广大专家学者的青睐.林存刚等[11]采用Pasternak地基模拟管-土相互作用,进一步考虑管线接头非线性特性,得到了上覆管线在盾构下穿影响下的变形解析;魏纲等[12]从欧拉梁和Pasternak地基出发,通过解析4阶方程的方法获得管-土相互作用解析解,解析结果与实测数据接近;可文海等[13]通过变分迭代的方法修正了Pasternak地基模型参数,进一步获得精确的管线受力变形响应,与既有方法相比,所提方法计算结果更贴近实测数据;何小龙等[14]基于考虑到管-土分离的工况,通过建立分离段和未分离段Pasternak地基模型控制方程,随后通过不同边界条件解析获得管线在邻近基坑开挖影响下受力变形响应.章李刚等[15]基于管廊存在残余轴向内力的工况,采用Pasternak地基模拟

管廊与土体相互作用,进一步获得既有管廊在盾构下穿影响下的变形响应,与工程实测数据比较,该方法接近实测数据.进一步,刘建文、唐汐等[16-17]采用Vlasov地基模拟隧-土相互作用,并分别基于Timoshenko梁和Euler-Bernoulli理论,获得了侧向土体影响下基坑开挖引起下卧隧道受力变形简化计算方法;冯国辉等[18-20]基于三参数Kerr地基模型,通过考虑侧向土体影响和管线剪切变形的影响,进一步获得管-土相互作用简化解析.专家学者对Kerr地基的研究也有一定的进展[21-22],然而Kerr地基模型参数较多,计算较为复杂,目前研究较少.

综上所述,Pasternak地基能够考虑到土体剪切变形影响,且较为简单,广泛应用于土与结构相互作用研究中.然而,很少有学者考虑到管线无限远端侧向土体对管-土相互作用的影响.基于此,本文将既有管线简化为无限长梁放置在Pasternak地基上,引入无限远端土体侧向位移对既有管线的影响,采用力学平衡法获得管线竖向受力变形控制方程,再通过有限差分法获得管线变形及其内力数值解析.该方法计算结果与工程实测数据接近,验证了该方法的可靠性.最后系统分析了隧道与管线竖向净距、地层损失率、管线抗弯刚度对既有管线受力变形的影响.

1 分析方法

1.1 自由位移场求解

基于文献[6]提出软土地区下盾构开挖引起周围土体自由竖向位移$f(x)$为:

$$f(x)=\varepsilon R^2\cdot \mathrm{e}^{-\frac{1.38x^2}{(H+R)^2}-\frac{0.69z^2}{H^2}}\left[-\frac{z-H}{x^2+(z-H)^2}+(3-4\nu)\frac{z+H}{x^2+(z+H)^2}-\frac{2z\left(x^2-(z+H)^2\right)}{\left(x^2+(z+H)^2\right)^2}\right] \tag{1}$$

式中:ε为等效地层损失比;R为隧道开挖半径;H为隧道埋深;z为距地表垂直距离;x为管线到隧道中心线的水平距离;ν为土体泊松比.

根据冯国辉等[19]的建议,若开挖隧道与既有管线成夹角θ时,式(1)中x将变成$x\sin\theta$.那么管线所受到的附加应力为:

$$q(x)=k_{\mathrm{c}}f(x)-G_{\mathrm{c}}\frac{\mathrm{d}^2f(x)}{\mathrm{d}x^2} \tag{2}$$

式中:$\frac{\mathrm{d}^2f(x)}{\mathrm{d}x^2}$为土体竖向变形曲率;$k_{\mathrm{c}}$为土体弹簧刚度,相当于Winkler地基模型弹簧参数;G_{c}为土体剪切层刚度.

对于Pasternak地基模型参数取值方面,林存刚等[11]考虑到地下管线的埋深及剪切刚度折减因素的影响,可文海等[12]采用能量变分迭代的方法获得地基模型参数.然而,过于复杂的地基模型取值方法较难应用于实际工程,大多数学者采用简单的取值方法具体如下[13-15,23]:

$$k_{\mathrm{c}}=\frac{1.3E_{\mathrm{s}}}{D(1-\nu^2)}\left(\frac{E_{\mathrm{s}}D^4}{EI}\right)^{1/12} \tag{3}$$

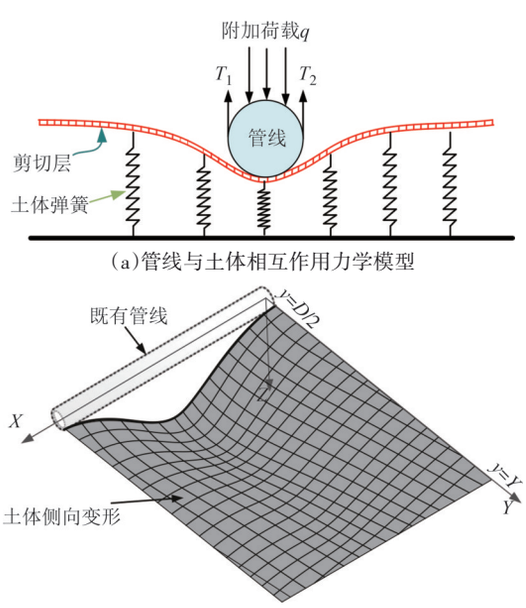
$$G_{\mathrm{c}}=\frac{E_{\mathrm{s}}t}{6(1+\nu)} \tag{4}$$

式中:D和EI分别为既有管线直径和抗弯刚度;E_{s}为土体弹性模量;ν为土体泊松比;t为土体剪切层厚度,取t=2.5D[11].

1.2 管线变形理论分析

管线受到附加荷载产生沉降变形时,不仅受到管线下方土体的地基反力,其侧向土体也同样对管线的变形存在约束作用,如图1(a)所示.管线侧向土体因管线受力而产生的变形如图1(b)所示.

为了分析侧向土体作用对既有管线的影响,做出如下假定:



(a)管线与土体相互作用力学模型

(b)侧向土体变形示意图

图1 侧向土体影响下管-土相互作用模型

Fig. 1 Pipeline-soil interaction considering the lateral soil effect

1) 半无限空间内的土体为均质土体,变形均在弹性范围内;

2) 侧向土体作用力 T_1 与 T_2 通过土体剪切层附加在管线两侧;

3) 假定管线为刚性管线,不考虑管线非线性变形,且管–土协调变形,两者无间隙.

对于任意 $x=x_0$ 的平面,管线侧向土体的变形平衡方程为:

$$k_c\overline{U}(y)-G_c\frac{\mathrm{d}^2\overline{U}(y)}{\mathrm{d}y^2}=0 \tag{5}$$

式中:$\overline{U}(y)$ 为侧向土体剪切层沿 y 轴方向上的变形,通过解方程可得式(5)通解为:

$$\overline{U}(y)=C_1\mathrm{e}^{-\sqrt{k_c/G_c}\,(y-D/2)} \tag{6}$$

如图 1(b)所示,当 $y=D/2$ 时,土体和管线正好接触,此时侧向土体位移就等于管线位移 $w(x)$;同时,距离管线足够远处($y=Y$ 处)的侧向土体位移为 $\overline{U}(Y)=0$,根据上述边界条件,可求得式(6)中待定系数 C_1,因此侧向土体的位移为:

$$\overline{U}(x,y)=w(x)\mathrm{e}^{-\sqrt{k_c/G_c}\,(y-D/2)} \tag{7}$$

进一步,侧向土体对管线的作用力为:

$$T_1=T_2=G_c\frac{\mathrm{d}\overline{U}(x,y)}{\mathrm{d}y}\Big|_{y=\frac{D}{2}}=w(x)\sqrt{k_cG_c} \tag{8}$$

考虑到管线受到的土体反力为:

$$p(x)=k_cw(x)-G_c\frac{\mathrm{d}^2w(x)}{\mathrm{d}x^2} \tag{9}$$

结合以上分析,在侧向土体力的约束作用下,管线变形控制方程为:

$$EI\frac{\mathrm{d}^4w(x)}{\mathrm{d}x^4}+p(x)D+T_1+T_2=q(x)D \tag{10}$$

综合式(8)~(10),既有管线竖向受力平衡控制方程为:

$$EI\frac{\mathrm{d}^4w}{\mathrm{d}x^4}-G_cD\frac{\mathrm{d}^2w}{\mathrm{d}x^2}+\left(k_cD+2\sqrt{k_cG_c}\right)w=qD \tag{11}$$

考虑到式(11)为 4 阶微分方程,可将既有管线简化成 n 个单元并采用差分法求解.如图 2 所示,既有管线被简化成 n 个长度为 $l=L/n$ 节点单元,L 为管线轴线长度.

此时,式(11)可差分为:

$$EI\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{l^4}-G_cD\frac{w_{i+1}-2w_i+w_{i-1}}{l^2}+\left(k_cD+2\sqrt{k_cG_c}\right)w_i=q_iD \tag{12}$$

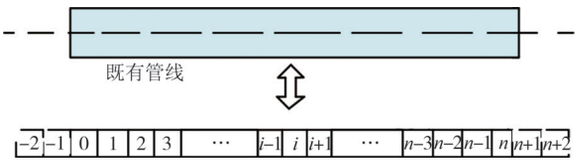


图 2　既有管线差分简化图

Fig. 2　Simplified differential diagram of the existing pipeline

那么管线弯矩和剪力可分别简化为:

$$M=-EI\frac{\mathrm{d}^2w}{\mathrm{d}x^2}=-EI\frac{w_{i+1}-2w_i+w_{i-1}}{l^2} \tag{13}$$

$$Q=-EI\frac{\mathrm{d}^3w}{\mathrm{d}x^3}=-EI\frac{w_{i+2}-2w_{i+1}+2w_{i-1}-w_{i-2}}{2l^3} \tag{14}$$

由于实际管线两端可简化成自由状态[11,15-18],即两端弯矩 $M=0$,剪力 $Q=0$:

$$\begin{cases}M_0=M_n=-EI\dfrac{\mathrm{d}^2w}{\mathrm{d}x^2}\Big|_{i=0/n}=0;\\ Q_0=Q_n=-EI\dfrac{\mathrm{d}^3w}{\mathrm{d}x^3}\Big|_{i=0/n}=0\end{cases} \tag{15}$$

那么式(12)可化为:

$$\left\{\left[K_1\right]+\left[K_2\right]-\left[G_0\right]\right\}\cdot\{w\}=D\{q\} \tag{16}$$

式(16)中的刚度矩阵分别为:

$$\left[K_1\right]=\frac{EI}{l^4}\begin{bmatrix}2&-4&2&&&&&&\\-2&5&-4&1&&&&&\\1&-4&6&-4&1&&&&\\&1&-4&6&-4&1&&&\\&&\cdots\cdots\cdots\cdots&&&&&&\\&&&1&-4&6&-4&1&\\&&&&1&-4&6&-4&1\\&&&&&1&-4&5&2\\&&&&&&2&-4&2\end{bmatrix}_{(n+1)\times(n+1)} \tag{17}$$

$$\left[K_2\right]=\left(k_cD+2\sqrt{k_cG_c}\right)\begin{bmatrix}1&&&&&&&&\\&1&&&&&&&\\&&1&&&&&&\\&&&1&&&&&\\&&&\cdots\cdots\cdots\cdots&&&&&\\&&&&1&&&&\\&&&&&1&&&\\&&&&&&1&&\\&&&&&&&1\end{bmatrix}_{(n+1)\times(n+1)} \tag{18}$$

$$[G_0]=\frac{G_c D}{l^2}\begin{bmatrix}0 & 0 & 0 & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & \cdots\cdots & & & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \\ & & & & 0 & 0 & 0\end{bmatrix}_{(n+1)\times(n+1)} \tag{19}$$

$$\{w\}=\{w_0\ w_1\ w_2\ \cdots\ w_{n-2}\ w_{n-1}\ w_n\}^{\mathrm{T}}_{n+1} \tag{20}$$

$$\{q\}=\{q_0\ q_1\ q_2\ \cdots\ q_{n-2}\ q_{n-1}\ q_n\}^{\mathrm{T}}_{n+1} \tag{21}$$

至此,得到管线竖向变形$w(x)$位移的数值解析,将得到的结果代入式(13)(14)即可得到管线的弯矩和剪力.

2 解析验证

文献[19-20]曾报道过深圳地铁下穿既有大直径电缆管线的工程,将管线变形实测数据与本文方法及其退化解进行对比验证.在该工况下,盾构开挖引起地层损失率为0.84%,其盾构下穿对上覆管线影响的简图如图3所示,其他具体计算参数见表1.

表1 实际工程计算参数

Tab. 1 Actual case study calculation parameters

R/m	H/m	z_0/m	ε_0/%	E_0/MPa	ν	D/m	EI/(kN·m^2)
3.0	14.4	8.7	0.84	8.2	0.3	3.0	5.87×10^7

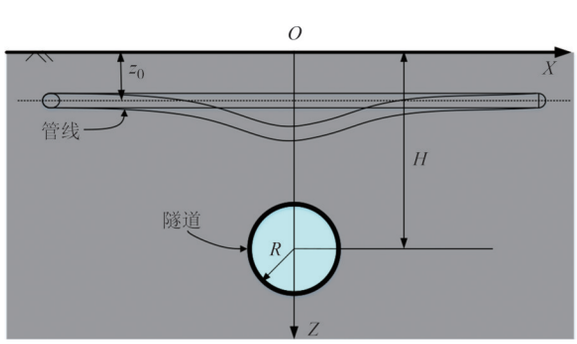


图3 隧道与管线位置简图

Fig.3 Simplified location diagram between tunnel and pipeline

图4为实际工程监测结果与本文方法及其退化解(即不考虑管线侧向土体力作用EB-P模型)计算对比图.由图4可见,本文方法及其退化解获得的管线沉降变形趋势与工程实际一致,且两种方法均能较好地预测管线变形.然而,相比于本文方法的结果,退化解将高估管线的实际变形.这是由于本文方法考虑到侧向土体的影响会使得管线所受附加应力减小,从而进一步引起管线变形减小.总的来说,考虑本文提出的管线侧向土体力能够提高管线变形的预测精度,使得Pasternak地基模型能更准确模拟管-土相互作用,也进一步证明了本文方法的正确性.

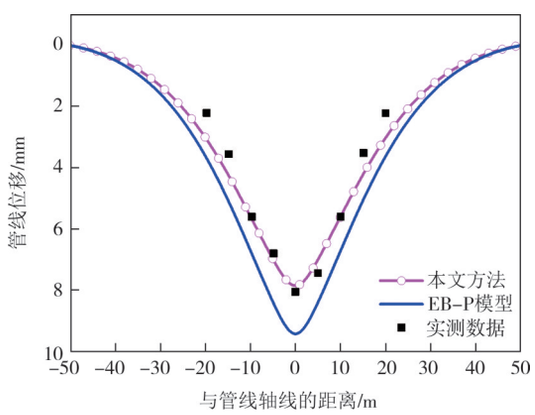


图4 管线变形计算结果与实际工程数据对比曲线

Fig.4 Comparison of the calculated and actual project results of deformation of tunnel

3 敏感参数分析

考虑到隧道-管线竖向净距、地层损失率及管线抗弯刚度对管-土相互作用的影响,假设隧道轴线和管线轴线垂直相交,其余参数见表2,并按照单一变量法进行参数分析.

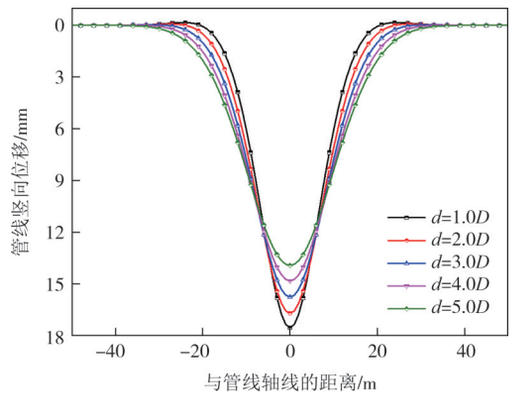
表2 基本计算参数

Tab. 2 Basic calculation parameters

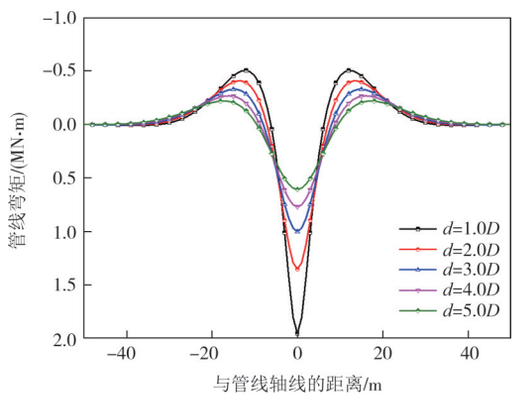
R/m	H/m	z/m	ε/%	E_s/MPa	ν	D/m	EI/(N·m^2)
3.0	15	8	1	20	0.3	3.0	3.36×10^9

3.1 隧道-管线竖向净距d

不同隧道-管线竖向净距d下盾构下穿引起既有管线变形及其弯矩变化如图5所示.可以看出,在隧道开挖的影响下,既有管线变形和弯矩峰值均出现在管线中心处,且两者都以管线轴线为对称轴呈轴对称分布.同时,随着隧道和管线竖向净距逐步增大,管线的位移和弯矩均呈现非线性减小的过程.这是由于管线的附加应力会随着两者距离的减小非线性增大,导致管线受力变形影响越来越大.因此,在实际工程中,在既有管线下穿隧道时,应尽可能地增大盾构深度,以减小盾构下穿对既有管线的影响.



(a)位移



(b)弯矩

图 5　隧道-管线竖向净距变化对管-土相互作用影响

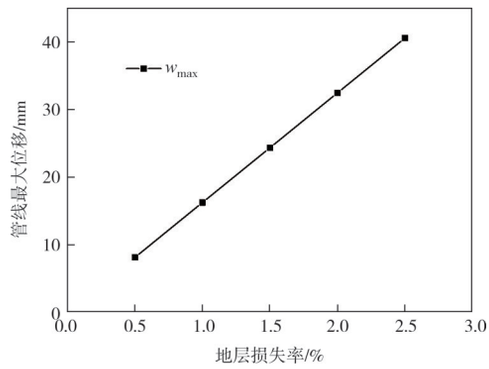
Fig. 5　The influence of pipeline-soil interaction with the variation of tunnel-pipeline vertical distance

3.2　地层损失率 ε

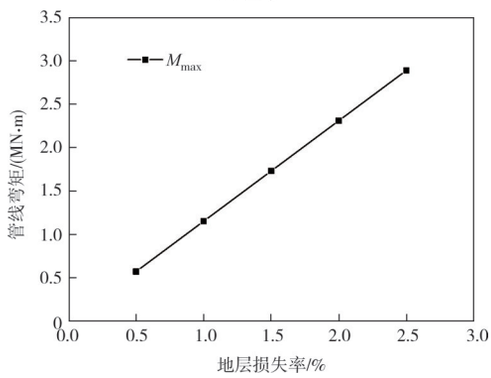
不同地层损失率 ε 下既有管线受到盾构下穿影响产生的最大位移和弯矩见图 6. 从图 6 可知,随着地层损失率从 0.5% 增加至 2.5% 的过程中,上覆管线位移峰值从 8.12 mm 增加到 40.59 mm,呈现线性增大的现象. 同样,随着地层损失率的逐渐增大,管线弯矩峰值从 0.578 MN·m 逐渐增大至 2.892 MN·m,也呈现出线性增大的情况. 说明地层损失率是管线受力变形的敏感参数,且随着地层损失率的增大,管线所受到的附加应力也会线性增大,进一步导致管线受力变形的线性增大. 故在实际工程中应尽可能加强盾尾注浆以减小盾构开挖引起的地层损失,并进一步遏制盾构开挖对周边环境的影响.

3.3　管线抗弯刚度 $(EI)_{new}$

不同管线抗弯刚度 $(EI)_{new}$ 下管线受到盾构下穿影响引起的最大位移和弯矩见图 7,其中新的管线抗弯刚度 $(EI)_{new}=S\cdot EI$,EI 为原管线抗弯刚度,S 为管线抗弯刚度系数. 从图 7 可知,随着管线抗弯刚度的逐渐增大,管线的最大位移从 19.04 mm 减小至 8.533 mm,降幅 55%;而管线的弯矩峰值从 0.05 MN·m



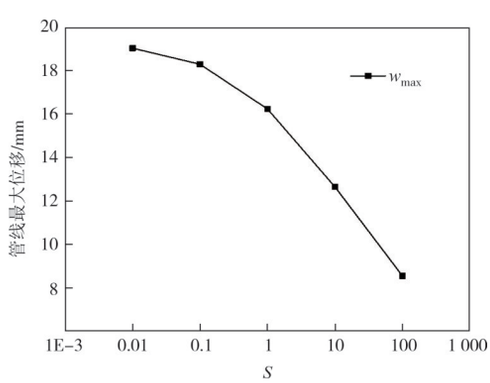
(a)位移



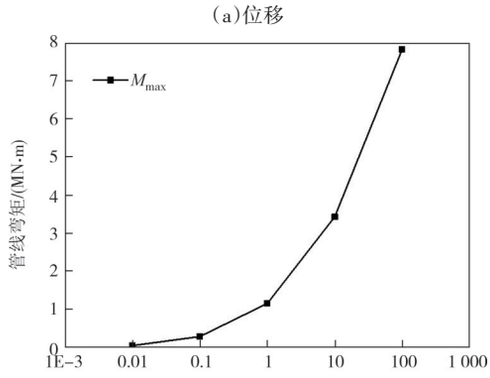
(b)弯矩

图 6　地层损失率变化对管-土相互作用影响

Fig.6　The influence of pipeline-soil interaction with the variation of volume loss



(a)位移



(b)弯矩

图 7　管线抗弯刚度变化对管-土相互作用影响

Fig.7　The influence of pipeline-soil interaction with the variation of pipeline bending stiffness

增加到7.83 MN·m,增幅近156倍.这是由于随着管线抗弯刚度的增大,管线抵抗变形的能力也会逐渐提高,相应所要受到的内力也会逐渐增大.从以上分析可以看出,相比于管线位移的变化,管线弯矩变化对管线抗弯刚度更为敏感,在实际工程中,应适当选取管线管片,以达到保护管线安全的要求.

4 结 论

本文基于理论研究的方法提出了一种盾构下穿对上覆管线变形影响的预测方法.得到的结论如下:

1) 考虑管线侧向土体对既有管线的影响,将管线简化成无限长的欧拉梁搁置在Pasternak地基上,随后采用差分形式获得管线变形简化计算结果.

2) 与该方法可退化的解析对比,本文方法计算结果更贴近既有工程实测数据,进一步证明了该方法的可靠性.

3) 增大隧道与管线的竖向净距会非线性地减小盾构下穿对上覆管线的影响;管线变形及其内力会随着地层损失率增大而线性增大;管线抗弯刚度的增大会引起管线变形减小,但会大幅增加管线弯矩.

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